

## APPENDIX E

### INVENTORY RECONCILIATION METHODS

#### I. EPA INVENTORY RECONCILIATION METHOD

EPA has developed a simple method<sup>1</sup> for monitoring underground motor fuel storage tank inventory records to detect a systematic deficit which may be attributable to a leak. The method is based on counts of the number of daily underages found in the inventory record and is simple enough to be implemented by a tank operator without excessive calculation or burdensome record-keeping. As originally formulated, the method is intended for application as the "first line of defense against leaks" in an on-going monitoring program. Thus, the approach is sequential in nature and involves making a decision on the presence or absence of an inventory deficit at the end of each 30-business-day period, based on a comparison between the cumulative count of daily underages and certain statistically-derived "action numbers"<sup>1</sup>. A cumulative number of underages in excess of the appropriate action number was to be interpreted as evidence of a deficit. The statistical model and calculations underlying the method were detailed in the report from Battelle Columbus Laboratories to EPA<sup>2</sup>. The basic method required modification for application to the inventory data collected in the survey because each sampled facility provided only a single, one-time record of

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<sup>1</sup>U.S. EPA, Office of Toxic Substances, "More About Leaking - Underground Storage Tanks: A Background Booklet for the Chemical Advisory," (October 1984).

<sup>2</sup>David C. Cox, "Performance of the Chemical Advisory Inventory Analysis Method Under Various Scenarios," Report from Battelle Columbus Laboratories to EPA under contract No. 68-01-6721 (April 1984).

30 days' inventory for analysis. The purpose of this section is to describe the statistical model on which the modified EPA method is based.

The decision rule for the proposed method will be defined by considering a well-run station where the only sources of discrepancy in the inventory records are (i) a daily leak of magnitude  $L$  and (ii) unavoidable random error in the daily stick measurement of the tank. Successive daily errors are assumed independent and identically normally distributed with mean zero; this assumption is supported by the research of Warren Rogers<sup>3,4</sup>. Hence, we can write:

$$X_i = x_i + e_i$$

where  $X_i$  is the  $i^{\text{th}}$  daily stick measurement,  $x_i$  is the true quantity of gasoline in the tank at the close of the  $i^{\text{th}}$  day, and  $e_i \sim N(0, \sigma^2)$  is the stick measurement error. Now consider a period of  $n$  days, assuming for simplicity that the station is open every day. The process of balancing inventory at the end of each day, as described in the literature<sup>5</sup> and assuming that there is no metering error at the pump<sup>6</sup>, leads to a set of daily variances (discrepancies),

$$d_i = -L + e_i - e_{i-1}, \quad i = 1, \dots, n.$$

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<sup>3</sup>"Inventory Reconciliation system," Warren Rogers Associates.

<sup>4</sup>Warren Rogers, personal communication.

<sup>5</sup>American Petroleum Institute: "Recording Practices for Bulk Liquid Stock Control at Retail Outlets," (1977).

<sup>6</sup>Metering error, if present, can be estimated and removed from the record, see American Petroleum Institute, "Recommended Practice for Bulk Liquid Stock Control at Retail Outlets," (1977).

Let  $N$  be the total number of negative daily variances,

$$N = \#\{i | 1 \leq i \leq n, d_i < 0\}.$$

Clearly, large values of  $N$  suggest that there is a leak, i.e.,  $L > 0$ . The exact probability distribution of  $N$  is, in general, very difficult to derive. However, of the special case of no leak, i.e.,  $L = 0$ , the calculation has been carried out<sup>7</sup>. Table E-1 shows the distribution for the case  $n = 30$  of most interest. In general, we must rely on a normal approximation to the distribution. This is derived as follows. We first find the mean  $E(N)$  and variance  $V(N)$  as follows. Define:

