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Ranking Distributions of Environmental Outcomes Across Population Groups

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Abstract

This paper develops methods for evaluating distributional impacts of alternative environmental policies across demographic groups. The income inequality literature provides a natural methodological toolbox for comparing distributions of environmental outcomes. We show that the most commonly used inequality indexes, such as the Atkinson index, have theoretical properties that make them inappropriate for analyzing bads, like pollution, as opposed to goods, like income. We develop a transformation of the Atkinson index suitable for analyzing bad outcomes. We also show how the rarely used Kolm-Pollak index is particularly convenient for ranking distributions of both good and bad health and environmental outcomes. We demonstrate these methods in the context of emissions standards affecting indoor air quality.

Keywords: environmental justice, distributional analysis, inequality indexes, Lorenz curves, benefit-cost analysis

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The distribution of environmental outcomes complements efficiency considerations in policy evaluation. Policy makers have long considered environmental justice (EJ), a particular type of distributional analysis, an important consideration in policy evaluation. However, while tools for conducting benefit-cost analysis are well established, those for quantitatively examining equity or distributional effects are less so.

This paper develops analytical tools for ranking policy alternatives from the perspective of how they affect distributions of environmental outcomes. It draws primarily from the rich theoretical literature on measuring income inequality. We show how the emerging practice of simply substituting environmental harm for income in an inequality index implies a ranking based on inconsistent social preferences. This result is particularly problematic since the inequality index specifications themselves were derived for the purpose of ranking distributions in a manner consistent with a plausible set of social preferences.

To address this issue, we propose a modification of the standard Atkinson income inequality index that allows a cardinal normative ranking of adverse outcomes. We also show how the seldom-used Kolm-Pollak inequality index and its associated social evaluation function can easily accommodate bad outcomes. We illustrate how these methods can be applied towards ranking environmental policy outcomes from an EJ perspective in the context of recent legislation to reduce formaldehyde emissions from composite wood products.

The relevance of this line of inquiry is not merely academic. Benefit-cost analysis of major new environmental regulations is widely applied throughout the Organization for Economic Cooperation and Development (OECD, 2002) and making inroads in many developing countries (Livermore and Revesz, 2013).[1] The standard focus of such analysis has been to identify whether proposed policies represent an improvement in efficiency under the potential compensated Pareto criterion. Discussion of how benefits and costs are distributed across population groups, economic sectors, or regions is secondary and often qualitative in nature.

In the U.S., however, Executive Order 12,898, Federal Actions to Address Environmental

[1]In the U.S., Executive Orders 12,866 and 13,563 require benefit-cost analysis for new economically significant regulations.

Justice in Minority Populations and Low-Income Populations, requires federal agencies to address "disproportionately high and adverse human health or environmental effects on minority populations and low-income populations." The U.S. Environmental Protection Agency (EPA) has interpreted this to mean fair-treatment and the same degree of environmental protection for all. Implementation of this Executive Order has been slow and typically relies on the assumption that any regulation that improves environmental outcomes necessarily implies a lack of EJ concerns. Assertions that outcomes will improve for everyone often lack rigorous analytical support, however. Moreover, simply ensuring that a policy does not make anyone worse off does not mean that it necessarily addresses a pre-existing disproportionate impact. A hypothetical policy that eliminates an environmental threat for high income groups while leaving low income groups unaffected, for example, could exacerbate a disparity in outcomes.

The tools developed here provide a framework for ranking distributions of adverse environmental and health outcomes under alternative policy scenarios. As such, our focus is on analyzing distributions of the benefits of environmental policy, rather than the costs.

This emphasis is motivated in part by the text of E.O. 12,898, and the traditional prominence of health and environmental outcomes in both the EJ movement and academic literature. It is also partly motivated by the fact that the equity of the distribution of monetary income is typically beyond the purview of agencies charged with environmental protection. Finally, although fairly sophisticated models have been developed to predict the distribution of benefits, less information is currently available regarding the distribution of costs. Suppose a plant emitting air pollution were to close, for example. It is possible to model the geographic dispersion of the pollutants, and link that to local air quality and the demographic composition of affected communities. It is much more difficult to identify the geographic distribution of lost wages, profits, and consumer surplus since workers may have commuted from other parts of the region and the plant's owners and consumers of its products may be spread across the globe. Nonetheless, we acknowledge that a complete analysis of the equity of environmental policy would examine distributions of both benefits and costs, and we conclude with a brief discussion of how tools developed here could be adapted to incorporate cost data.

The basic social welfare criterion we employ is akin to the Rawlsian "veil of ignorance" question (Rawls, 1971). Suppose a risk averse individual knew the distribution of environmental outcomes for distinct demographic groups in a society and were forced to choose one to which to belong. If she were to be randomly assigned an outcome in her chosen group, which group would she prefer? By applying additional structure to preferences, this approach can be extended to explicitly quantify the differences between distributions in cardinal units.

The rest of the paper is organized as follows. The next section discusses how two sets of tools developed for ranking income distributions, Lorenz curves and index numbers, can be adapted to the EJ context. We then discuss in greater detail the theoretical implications of using inequality indexes to analyze distributions of bad environmental outcomes focusing on the Atkinson and Kolm-Pollak indexes. Next, we analyze the distributional impacts of the 2010 Formaldehyde Standards for Composite Wood Products Act, and offer concluding comments.

1 Ranking Distributions

In the context of regulatory impacts, distributional analysis provides policy makers and the public with information regarding the degree to which options under consideration ameliorate or worsen previous disparities in environmental outcomes for vulnerable communities, or create new disparities where none existed. As such, it is important to analyze changes in distributions of environmental outcomes between baseline and various policy options, rather than just the distribution of changes (since an unequal distribution of environmental changes may actually help alleviate existing disparities).[2]

Maguire and Sheriff (2011) recommend three fundamental questions to address in a regulatory EJ analysis:

- What is the baseline distribution of the environmental outcome?

[2]This perspective also has normative welfare meaning since the arguments of a standard utility function are the outcomes, not the change in outcomes.

- What is the distribution of the environmental outcome for each regulatory option?
- How do the policy options being considered improve or worsen the distribution of the environmental outcome with respect to vulnerable subgroups?

To develop a tool that could address these questions we draw on the economic development and public finance literature exploring how to compare income distributions. We focus on two commonly used approaches, Lorenz curves and inequality index numbers. Both tools are summary measures of outcomes for all individuals in a population. They allow analysts to rank alternatives in a way that can provide concise, yet useful, information to the decision-maker. Lorenz curves have the advantage of requiring minimal assumptions on social preferences to generate a ranking. Their primary disadvantage is that they are unable to rank two or more alternative distributions if the curves cross. Inequality index numbers were developed in part to overcome this problem. They provide a complete ordering, but at the cost of imposing more restrictions on admissible social preferences.

Throughout the text we adopt the following notational conventions. N denotes the size of the total population. The vector $\mathbf{x} \in \mathbb{R}^N$ is the distribution of an outcome (e.g., cancer risk) across the population, with x_n denoting the allocation of the outcome to individual n. For any given distribution, the elements of $\mathbf{x}$ are ordered such that $x_1 \leq x_2 \leq ... \leq x_n \leq ... \leq x_N$, the average allocation of the outcome is $\bar{x}$, the cumulative population proportion is $p = n/N$, and $\mathbf{1}$ is an N dimensional vector $(1,...,1)'$. Preferences are defined by a continuous social evaluation function of $\mathbf{x}, W : \mathbb{R}^N \mapsto \mathbb{R}$, such that $W(\mathbf{x}) > W(\tilde{\mathbf{x}})$ if and only if $\mathbf{x}$ is preferred to $\tilde{\mathbf{x}}$. We normalize $\mathbf{x}$ such that a positive value of x_n indicates the quantity of a desirable outcome, while a negative value indicates an undesirable outcome.

2 Lorenz Curves

Lorenz curves are a tool for visually comparing different distributions of an outcome variable. They display a curve for each distribution being analyzed. The analyst can infer the relative equity of a given allocation based on its distance from a line corresponding to a perfectly equal distribution. One can unambiguously conclude that an allocation Lorenz dominates another only if their curves do not cross. We follow the taxonomy of Moyes (1987) and describe three classes of Lorenz curves: Relative Lorenz (RL) curves, Generalized Lorenz (GL) curves, and Absolute Lorenz (AL) curves. These three curves share the characteristic that the support lies between zero and one, representing the cumulative proportion of the population, p (ordered as described above from $n = 1$ to N). They differ in how cumulative allocations are represented on the vertical axis. Here we provide a brief discussion of the three classes of Lorenz curves, and under what conditions one can use Lorenz dominance to infer that one distribution is preferable to another.

2.1 Relative Lorenz Curve

The original curve developed by Lorenz (1905) compared cumulative fractions of total income received by cumulative fractions of the population. The RL curve can be defined as

$$RL(\mathbf{x}, p) = \sum_{i=1}^{n} \frac{x_n}{N\bar{x}} \qquad n = 1, 2, ..., N \tag{1}$$

with $RL(\mathbf{x}, 0) = 0$ and line segments connecting adjacent points. The height of this curve is the cumulative percent of the total outcome variable allotted to a given population fraction. For example, Panel (a) of Figure 1 displays the RL curve for formaldehyde-induced cancer risk for the entire population. The curve is anchored at zero for $p = 0$ and one hundred for $p = 1$. A perfectly equal distribution would be represented by a straight line connecting these two points.

RL curves express individual allocations of the outcome relative to the total. They facilitate comparisons of distributions independent of the mean. RL curves are unchanged by multiplying all outcomes by a positive constant (scale invariant), and therefore convenient for comparisons with different units of measurement (e.g., income distributions denominated in different currencies).

RL dominance has no normative implications for distributions with different means unless one is willing to discard the Pareto principle.[3] Nonetheless, relative Lorenz dominance conveys

[3]Satisfaction of the Pareto principle requires that social welfare not increase if at least one individual is made

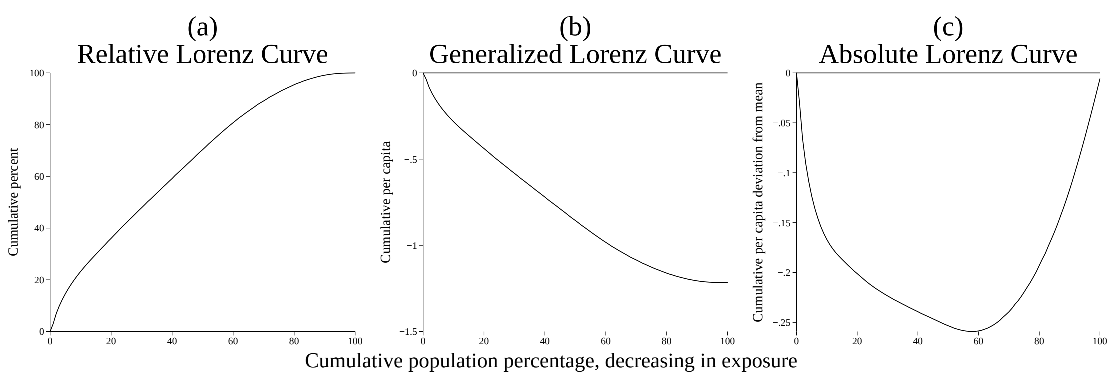
potentially useful information regarding the equity of various options. For distributions sharing the same *positive* mean outcome, RL dominance has the following normative interpretation:

$$W(\mathbf{x}) \geq W(\mathbf{x}') \iff \begin{cases} RL(\mathbf{x}, p) \geq RL(\tilde{\mathbf{x}}, p) & \bar{x} > 0 \\ RL(\mathbf{x}, p) \leq RL(\tilde{\mathbf{x}}, p) & \bar{x} < 0, \end{cases} \tag{2}$$

for all p. Care must be taken when using relative Lorenz analysis for bad outcomes. If the mean outcome is negative, then the relative Lorenz curves lie above the line of perfect equality, and the dominance criterion is reversed.

Moreover, simply redefining a bad outcome in terms of a complementary good outcome can affect the ranking of distributions. For example, converting a probability of contracting cancer (a bad) to a probability of *not* contracting cancer (a good) by adding one to each negative outcome does not necessarily preserve the RL dominance relationship for two distributions. This sensitivity in ranking is due to the fact that RL dominance is not translation invariant, i.e., the ranking may change if a constant value (in this case one) is added to each element of **x**.

Figure 1: Population Lorenz curves for cancer risk in cases per million



2.2 Generalized Lorenz Curve

Shorrocks (1983) introduced the GL curve as a tool for comparing distributions of cumulative monetary income received by a cumulative fraction of the population. Unlike RL dominance, GL

worse off and no one is made better off.

dominance takes both the average outcome and equality of outcomes into account when ranking distributions. The GL curve can be defined

$$(3) \qquad GL(\mathbf{x},p) = \sum_{i=1}^{n} \frac{x_n}{N} \qquad n = 1,2,...,N$$

with $GL(\mathbf{x},0) = 0$ and line segments connecting adjacent points.

The height of the GL curve is the cumulative value of the outcome variable in a given percentile, normalized by the size of the total population (see Figure 1, Panel (b)). This normalization facilitates comparisons of distributions for different size populations.[4] The GL curve is anchored at zero for $p = 0$ and $\bar{x}$ for $p = 1$. A perfectly equal distribution is represented by a straight line connecting these two points. A distribution GL dominates another if its curve lies somewhere above and nowhere below the other.

A normative interpretation of GL dominance requires two key assumptions regarding social preferences: (i) the social welfare function satisfies the Pareto property, and (ii) the social welfare function is Schur concave, i.e., a mean-preserving regressive reallocation does not increase social welfare.[5] The first assumption expresses a preference for Pareto efficiency and the second expresses a preference for equity.

Given these assumptions, distribution $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff GL(\mathbf{x},p) \geq GL(\tilde{\mathbf{x}},p)$, for all p. GL dominance accounts for both the per capita outcome and the distribution across individuals.[6] Thus, for example, a population with a very unequal distribution of income can dominate a population with a more egalitarian distribution if each percentile of the former population is wealthier than the corresponding percentile of the latter. In addition to being scale invariant, GL ranking is unaffected by adding a constant amount to all outcomes (translation invariant): $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff GL(\beta\mathbf{x},p) \geq GL(\beta\tilde{\mathbf{x}},p)$, for all $\beta > 0$ and p, and $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff$

[4] For example, a super-population generated by duplicating a distribution $\mathbf{x}$ has the same value for GL as the original population.

[5] Let $\mathbf{Q}$ be a square matrix composed of non-negative real numbers whose rows and columns each sum to 1. A function $F(\mathbf{x})$ is Schur concave if $\mathbf{Qx}$ is not a permutation of $\mathbf{x}$ and $F(\mathbf{Qx}) \geq F(\mathbf{Qx})$. All symmetric quasiconcave functions are Schur concave, although the converse is not true.

[6] Thistle (1989) shows that GL dominance is equivalent to second order stochastic domination.

$GL(\mathbf{x}+\gamma\mathbf{1},p) \geq GL(\tilde{\mathbf{x}}+\gamma\mathbf{1},p)$, for all γ and p.

The GL curve highlights the importance of correctly specifying bad outcomes. If one were to simply define bad outcomes with increasing positive values of x, the normative implications of the standard GL curve break down since the Pareto principle would be violated: A GL could lie above another because it has a more equal distribution (a good thing), or more bad outcomes per capita (a bad thing). This problem does not arise if bad outcomes are specified as decreasing in x.

2.3 Absolute Lorenz Curve

Moyes (1987) introduced the concept of the AL curve to compare cumulative differences from the mean income received from cumulative fractions of the population. It can be defined

(4) $$AL(\mathbf{x},p) = \sum_{i=1}^{n} \frac{x_n - \bar{x}}{N} \qquad n = 1,2,...,N$$

with $AL(\mathbf{x},0) = 0$ and line segments connecting adjacent points.

The height of the AL curve is the cumulative gap between the mean allocation of the outcome variable and the amount allotted to a given fraction of the population, normalized by the total population (see Figure 1, Panel (c)). Note that this gap is measured in absolute terms, not as a percent of the total outcome. This curve is anchored at zero both for $p = 0$ and $p = 1$. A perfectly equal distribution is represented by a straight line along the horizontal axis connecting these two points. A distribution dominates another in an AL sense if its curve lies somewhere above and nowhere below the other.

Like RL analysis, for social evaluation functions satisfying the Pareto principle, AL dominance has normative meaning only if distributions share the same mean.[7] In this case, $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff AL(\mathbf{x},p) \leq AL(\tilde{\mathbf{x}},p)$, for all p.

The AL curve is not scale invariant, therefore its ranking of two distributions may be affected by purely nominal transformations such as changing units of measurement. Since the height of the AL curve represents the difference from the mean it is translation invariant. As such, it

[7]These two approaches yield identical rankings for distributions with the same mean.

easily accommodates negative values for the outcome variable without redefining the dominance criterion.