$$p = \Pr(d_i < 0)$$

$$p_1 = \Pr(d_i < 0, d_{i+1} < 0)$$

$$I_i = 1, \text{ if } d_i < 0 \\ 0, \text{ else}$$

Then  $E(I_i) = p$ ,  $E(I_i I_j) = p^2$  if  $|j-i| > 1$  (because then  $I_i, I_j$  are independent),  $E(I_i I_{i+1}) = p_1$ . Thus

$$E(N) = E\left(\sum_{i=1}^n I_i\right) = np. \text{ Also,}$$

$$\begin{aligned} E(N^2) &= E\left(\sum_{i=1}^n I_i^2 + 2\sum_{i < j} I_i I_j\right) \\ &= E\left(\sum_{i=1}^n I_i^2 + 2\sum_{i=1}^{n-1} I_i I_{i+1} + 2\sum_{i < j-1} I_i I_j\right) \\ &= np + 2(n-1)p_1 + [n(n-1) - 2(n-1)]p^2 \end{aligned}$$

Therefore

$$\begin{aligned} V(N) &= E(N^2) - (E(N))^2 && [1] \\ &= np(1-p) - 2(n-1)(p^2 - p_1) \\ &= \sigma(L)^2 \end{aligned}$$

<sup>7</sup>Warren Rogers, "The Exact Null Distribution of the Number of Negative Daily Variances," Report from Warren Rogers Associates to EPA, (September 1984).

Table E-1. Probability distribution of the number of negative daily variances, N, for the no-leak case, based on 30-day inventory

No. of negative variances	Probability of occurrence
≤ 10	0.0024
11	0.0121
12	0.0456
13	0.1161
14	0.2022
15	0.2432
16	0.2022
17	0.1161
18	0.0456
19	0.0121
≥ 20	0.0024

We approximate  $N$  by a normal distribution with mean  $np + 0.5$  and variance  $\sigma(L)^2$ . The mean is taken as  $np + 0.5$  to provide an approximate continuity correction for use in the upper tail of the distribution, in which our greatest interest lies.

To check the accuracy of the approximation, consider the case  $L = 0$ . Then,

$$\begin{aligned} p_i &= P_r(i < 0, d_i + 1 < 0) = P_r(e_i < e_{i-1}, e_{i+1} < e_i) \\ &= P_r(e_i + 1 < e_i < e_{i-1}) \\ &= 1/6 \end{aligned}$$

since all six orderings of  $e_{i-1}, e_i, e_i + 1$  are equally likely. Thus, from Equation [1],

$$\sigma(L)^2 = n/4 - 2(n-1)/12 = (n+2)/12$$

Setting  $n = 30$  we have the approximation,

$$N \sim N(15.5, 2.67)$$

Table E-2. shows the accuracy of the approximation.

Table E-2. Accuracy of normal approximation to distribution of  $N$  for the case  $L = 0$  (no leak)

$n$	$P_r(N \geq n)$ (exact)	$P_r(N \geq n)$ (approximate)
15	0.6216	0.6217
16	0.3784	0.3783
17	0.1762	0.1788
18	0.0601	0.0630
19	0.0145	0.0162
20	0.0024	0.0029

Clearly the approximation is sufficiently accurate over the range of  $n$  reported. For  $L \neq 0$ , the exact distribution of  $N$  has not been derived. We will rely on the normal approximation in such cases. The mean and standard deviation of the approximating distribution have been calculated and are shown in Table E-3.

Table E-3. Mean and standard deviation of normal approximation to the distribution of  $N$ , the number of negative daily variances, for various values of the daily leak rate  $L$ , for a 30-day inventory

L (gallons)	Mean	Standard deviation
2	16.46	1.636
3	16.93	1.641
4	17.41	1.647
5	17.88	1.654
6	18.34	1.665
7	18.81	1.678
8	19.27	1.684
9	19.72	1.699
10	20.16	1.707

The final feature for which we must account before we can determine the decision rule is round-off error. In practice, inventory values are typically reported to the nearest gallon so that an exact inventory balance, i.e., a zero variance, can occur due to round-off. This is fairly common in actual inventory data. We will assume that a zero variance is reported if the actual variance is less than 0.5 gallons in absolute value. Thus, a negative variance is reported only if the actual variance is less than -0.5 gallons. Let  $N^*$  be the number of negative variances actually reported and assume  $\sigma\sqrt{2} = 25$  gallons. Then the distribution of  $N^*$  should be approximated by a normal distribution with mean and standard deviation shown in Table E-4.