2.4 Inter-group Inequality

Applying Lorenz curve analysis to compare alternate distributions of outcomes for individuals within a sub-population (based on income or race, for example) is straightforward. One need only calculate the curve using the relevant elements of $\mathbf{x}$, and respecify n and N accordingly. Such intra-group analysis is of limited use in the context of EJ, where one may be interested in disparities *across* groups.

Conducting Lorenz analysis across such groups is more problematic. One might proceed by assigning the same representative value of the outcome variable to all individuals within each group, re-order the elements of $\mathbf{x}$ such that they are non-decreasing in outcomes, and use these values to construct a Lorenz curve. The key question then arises as to what constitutes a "representative" value.

A natural choice might seem to be a group's mean outcome. A ranking based on such a choice, however, implicitly assumes that social welfare is unaffected by distributions of outcomes within groups. This approach leads to the inconsistent result that for a given distribution $\mathbf{x}$ the inter-group Lorenz curve necessarily (weakly) dominates the whole-population Lorenz curve for the same distribution. That is, under these implied preferences, for an identical distribution of outcomes, society can be made better off simply by classifying individuals into fewer and fewer groups.[8] In addition, such an approach would violate the Schur-concavity assumption of the underlying social welfare function since one could improve social welfare with a regressive reallocation from an individual with a low outcome who belongs to a high-mean group to an individual with a high outcome in a low-mean group.

An alternative approach would be to adjust group means such that, all else equal, groups with a less equal intra-group distribution would have a lower representative outcome. This representative

[8] At the extreme, perfect equality can be achieved by simply placing all individuals into one group.

outcome could answer the question "What allocation, if given to everyone in the group, would generate the same social welfare as the actual distribution of allocations?" This adjusted mean is commonly referred to as the Equally Distributed Equivalent (EDE) outcome.

The EDE approach clearly imposes stronger assumptions on social preferences than Schur concavity and the Pareto property. It lays the foundation, however, for a method of ranking distributions in terms of inter-group inequality in a manner that is consistent with rankings for the population as a whole. In addition, it allows the analyst to rank distributions in cases where Lorenz curves intersect. In the next section, we discuss how the EDE can be used in EJ analysis to generate complete and consistent rankings for entire populations as well as within and between subgroups. We also discuss tradeoffs in terms of restrictions on permissible social welfare functions.

3 Equally distributed equivalent outcomes and inequality indexes

The inability of Lorenz curve analysis to provide a complete ordering of outcomes combined with its inappropriateness for analyzing distributions across population subgroups limit its usefulness for EJ analysis. Here we describe an alternative approach based on the EDE and the related concept of an inequality index. The EDE provides a complete, consistent, and transparent system for ranking distributions taking into consideration average outcomes as well as within group, across group, and total inequality. The EDEs discussed here impose the same restrictions on preferences as the GL ranking (the Pareto property and Schur concavity) as well as additional restrictions discussed below. Thus, if one distribution GL dominates another, it will also dominate the other using a ranking method based on the EDE, but the reverse is not generally true. At the cost of imposing additional structure on social preferences, the EDE approach allows the analyst to rank distributions for which the GL framework is unable to provide a ranking (e.g., if GL curves intersect).

Let $\Xi(\mathbf{x})$ denote the EDE of distribution $\mathbf{x} \in \mathbb{R}^N$, such that

$$W(\mathbf{x}) = W(\Xi(\mathbf{x})\mathbf{1}). \tag{5}$$

Note that although social welfare functions are ordinal, $\Xi(\mathbf{x})$ provides a cardinal measure of social preferences. The EDE for a given distribution is the amount of the outcome variable which, if given equally to every individual in the population, would leave society just as well off as the actual distribution. It thus embodies a set of social and individual preferences and is a measure of social welfare. Like the GL curve, the EDE takes into account both the equity of a distribution and the total amount of the outcome variable. The EDE is therefore able to provide a consistent ordering of alternative distributions with different means.

First let us consider distributions that share the same mean outcome. Ideally an index $I(\mathbf{x})$ would enable normative comparisons of any such distributions $\mathbf{x}$ and $\tilde{\mathbf{x}}$ along the lines of $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff I(\mathbf{x}) \leq I(\tilde{\mathbf{x}})$. The literature commonly considers two general classes of inequality indexes defined on the basis of how common changes to the distributions affect the index values.[9]

Analogous to their counterparts in Lorenz curve analysis, a relative index, *RI*, is scale invariant,

$$RI(\mathbf{x}) = RI(\beta\mathbf{x}), \qquad \beta \in \mathbb{R}_{++}, \tag{6}$$

and an absolute index, *AI*, is translation invariant,[10]

$$AI(\mathbf{x}) = AI(\mathbf{x} + \gamma\mathbf{1}), \qquad \gamma \in \mathbb{R}. \tag{7}$$

For $\mathbf{x} \in \mathbb{R}^N_+$, Blackorby and Donaldson (1978) show that if a continuous, increasing, Schur-concave social evaluation function is also homothetic then it has a unique relative inequality index of the general Atkinson-Kolm-Sen (AKS) form

$$RI(\mathbf{x}) = 1 - \frac{\Xi(\mathbf{x})}{\bar{x}}. \tag{8}$$

Moreover, under these assumptions, *RI* has an associated family of ordinally equivalent homothetic

[9]The value of an inequality index increases as the distribution becomes less equal, with a value of zero indicating a perfectly equal distribution.

[10]These are not the only categories of indexes; for more examples, see Kolm (1976a).

social evaluation functions. These results establish the normative implication that for distributions of *goods* with a common mean, $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff RI(\mathbf{x}) \leq RI(\tilde{\mathbf{x}})$.

RI lies between zero and one, with zero indicating perfect equality. It has the interpretation as the maximum percent of the outcome society would be willing to give up if the remainder were distributed equally. Note that for $\mathbf{x} \in \mathbb{R}^N_-$, the normative implication of *RI* reverses; if $\bar{x} < 0$, $\Xi(\mathbf{x})/\bar{x} > 1$ and *RI* would be negative and *decreasing* in inequality. Consequently, for distributions with the same negative mean, social welfare would be *increasing* in *RI*.

We therefore propose the following transformation of *RI* for $\mathbf{x} \in \mathbb{R}^N_-$:

$$RI_-(\mathbf{x}) = \frac{\Xi(\mathbf{x})}{\bar{x}} - 1. \tag{9}$$

This transformation preserves the intuitive interpretation of the index for bads. It represents the maximum percent increase (in absolute value) of the bad outcome society would be willing to *accept* if the resulting total amount were distributed equally. Consequently welfare is decreasing in RI_-, as desired.

Blackorby and Donaldson (1980) show that if a continuous, increasing, Schur-concave social evaluation function is also translatable then it has a unique absolute inequality index of the general Kolm form[11]

$$AI(\mathbf{x}) = \bar{x} - \Xi(\mathbf{x}). \tag{10}$$

An index *AI* has an associated family of ordinally equivalent translatable social evaluation functions. These results establish the normative implication that for distributions with a common mean, $W(\mathbf{x}) \geq W(\tilde{\mathbf{x}}) \iff AI(\mathbf{x}) \leq AI(\tilde{\mathbf{x}})$. This index has the interpretation of the maximum amount per capita of the outcome society would be willing to give up if the remainder were distributed equally. Note that, unlike *RI*, this index is well behaved for all $\mathbf{x} \in \mathbb{R}^N$.

[11] Blackorby and Donaldson (1980) define a function as translatable if it can be expressed as $W(\mathbf{x}) = \phi(\bar{W}(\mathbf{x}))$ where $\phi(\cdot)$ is an increasing function and $\bar{W}(\mathbf{x} + \gamma\mathbf{1}) = \bar{W}(\mathbf{x}) + \gamma$ for all $\mathbf{x}, \mathbf{x} + \gamma\mathbf{1} \in \mathbb{R}^N$.

An advantage of initially focusing on inequality indexes is that it allows one to systematically develop a list of desirable axiomatic properties an index should possess in the relatively simple context of comparing distributions with equivalent means, and based on these properties exclude candidate mathematical representations of indexes from consideration. Eqs. (8) - (10) imply that for relative and absolute indexes it is then straightforward to derive the EDE, which in turn can be used to rank distributions with different means.

In addition to being a relative or absolute measure, some commonly used axioms for a general inequality index are:[12]

i. *Transfer Principle*. Index does not decrease with a mean-preserving regressive transfer.

ii. *Diminishing Transfer Principle*. A transfer of a unit of a good between two individuals who have relatively low values of x affects the index value more than an identical transfer among two individuals who are the same distance apart, but with higher values of x.

iii. *Welfare Independence*. Society's willingness to trade an increase in one individual's outcome for a decrease in another's does not depend on the unchanged outcome level of a third individual.

iv. *Impartiality*. No variable besides x affects the value of the index.

The transfer principle is a fundamental requirement for an inequality index. It states that increasing the dispersion of the outcome across the population should increase the measure of inequality. The diminishing transfer principle is an extension that incorporates the normative belief that an inequality index should be more sensitive to changes in the allocations to people who are less well-off.[13] This additional restriction on social preferences enables one to rank distributions whose Lorenz curves cross.

The intuition behind welfare independence can be most easily understood in the case of a simple transfer that does not change the average outcome variable for a population. This property

[12] Kolm (1976a,b), among others, provides a detailed treatment of axioms i-iv.

[13] The Gini coefficient violates this property since changes in its value depend on the relative positions of the two individuals in the total population, rather than the magnitude of each x_n (see, for example, Chakravarty, 1990).

implies that the change in the index number arising from a transfer between two individuals is not affected by the distribution of the outcome variables for the rest of the population.[14]

Impartiality means that all individuals are treated symmetrically in calculating the inequality index, regardless of other attributes besides the outcome of interest. Note that this property does not preclude analysis of sub-populations differentiated by such attributes. For example, one could calculate an impartial inequality index for an entire population in which race does not affect the value, and one could calculate an index for various racial groups.

For the rest of this discussion, we focus on inequality indexes that satisfy axioms i-iv.[15] Kolm (1976a,b) shows that the Atkinson and Kolm-Pollak indexes are the only ones that do so for $\mathbf{x} \in \mathbb{R}_{+}^{N}$.

Blackorby and Donaldson (1978) introduce an axiom that enables analysis of subgroups within a population:

v. *Consistency in Aggregation*. An inequality index can be used to analyze subpopulations such that social evaluations made using the entire population arrive at the same result as those made applying the same preference structure to the collection of sub-populations.

This axiom requires that the EDE calculated for the entire population yields the same result as the EDE calculated on the basis of the EDEs of each subgroup (Blackorby and Donaldson, 1978, 1980). This property ensures that the same set of preferences are used to rank subpopulations as ranking an entire population. It also guarantees that total inequality for a population can be completely decomposed into within and between group inequalities. Satisfaction of this property is particularly useful in the EJ context. Blackorby and Donaldson (1978, 1980) show respectively that the Atkinson and Kolm-Pollak indexes satisfy consistency in aggregation.[16]

Finally, we add the requirement that the inequality index be able to rank allocations of any real-valued outcomes:

[14]Importantly from a technical perspective, this property is equivalent to requiring that there be a function of the index that can be expressed as a sum of functions of each x_n (Kolm, 1976a). The Gini coefficient also violates this property.

[15]For a discussion of other inequality indexes, see Chakravarty (1990).

[16]The Gini coefficient fails to satisfy this property, making it particularly unsuitable for EJ analysis.

vi. *Unlimited Domain*. An inequality index can be used to evaluate any distribution $\mathbf{x} \in \mathbb{R}^N$.

This axiom has not received much emphasis in the context of studies of distributions of income or other goods, for which the domain has been restricted to $\mathbb{R}^N_+$. It is, however, of obvious importance for analysis of pollution, adverse health impacts, or other negative outcomes that are the subject of environmental regulations. Blackorby and Donaldson (1982) show that the Kolm-Pollak index satisfies this property, while the Atkinson index does not.

Some authors (e.g., Levy, Wilson, and Zwack, 2007) have advocated presenting inequality indexes alongside average outcomes so as not to impose normative assumptions on the efficiency-equity tradeoff. This approach does not, however, avoid the imposition of normative assumptions. By using a given inequality index to rank distributions with the same mean, Blackorby and Donaldson (1978, 1980) show that one has already adopted a normative position that implies a specific ranking of distributions with different means. Put another way, if one is not comfortable with the ranking of distributions with different means implied by the social evaluation function associated with a given index, one should not be using that index as a measure of inequality in the first place. Kaplow (2005) goes so far as to argue that inequality indexes are not particularly useful for comparing distributions with different means; instead all comparisons should be based on the underlying EDE.

Although the EDE may be sufficient for comparisons of given distributions, we believe inequality indexes can be useful for fine-tuning policy instruments. Knowing that a particular option results in large average gains with a regressive distribution, for example, may provide a signal to look for ways of modifying the policy to make it more equitable. Such information would be lost by focusing exclusively on the EDE.

Similarly, with respect to EJ analysis, it can be informative to decompose an inequality index by population subgroups. The index allows the analyst to determine whether a welfare change for a subgroup is due to a change in average outcomes or or their distribution. The index may also be useful in identifying the potential for "hotspots" (concentration of adverse outcome in a small set of people) in a demographic group. Since it conflates information about average outcomes with information regarding the distribution, the EDE alone would not allow such analysis.

In the next sections, we examine the Atkinson and Kolm-Pollak inequality indexes in greater detail, focusing on their suitability for EJ analysis. We also develop a transformation of the Atkinson index which is suitable for analyzing distributions of non-positive outcomes. Table 1 summarizes the main differences between these indexes.

3.1 Atkinson Index

For $\mathbf{x} \in \mathbb{R}_+^N$, the Atkinson index is defined as:

$$I_A(\mathbf{x}) = \begin{cases} 1 - \left[\frac{1}{N}\sum_{n=1}^{N}\left[\frac{x_n}{\bar{x}}\right]^{1-\alpha}\right]^{\frac{1}{1-\alpha}}, & \alpha \geq 0, \alpha \neq 1 \\ 1 - \prod_{n=1}^{N}\left[\frac{x_n}{\bar{x}}\right]^{\frac{1}{N}}, & \alpha = 1. \end{cases} \tag{11}$$

Atkinson (1970) derived this index based on the underlying assumption that individual preferences are consistent with a utility function for individual n that is increasing and concave in income and exhibits constant relative risk aversion,

$$U_A(x_n) = x_n^{1-\alpha}/[1-\alpha], \qquad \alpha \geq 0. \tag{12}$$

The (constant) elasticity of the marginal social value placed on increasing the outcome variable for any given individual is $-\alpha$. I_A takes a value between 0 and 1 (with the interpretation as the percent reduction in a good that society would be willing to give up in order to have a perfectly equal distribution of the rest) since the EDE, $\Xi_A(\mathbf{x})$, is necessarily lower than the mean.

It is important to stress that x_n is a *desirable* outcome variable in the sense that the EDE of the Atkinson index,

$$\Xi_A(\mathbf{x}) = \begin{cases} \left[\frac{1}{N}\sum_{n=1}^{N}[x_n]^{1-\alpha}\right]^{\frac{1}{1-\alpha}}, & \alpha \geq 0, \alpha \neq 1 \\ \prod_{n=1}^{N}[x_n]^{\frac{1}{N}}, & \alpha = 1, \end{cases} \tag{13}$$

is increasing in x_n.

Table 1: Differences between measures of Equally Distributed Equivalent outcomes

Name	Functional Form	Domain	Invariance
Atkinson	$\Xi_A(\mathbf{x}) = \begin{cases} \left(\frac{1}{N}\sum_{n=1}^{N}[x_n]^{1-\alpha}\right)^{\frac{1}{1-\alpha}} & \alpha \geq 0, \alpha \neq 1 \\ \prod_{n=1}^{N}[x_n]^{\frac{1}{N}} & \alpha = 1 \end{cases}$	$\mathbf{x} \in \mathbb{R}_+^N$	Scale
Transformed Atkinson	$\Xi_{A-}(\mathbf{x}) = -\left(\frac{1}{N}\sum_{n=1}^{N}[-x_n]^{1+\alpha}\right)^{\frac{1}{1+\alpha}}$	$\mathbf{x} \in \mathbb{R}_-^N$	Scale
Kolm-Pollak	$\Xi_K(\mathbf{x}) = \begin{cases} -\frac{1}{\kappa}\ln\frac{1}{N}\sum_{n=1}^{N}\mathrm{e}^{-\kappa x_n} & \kappa > 0 \\ \bar{x} & \kappa = 0 \end{cases}$	$\mathbf{x} \in \mathbb{R}^N$	Translation