Table E-4. Mean and standard deviation of normal approximation to the distribution of  $N^*$ , the number of negative daily variances accounting for round-off error, for various values of the leak rate  $L$ , for a 30-day inventory

L (gallons)	Mean	Standard deviation
0	15.26	1.633
1	15.74	1.634
2	16.22	1.635
3	16.69	1.638
4	17.17	1.644
5	17.64	1.650
6	18.11	1.660
7	18.58	1.672
8	19.04	1.681
9	19.49	1.687
10	19.94	1.703

Now suppose we have 30 days' inventory and there is no leak. Using the approximating distribution from Table E-4 the number of daily variances observed should have the distribution shown in Table E-5.

Table E-5. Probability distribution of the number of negative daily variances,  $N^*$ , observed when no leak is present

n = number of negative variances	$(P_r(N^* \geq n))$
15	0.564
16	0.326
17	0.142
18	0.047
19	0.011
20	0.002

Thus, if we make 18 or more negatives our criterion for deciding that a deficit is present, there is approximately a five percent false-positive rate. That is, a tank with no leak and no source of error in inventory other than random measurement error due to sticking has approximately a five percent chance of being erroneously classified as a leaker. Note that false-positives due to other factors such as theft are not accounted for here. The detection capability of this version of the EPA inventory analysis method can now be calculated using the values given in Table E-4. Results are shown in Table E-6.

Table E-6. Probability of detection of leaks of various sizes using the modified EPA inventory method based on 30 days' data

Actual leak		Detection probability
Gallons/day	Gallons/hour	
1	.04	0.08
2	.08	0.14
3	.12	0.21
4	.17	0.31
5	.21	0.41
6	.25	0.53
7	.29	0.64
8	.33	0.73
9	.37	0.81
10	.42	0.87

Thus, leaks of at least nine gallons per day or more have better than 80 percent chance of detection. It should be noted that the detection capability of the simple inventory method based on only 30 days' data would be expected to be poor. The method was designed, as explained previously, for use as a tool for on-going monitoring programs.

## II. WARREN ROGERS ASSOCIATES' INVENTORY RECONCILIATION METHOD

Warren Rogers Associates (WRA) has developed a computerized system for analyzing daily inventory data from underground storage tanks in order to identify leaks<sup>8</sup>. The details of the method are proprietary. This section provides a brief description of publicly-available information on the model and should not be interpreted as an evaluation or endorsement by EPA.

The WRA system was developed in response to the perceived inadequacy of conventional, routine inventory accounting in detecting small or moderate leaks. Typically, such leaks are masked in the data by a variety of errors. For example, a single delivery error of 300 gallons could mask a 10 gallon-per-day leak based on 30 days' inventory. The purpose of the model is to isolate, identify, and quantify these errors.

Errors accounted for include:

- Delivery errors;
- Unexplained additions;
- Pump meter error;
- Temperature effects;
- Stick error; and
- Tank or line leaks.

Occasionally, other, rarer, errors will appear, e.g., use of an incorrect tank conversion chart, or theft. The data required by the model include only daily stick readings, deliveries, and sales.

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<sup>8</sup>Warren Rogers Associates, Inc., "Inventory Reconciliation System," (undated).