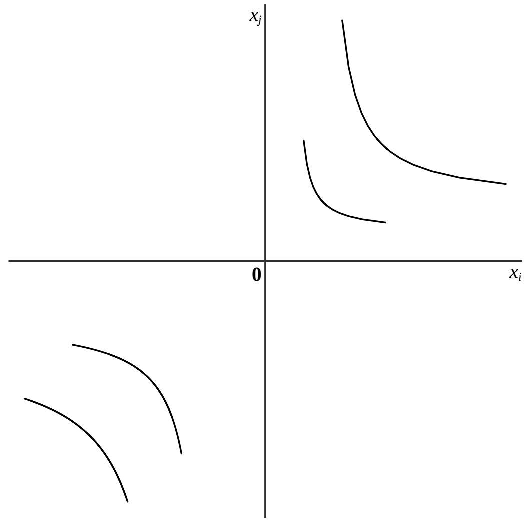
The parameter α is commonly referred to as the inequality aversion parameter. It allows the analyst to specify the amount society is willing to trade a reduction in the outcome variable for one individual for an increase for another. Note that this formulation nests the extremes of Benthamite utilitarian and Rawlsian maximin preferences. Setting $\alpha = 0$ implies that $\Xi_A = \bar{x}$; society is indifferent between mean-preserving transfers among any two individuals so that $I_A(\mathbf{x}) = 0$ for any $\mathbf{x}$. As α increases, society places greater weight on transfers to individuals with lower outcomes. At the limit, as $\alpha \to \infty$ then $\Xi_A = \min_n\{x_n\}$ so that $I_A(\mathbf{x}) = 1 - \min_n\{x_n\}/\bar{x}$ and a reallocation that does not benefit the worst off does not increase social welfare (Chakravarty, 1990). Since the choice of α reflects a normative judgment, it is common to calculate I_A for several values in order to determine how sensitive rankings are to the choice.[17]

A natural question arises as to how to extend the Atkinson index to distributional analysis of bad outcomes such as pollution or adverse health impacts. I_A is not defined for negative numbers for all $\alpha > 0$, thus precluding a simple redefinition of bads in this way.[18] Moreover, Blackorby and Donaldson (1982) show that even when well-defined I_A generates the perverse result that a progressive redistribution among individuals with negative outcomes *reduces* social welfare. Figure 2 depicts social indifference curves for the Atkinson social welfare function for $\alpha = 2$. Here it is clear that in the negative orthant, the social welfare function is Schur convex. Thus, a mean-preserving progressive redistribution reduces social welfare. In this case, the I_A also violates

[17] In its publications for example, the U.S. Census Bureau often reports Atkinson indexes calculated with α equal to 0.25, 0.5, and 0.75 (e.g., DeNavas-Walt, Proctor, and Smith, 2012).

[18] Nor could one simply subtract this negative value from an arbitrarily high benchmark value. Since the Atkinson index is not translation invariant, the index value and hence the ranking may be different for different benchmarks.

Figure 2: Social indifference curves associated with $I_A(\mathbf{x})$



the diminishing transfer principle for $\mathbf{x} \in \mathbb{R}^N_-$.[19]

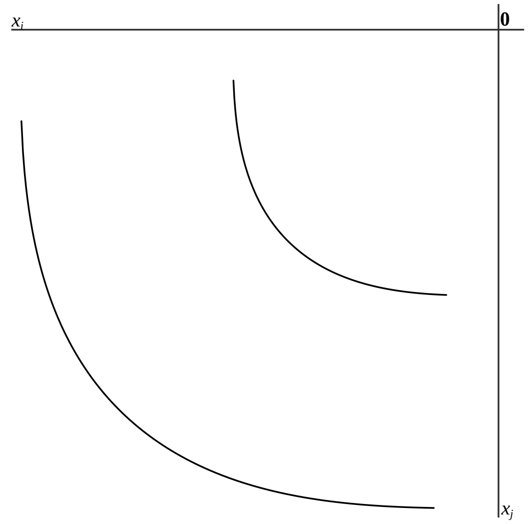
A second approach could be simply to use the absolute value of $\mathbf{x}$ in the Atkinson formula when $\mathbf{x} \in \mathbb{R}^N_-$.[20] This approach has similar drawbacks as using the negative values themselves. The implied social welfare function does not satisfy both the Pareto principle and Schur concavity. Referring to the positive orthant of Figure 2, either welfare decreases with increases in $\mathbf{x}$ (satisfying the Pareto principle) and a progressive redistribution of the bad outcome reduces social welfare (violating Schur concavity), or a progressive redistribution increases welfare, but so does an increase in the vector of bad outcomes. The index calculated with this approach would also violate the diminishing transfer principle since it implies a social welfare function that is more sensitive to transfers among the well off (those with low absolute values of the bad outcome).

A third alternative seen in the literature (e.g., Levy et al., 2009) is to transform a bad outcome into a good outcome by replacing it with its reciprocal. By substituting x_n^{-1} for x_n in Eq. (12), it is clear that the associated social evaluation function satisfies the Pareto property (is decreasing in $\mathbf{x}$), but is not Schur concave for $\alpha < 2$. In such cases, a progressive redistribution of $\mathbf{x}$ reduces social

[19] Kolm (1976b) shows that the diminishing transfer principle is satisfied for the Atkinson index if and only if $(1-\alpha)x^{-\alpha}$ and $(\alpha+\alpha^3)x^{-(\alpha+2)}$ have the same sign.

[20] See, for example, Levy, Chemerynski, and Tuchmann (2006); Levy, Wilson, and Zwack (2007); Levy et al. (2009); Fann et al. (2011); Post, Belova, and Huang (2011).

Figure 3: Social indifference curves associated with $I_{A-}(\mathbf{x})$



welfare, and the index violates the diminishing transfer principle.

Here, we propose a simple transformation of the Atkinson index that allows a sensible ranking of distributions of bads, while preserving its interpretation as a cardinal relative index. This method does not satisfy the unrestricted domain property, but defines a relative index satisfying axioms i-vi for allocations consisting only of bads. For $\mathbf{x} \in \mathbb{R}^N_-$, we respecify the individual utility function in Eq. (12) with $U_{A-}(x_n) = -[-x_n]^{1+\alpha}/(1+\alpha)$. Social welfare is decreasing in the bad outcome, ensuring satisfaction of the Pareto principle. Changing the sign of α in the utility function generates social indifference curves that are bowed outwards from the origin as in Figure 3, ensuring that the social welfare function is Schur-concave and that the index satisfies the diminishing marginal transfer principle.

The EDE for the social welfare function based on U_{A-} is

(14)
$$\Xi_{A-}(\mathbf{x}) = -\left[\frac{1}{N}\sum_{n=1}^{N}[-x_n]^{1+\alpha}\right]^{\frac{1}{1+\alpha}}.$$

In contrast to the case with good outcomes, note that $\Xi_{A-} \geq \bar{x}$. To preserve the property that an inequality index is increasing in inequality, we use the modified definition of a relative index in

Eq. (9) so that the transformed Atkinson index is

$$(15) \qquad I_{A-}(\mathbf{x}) = -\left[1+\frac{\Xi_{A-}(\mathbf{x})}{\bar{x}}\right] = \left(\frac{1}{N}\sum_{n=1}^{N}\left(\frac{x_n}{\bar{x}}\right)^{1+\alpha}\right)^{\frac{1}{1+\alpha}} - 1.$$

Its cardinal interpretation is the percent increase in the average bad outcome that, if equally distributed, would result in the same social welfare as the actual distribution **x**.

3.2 Kolm-Pollak index

For $\mathbf{x} \in \mathbb{R}^N$ the Kolm-Pollak index is defined as:

$$(16) \qquad I_K(\mathbf{x}) = \begin{cases} \frac{1}{\kappa}\ln\frac{1}{N}\sum_{n=1}^{N} \mathrm{e}^{\kappa[\bar{x}-x_n]}, & \kappa > 0 \\ 0 & \kappa = 0. \end{cases}$$

As shown by Blackorby and Donaldson (1980), using this index to rank outcomes is equivalent to assuming an additively separable social welfare function composed of individual utility functions of the form developed by Pollak (1971), $U(x) = -\mathrm{e}^{-\kappa x}$. Figure 4 illustrates social indifference curves associated with Kolm-Pollak social preferences. Unlike the Atkinson preferences discussed above, these satisfy Schur concavity and the Pareto property for all real-valued **x**.

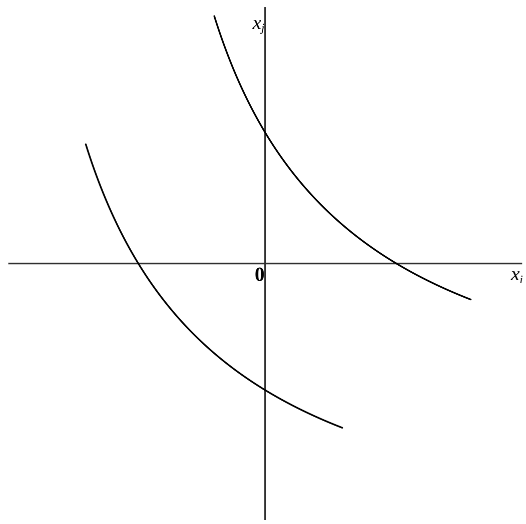
The EDE for the Kolm-Pollak index is:

$$(17) \qquad \Xi_K(\mathbf{x}) = \begin{cases} -\frac{1}{\kappa}\ln\frac{1}{N}\sum_{n=1}^{N} \mathrm{e}^{-\kappa x_n}, & \kappa > 0 \\ \bar{x} & \kappa = 0. \end{cases}$$

Analogous to α, κ can be interpreted as an inequality aversion parameter. For Kolm-Pollack social preferences, the elasticity of marginal social welfare with respect to a change in an individual's allocation is $-\kappa x_n$. As with I_A, social preferences vary from Benthamite to Rawlsian as κ takes values from zero to infinity (Chakravarty, 1990).

Unlike the case with the social evaluation functions associated with I_A or I_{A-}, this elasticity varies with an individual's allocation of the outcome variable. Consequently, the inequality

Figure 4: Social indifference curves associated with $I_K(\mathbf{x})$



aversion parameter needs to be appropriately normalized to maintain comparability across different units of measurement. To change units, one can simply divide κ by the conversion factor. If one had used $\kappa = 0.5$ to calculate I_K for an outcome measured in pounds, for example, and wished to convert to kilograms, it would be necessary to divide $\mathbf{x}$ and multiply κ by 2.2. This conversion ensures that the elasticity of marginal utility remain unchanged for each individual, and the values of I_K and Ξ_K would be 2.2 times larger, as desired.[21]

Less straightforward is the problem comparing welfare rankings obtained using Ξ_{A-} (or Ξ_A in the case of desirable outcomes) and Ξ_K for "similar" levels of social inequality aversion. In a sense, such a comparison is impossible since the implied elasticity of marginal utility is constant in one case and varies in the other. Nonetheless, the analyst may be interested in how sensitive results are to using a relative versus absolute inequality index. If one takes as a starting point the values of α typically used in the income inequality literature, the question becomes one of what value of κ generates a set of elasticities of marginal utility defined by the Pollak utility function that is closest to that generated by the Atkinson utility function for a given value of α.

To that end, we propose choosing κ to minimize the sum of squared differences between

[21]Changing units of $\mathbf{x}$ without modifying κ can cause the social ranking of two distributions to reverse depending on the unit of measurement (see, for example Zheng, 2007).

individual elasticities and α:

$$\kappa = \arg\min_{\hat{\kappa}} \left\{ [\hat{\kappa}\mathbf{x} - \alpha\mathbf{1}]'[\hat{\kappa}\mathbf{x} - \alpha\mathbf{1}] \right\} = \frac{\alpha \sum_{n=1}^{N} x_n}{\sum_{n=1}^{N} x_n^2}. \tag{18}$$

It is important that this calibration of κ be undertaken only once for each outcome under consideration (cancer risk, mortality risk, etc.), not separately for each alternative distribution of that outcome.[22] When comparing J allocation vectors of the same outcome (e.g., the distribution of cancer risk under different policy options) we replace $\mathbf{x}$ in Eq. (18) with the NJ dimensional vector $\hat{\mathbf{x}} = (\mathbf{x}_1', \mathbf{x}_2', ..., \mathbf{x}_J')'$.

3.3 Inter-group analysis

The methods described thus far allow analysis of inequality within an entire population or within distinct sub-populations. To characterize inter-group inequality, it is necessary to introduce further notation. Let $\mathbf{x}$ be partitioned into M distinct groups indexed $m = 1, 2, ..., M$, with N_m denoting the number of individuals in group m. A group's outcome vector is $\mathbf{x}_m$ with mean $\bar{x}_m$. The index numbers and EDEs for each group are indicated by a superscript m. The only difference between their formulas and the whole population formulas are that N_m and $\bar{x}_m$ replace N and $\bar{x}$.

For $\mathbf{x} \in \mathbb{R}_+^N$, Blackorby, Donaldson, and Auersperg (1981) derive a measure of inter-group inequality for I_A,

$$I_A^R(\mathbf{x}) = 1 - \frac{\Xi_A(\mathbf{x})}{\frac{1}{N}\sum_{m=1}^{M} N_m \Xi_A^m(\mathbf{x}_m)}. \tag{19}$$

Its can be interpreted as follows. Suppose that instead of $\mathbf{x}$, the good was distributed such that for each group m every individual in that group received $\Xi_A^m(\mathbf{x}_m)$. This second distribution has no intra-group inequality. Since I_A satisfies axiom vi, the aggregate EDE of this second distribution

[22] I_K would not be translation invariant if $\hat{\kappa}$ were recalculated for each distribution of a given outcome.

is equal to $\Xi_A(\mathbf{x})$, which by definition equals the EDE of the vector $\Xi_A(\mathbf{x})\mathbf{1}$. The inter-group inequality index I_A^R measures society's willingness to pay to move from the distribution with no intra-group inequality to a distribution in which every individual receives $\Xi_A(\mathbf{x})$. This willingness to pay is expressed as a percent of the total amount of the good in the distribution with no intra-group inequality.

It is straightforward to show that the corresponding inter-group inequality index for I_{A-} is

$$I_{A-}^R(\mathbf{x}) = \frac{\Xi_{A-}(\mathbf{x})}{\frac{1}{N}\sum_{m=1}^{M} N_m \Xi_{A-}^m(\mathbf{x}_m)} - 1. \tag{20}$$

For I_K, Blackorby, Donaldson, and Auersperg (1981) similarly show that inter-group inequality can be expressed as

$$I_K^R = \frac{1}{N}\sum_{m=1}^{M} N_m \Xi_K^m(\mathbf{x}_m) - \Xi_K(\mathbf{x}). \tag{21}$$

Its interpretation is society's per capita willingness to pay (in units of x) to move from a distribution in which there is no intra-group inequality to one in which the remainder of the total outcome is divided equally among all individuals.

4 Illustrative application

Despite being the subject of several theoretical papers, I_K has seldom been used in empirical income distribution studies (exceptions include Blackorby, Donaldson, and Auersperg, 1981; Atkinson and Brandolini, 2010). To our knowledge, neither it nor I_{A-} have been used to evaluate distributions of adverse environmental outcomes. Here we provide an illustrative application to regulation of indoor air pollution.

Resins commonly used in pressed wood products can off-gas significant amounts of formaldehyde. This chemical has been linked to numerous adverse health outcomes including nasopharyngeal cancer and eye irritation. Partly in response to concerns over high levels of formaldehyde in temporary trailers used by the Federal Emergency Management Agency (FEMA) to house people dislocated in the wake of Hurricane Katrina, Congress enacted the 2010 Formaldehyde Standards for Composite Wood Products Act. This legislation amended the Toxic Substances Control Act to set national emissions standards for pressed wood components common in home construction and furniture.

In its economic analysis of the act, the EPA uses a three-stage model to evaluate benefits of the standards. The first stage generates models of nine housing types in various climate zones, and stocks them with a representative selection of pressed wood products. These engineering models simulate indoor air concentrations of formaldehyde at baseline and under alternative policy scenarios. The second stage introduces epidemiological concentration-response functions to model the impact of formaldehyde concentrations on the probability of incurring eye irritation and cancer (fatal and non-fatal). Concentrations vary by age of home (due to a decay in off-gassing over time) and responses vary by age of the individual.[23] The economic model uses data from the American Community Survey and American Housing Survey to populate the model homes and simulate health benefits for the various demographic groups. In the final stage, the EPA uses results from willingness-to-pay studies to monetize health outcomes. For details on the modeling approach used in the benefit-cost analysis see EPA (2013).[24]

The EPA's analysis includes several emissions levels, here we focus on three: the baseline that would have occurred in the absence of the legislation (Base), the level that was assumed would prevail under the EPA's proposed implementation of the legislated standards (Act), and a level that corresponded to a hypothetical stricter standard that would require manufacturers to use resins with no added formaldehyde (NAF).

The standards introduced by the Act raise some interesting questions in the context of EJ. Formaldehyde off-gases at an exponentially decaying rate over time. Concentrations tend to be

[23] In principle, the concentration-response function could vary by race or income, but the EPA's analysis assumes it does not.

[24] In this example we use a seven percent discount rate and conservative concentration-response function estimates. These functions correspond to the EPA's low estimate for cancer risk and the EPA's high estimate for eye irritation. For eye irritation, the concentration response function for the high estimate is more conservative than for the low estimate. To obtain a low estimate for the change in cases, the EPA assumed a relatively high response to formaldehyde at very low concentrations to ensure that reducing exposure at these concentrations would not generate any benefits.

highest in new or newly remodeled homes with new furniture. Although the legislation was motivated by adverse outcomes experienced by poor and minority individuals using FEMA trailers, these groups may not be the main beneficiaries if they are less likely to live in homes with high formaldehyde levels.