The basis for the model is that the major errors and discrepancies in the inventory data are very distinct in their characteristics and thus in the way they contribute to the total record. Thus, for example, an unrecorded over-delivery or an unrecorded removal will cause a permanent shift in the record which remains as a fixed component in all future observations. This effect can be estimated and removed from consideration when evaluating the possibility of a continuing day-to-day trend indicative of a leak. By contrast, a large stick error caused by a mistake in reading the stick or conversion chart will typically cause a large discrepancy in that day's inventory which will be followed the next day by a discrepancy of similar size in the opposite direction. The two discrepancies will tend to cancel out in the cumulative inventory record. The "signature" of a pump meter error is different: such an error will induce day-to-day errors of constant sign proportional to the through-put of the tank.

WRA's report to clients includes a record of day-to-day variances and the cumulative variance between book inventory and stick measurement for the period. It also provides:

- Over- or under-deliveries by date of occurrence and amount. That is, the discrepancy between the amount of product actually delivered as opposed to the amount reported;
- Unexplained one-time gains or losses also by date and amount;
- Meter errors at the pump;
- Trends which are indicative of either a tank or line leak; and
- Effects of possible disparities between the ambient air temperature and underground temperature.

As a special contribution to this study, WRA also provided a "data quality code" based on professional interpretation and experience. The data quality code is explained in Table E-7. A sample WRA inventory report is shown in Figure E-1. Based on a discussion with the developers of the WRA model, the false-positive rate is five percent, comparable to the modified EPA method.

Table E-7. WRA data quality code

Category	Definition
1	Confident of the result
2	The trend could have been delivery-induced
3	The trend is noisy but believable
4	No confidence in the trend due to the data
5	Data is questionable and requires further investigation.

### III. ENTROPY LIMITED INVENTORY RECONCILIATION SYSTEM

Entropy Limited has developed the Precision Tank Inventory Control (PTIC) system<sup>9</sup>. The analysis is based on principles similar to the WRA system and accounts for the same types of errors and discrepancies. Entropy appears to consider thermal effects and vapor losses more comprehensively than does WRA. However, additional input data to the system is required for these analyses.

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<sup>9</sup>Entropy Limited, "Precision Tank Inventory Control," (1984).

10/27/85

INVENTORY ANALYSIS

TANK ID : 1  
 PRODUCT : UNLEADED  
 TANK SIZE : 10000

MONTH	DAY	SALES	DELIVERIES	STICK	BOOK	DAILY VARIANCE	CUM. VARIANCE	VARIANCE
4	3	1150.	0.	2926.	2901.	25.	25.	25.
4	4	1163.	0.	1733.	1738.	-30.	-30.	-5.
4	5	1628.	0.	1098.	1110.	-7.	-37.	-12.
4	6	1444.	410.	3759.	3766.	5.	-32.	-17.
4	7	902.	0.	2864.	2864.	7.	-25.	-10.
4	8	466.	0.	2373.	2398.	-25.	-50.	-25.
4	9	1055.	0.	1301.	1343.	-42.	-92.	-42.
4	10	856.	0.	437.	487.	-50.	-142.	-92.
4	11	524.	4230.	4152.	4193.	-41.	-183.	-142.
4	12	1285.	2060.	4948.	4968.	-20.	-203.	-163.
4	13	1195.	0.	3759.	3773.	-14.	-217.	-177.
4	14	1158.	0.	2616.	2615.	1.	-216.	-176.
4	15	630.	3750.	5744.	5735.	9.	-207.	-167.
4	16	921.	0.	4815.	4814.	1.	-206.	-166.
4	17	784.	0.	4021.	4030.	-9.	-215.	-175.
4	18	1453.	0.	2554.	2577.	-23.	-238.	-198.
4	19	1532.	5110.	7102.	7155.	-53.	-291.	-251.
4	20	1028.	0.	6073.	6127.	-54.	-345.	-305.
4	21	881.	0.	5214.	5246.	-32.	-377.	-337.
4	22	992.	0.	4218.	4254.	-36.	-413.	-373.
4	23	488.	0.	3693.	3766.	-73.	-486.	-446.
4	24	1169.	4100.	6594.	6697.	-103.	-589.	-549.
4	25	1528.	0.	6073.	6169.	-96.	-685.	-645.
4	26	827.	0.	5214.	5342.	-128.	-813.	-773.
4	27	786.	0.	4416.	4556.	-140.	-953.	-913.
4	28	661.	0.	3759.	3895.	-136.	-1089.	-1049.
4	29	842.	5240.	8125.	8293.	-168.	-1257.	-1217.
4	30	827.	0.	7289.	7466.	-177.	-1434.	-1394.
5	1	1401.	0.	5876.	6065.	-189.	-1623.	-1583.
5	2	1051.	0.	4815.	5014.	-199.	-1822.	-1623.