With respect to the three EJ questions identified in the introduction, the goals of this analysis are to develop an understanding of the degree to which there is expected to be a disparity in formaldehyde-induced health outcomes in the absence of the standards, the way the two policy options affect the distribution, and how the various distributions can be ranked. To this end, we use information available in the rulemaking docket to generate GL, RL, and AL curves, and calculate I_{A-} and I_K for each policy scenario and demographic group.[25] Demographic groups are defined by self-reported race, ethnicity and income in the American Community Survey.[26]

4.1 Lorenz curve analysis

Figure 5 presents generalized Lorenz (GL) curves for formaldehyde average daily concentrations for the three policy scenarios by population group. It is immediately apparent that the NAF scenario GL dominates the Act which dominates Base for each group. The GL curves for the various groups are remarkably similar, with Native Americans having the lowest curves and Black and Hispanic having the highest. Thus from the perspective of formaldehyde concentrations it seems that some minorities stand to have high benefits from the standards relative to whites. Poverty status, in contrast does not seem to be correlated with reductions in concentrations.

Figure 6 presents relative Lorenz (RL) curves for concentrations. In a relative sense, the Act does not seem to affect the distribution of concentrations. Concentrations are reduced by roughly the same proportion within each population group. The NAF scenario, however, largely eliminates any relative inequality.

The absolute Lorenz (AL) curves in Figure 7 present a slightly different perspective. Hispanics

[25] The rule's Docket ID is EPA-HQ-OPPT-2012-0018.

[26] The Hispanic category included individuals of any race, such that Black, White, Native American, and Other are properly interpreted as non-Hispanic Black, etc. The Below poverty category consists of individuals belonging to households below the U.S. Census Bureau poverty threshold. For more details, see EPA (2013).

and Blacks clearly have the greatest intra-group inequality at baseline, and Native Americans the least. Although both policies improve the equity of the distribution, only the NAF scenario results in similar intra-group inequality for all racial groups. The distribution of concentrations for people below poverty is slightly worse than that for people above poverty at baseline and under the Act, but this situation reverses for the NAF scenario.

Although these results shed some light on the extent to which the emissions standards affect the distribution of formaldehyde exposure, average daily concentrations are not of direct welfare significance. At best they are a proxy for health outcomes that affect utility. Since concentration-response functions can be highly nonlinear, the distribution of formaldehyde can be a misleading indicator for the distribution of health outcomes. Figures 8-10 present the distribution of the modeled probability of eye irritation cases. As was the case with concentrations, Figure 8 shows that there is no tradeoff between increased stringency in the standard and equity; NAF clearly outperforms the Act, which outperforms the baseline for all groups. Compared with Figure 5, in Figure 8 the Native American and White groups are better off than Black and Hispanic groups based on eye irritation risk. Income, however, does not appear to play a major role in the distribution of eye irritation.

Comparison of Figures 6 and 9 clearly shows the effect of the nonlinear concentration response function. While Figure 6 suggests that the Act results in a roughly equiproportionate reduction in formaldehyde levels for the entire population, Figure 9 shows that the people with the highest probability of irritation received proportionately higher benefits. This effect is due to the fact that the marginal probability of irritation is increasing in concentration levels. Comparing Figures 7 and 10 shows that although there were similar improvements in equity within groups for the two outcomes, the distribution of above-average (in absolute value) outcomes is more highly concentrated for irritation.

With respect to cancer risk, we use an EPA estimate that assumes formaldehyde exposure in a given year affects lifetime risk through a linear unit-risk function. Since current formaldehyde exposure cannot affect past cancer risk, the EPA adjusts lifetime risk by an individual's age.

Figure 5: Generalized Lorenz curves for formaldehyde average daily concentrations in $\mu g/m^3$

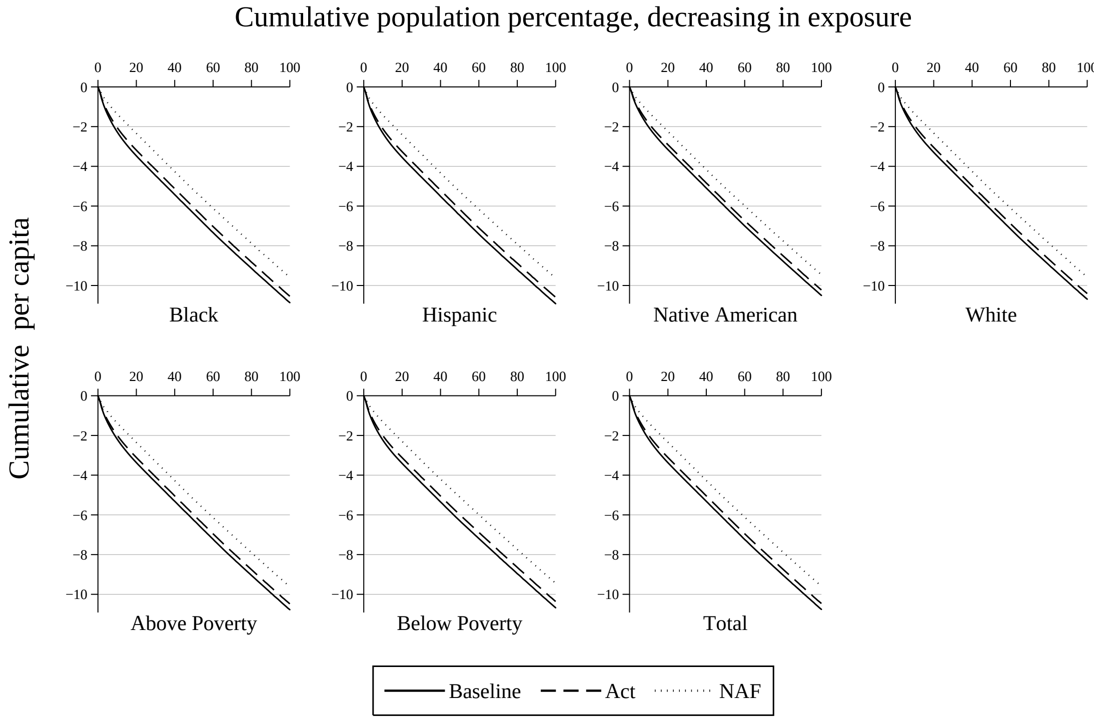


Figure 6: Relative Lorenz curves for formaldehyde average daily concentrations

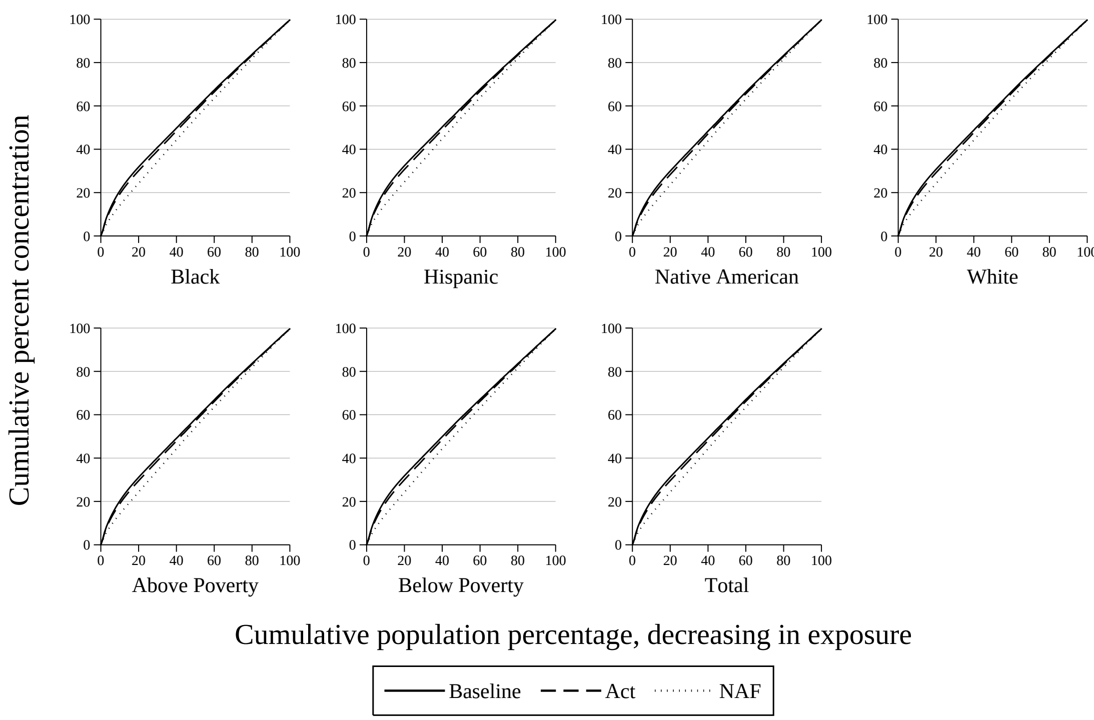


Figure 7: Absolute Lorenz curves for formaldehyde average daily concentrations in $\mu g/m^3$