END OF PERIOD CUMULATIVE VARIANCE

-199.

NUMBER OF NEGATIVE DAILY VARIANCES

19

DELIVERY DISCREPANCIES

MONTH	DAY	AMOUNT
4	12	46.

Figure E-1. WRA Inventory Report

WARREN ROGERS ASSOCIATES, INC  
65 BELLEVUE AVENUE  
NEWPORT, RI 02840  
(401) 846-4747

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INVENTORY ANALYSIS

TANK ID : 1  
PRODUCT : UNLEADED

TANK SIZE : 10000

MONTH DAY AMOUNT  
4 29 -17.

UNEXPLAINED ONE TIME GAINS AND LOSSES

MONTH DAY AMOUNT  
4 14 39.  
4 21 19.  
4 23 -36.  
4 26 -19.

PUMP METER ERROR

NONE

DAILY TREND

-8.37

AVERAGE STICK ERROR

6.95

LARGE STICK ERRORS

MONTH DAY  
4 19

TEMPERATURE DIFFERENTIAL

-2.66

SIGNIFICANCE OF TREND

0.00

1

The PTIC system reports its leak findings as an estimated leak rate, in gallons per day, and as a "probability of leak" (see the sample inventory report in Figure E-2). According to the model's developers, the probability of leak is based on a Bayesian-type analysis which accounts for various factors including the quality of the inventory data. Details are proprietary. Typically, the decision rule is phrased in terms of the leakage probability as follows:

<u>Leak probability</u>	<u>Decision</u>
< 10%	Tank is tight
10% - 50%	Inconclusive
≥ 50%	Tank is leaking

The 50 percent cutoff point corresponds to a false-positive rate of approximately two percent. To obtain a more typical five percent false-positive rate, a cutoff of 30 percent leak probability should be used to decide that the tank is leaking.

TANK # = 2 STAGE II VAP CONTROL LEVEL (9=NO VAP LOSS) = 1  
 TANK SIZE = 4000. 3 = NEG UNLD; INSTYR=1973

INVENTORY RECORDED GALLOWS: 9 = GROSS, 1 = NET  
 0 DELIVERIES  
 0 STICK INVENTORY  
 0 DISPENSED PRODUCT

N.B.!! STICK-READING AND TOTALIZING OCCURS ON THE MORNING AFTER THE LISTED DATE

I	DATE	DISPENSED	STORAGE	DELIV	DISCREP
1	1/2/85	147.8	1374.0	0.0	0.0
2	1/3/85	147.8	1221.0	0.0	-5.2
3	1/4/85	189.4	925.0	0.0	-115.6
4	1/5/85	239.8	584.0	0.0	-119.2
5	1/7/85	202.8	1846.0	1100.0	364.8
6	1/8/85	159.4	1769.0	0.0	76.4
7	1/9/85	86.0	1569.0	0.0	-111.0
8	1/10/85	121.5	1374.0	0.0	-73.5
9	1/11/85	179.8	1103.0	0.0	-86.2
10	1/12/85	134.4	925.0	0.0	-48.6
11	1/13/85	192.9	649.0	0.0	-83.1
12	1/15/85	230.3	209.0	0.0	-129.7
13	1/16/85	232.5	1569.0	1100.0	412.5
14	1/17/85	160.1	1297.0	0.0	-111.9
15	1/18/85	84.1	1221.0	0.0	0.1
16	1/19/85	180.8	961.0	0.0	-79.2
17	1/21/85	174.7	746.0	0.0	-70.3
18	1/22/85	111.7	521.0	0.0	-83.3
19	1/23/85	291.5	1687.0	1100.0	357.5
20	1/24/85	177.5	1451.0	0.0	-59.5
21	1/25/85	291.5	1145.0	0.0	-104.5
22	1/26/85	130.2	961.0	0.0	-53.8
23	1/28/85	181.6	682.0	0.0	-97.4
24	1/29/85	184.2	429.0	0.0	-68.8
25	1/30/85	283.9	1539.0	1100.0	281.9
26	1/31/85	224.5	1297.0	0.0	-8.5
27	2/1/85	46.1	1221.0	0.0	-29.9
28	2/2/85	112.0	1034.0	0.0	-75.0
29	2/3/85	194.5	784.0	0.0	-55.5
30	2/4/85	239.7	2885.0	1100.0	360.7
31	2/5/85	120.5	1806.0	0.0	-78.5

TOTALS  
 EVCT 30.0  
 DISPENSED 5191.7  
 STORAGE 432.0  
 DELIV 5500.0  
 DISCREP 123.7

DISCREP DISTRIBUTION  
 NUMBER OF DAYS WITH (+) DISCREP = 7 ( 29.3%)  
 NUMBER OF DAYS WITH (-) DISCREP = 23 ( 76.7%)

DV STATS  
 DV = DISCREP (GAL) CORRECTED FOR TEMP, VAPOR LOSS AND WATER INFLOW  
 DV COUNTS = 30.94009  
 DV AVG = 4.00511  
 DV S.D. (BIASED) = 164.96260  
 DV VARIANCE (BIASED) = 27212.47  
 LAG-1 AUTOCORR = -0.19435

Figure E-2. Entropy Limited Inventory Report

DV DISTRIBUTION  
 NUMBER OF DAYS WITH (+) DV = 7 ( 23.3%)  
 NUMBER OF DAYS WITH (-) DV = 23 ( 76.7%)

DIPSTICK READ ERRORS  
 NONE

DELIVERY DISCREPANCIES

5	1/ 7/85	399.75	= DELIVERY OVER(+)	OR UNDER(-)
13	1/16/85	423.84	= DELIVERY OVER(+)	OR UNDER(-)
19	1/23/85	411.26	= DELIVERY OVER(+)	OR UNDER(-)
25	1/30/85	347.07	= DELIVERY OVER(+)	OR UNDER(-)
30	2/ 4/85	491.91	= DELIVERY OVER(+)	OR UNDER(-)

UNEXPLAINED ONE-TIME DISCREPANCY  
 NONE

UNMODIFIED DISCREPANCIES  
 DISCREPANCIES CORRECTED FOR TEMP. VAP. CHART CALIB AND METER ERR

SOURCES OF INVENTORY DISCREPANCIES

1	LEAK RATE (GAL/DAY) =	ESTIMATE	UNCER
2	CALIB CHART ERR (100%*GAL/GAL-THIRD) =	53.168	6.576
3	TOTAL METER ERR (100%*GAL/GAL-THIRD) =	3.15	1.51
4	THERMAL SHRINKAGE LOSS (GAL) =	99.00	69.00
5	VAPOR LOSS (GAL) =	-9.0	17.3
6	NET DELIVERY DISCREPANCY (GAL) =	5.5	9.8
7	ONE-TIME UNEXPLAINED GAIN/LOSS (GAL) =	1975.8	43.6
8	WATER INFLOW TO TANK (GAL) =	0.0	0.0
9	WATER OUTFLOW FROM TANK (GAL) =	0.0	0.0

RECORDKEEPING FLAHS:  
 EXCESSIVE DELIVERY DISCREPANCIES  
 EXCESSIVE VARIANCE IN STICK READINGS

PROBABILITY OF TANK LEAKAGE=  
 PROBABLE TANK LEAKAGE 100.00%