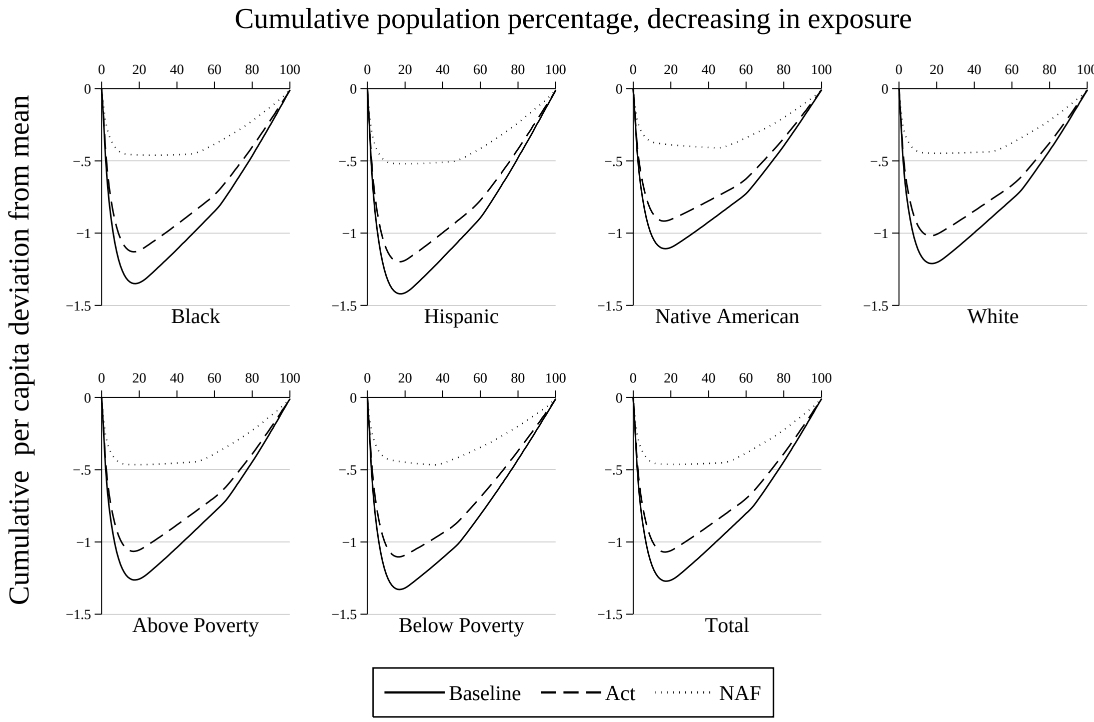


Figure 8: Generalized Lorenz curves for formaldehyde-induced eye irritation risk in cases/10^3

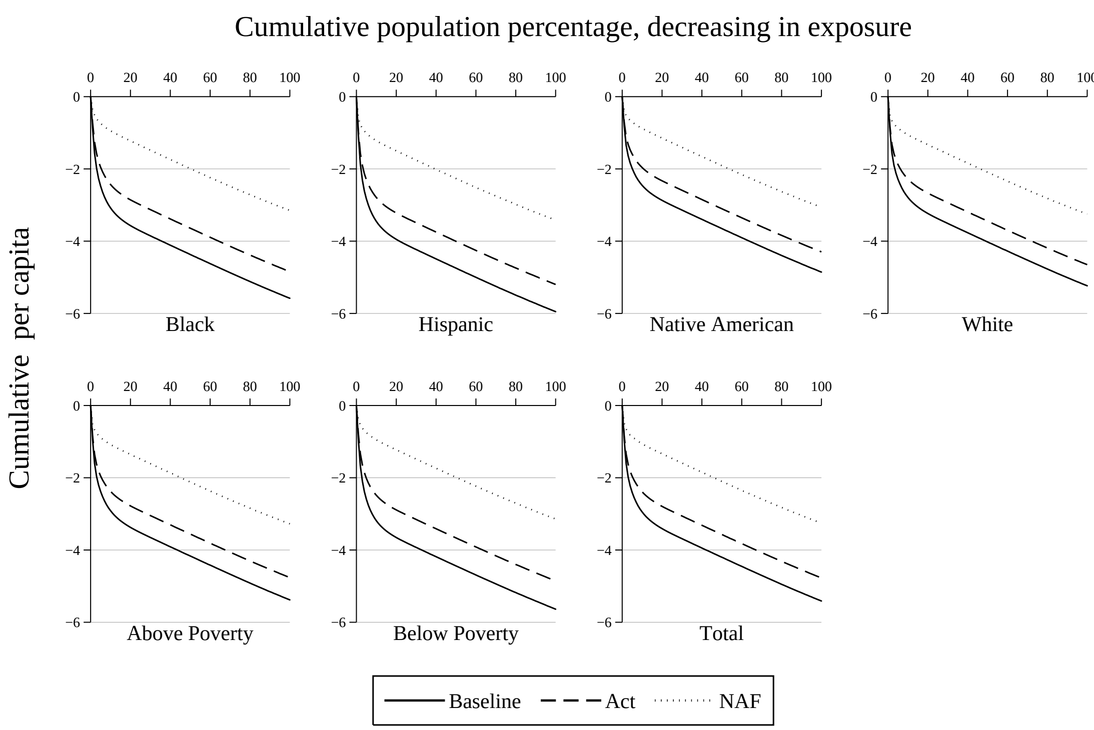


Figure 9: Relative Lorenz curves for formaldehyde-induced eye irritation risk

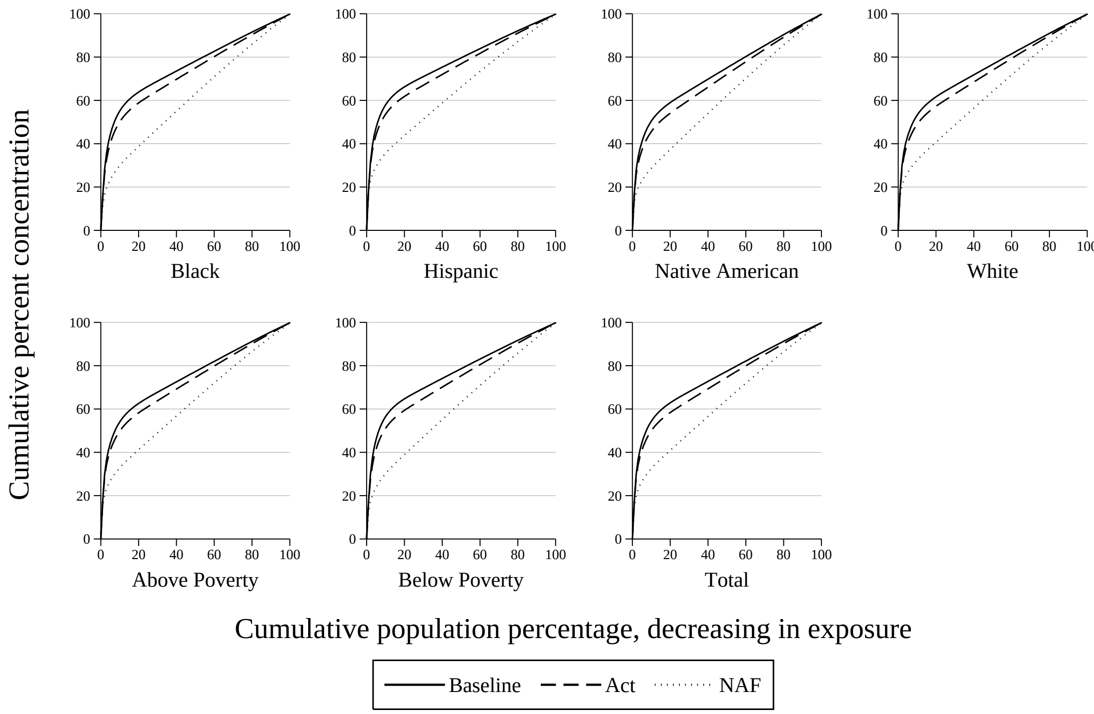


Figure 10: Absolute Lorenz curves for formaldehyde-induced eye irritation risk in cases/10^3

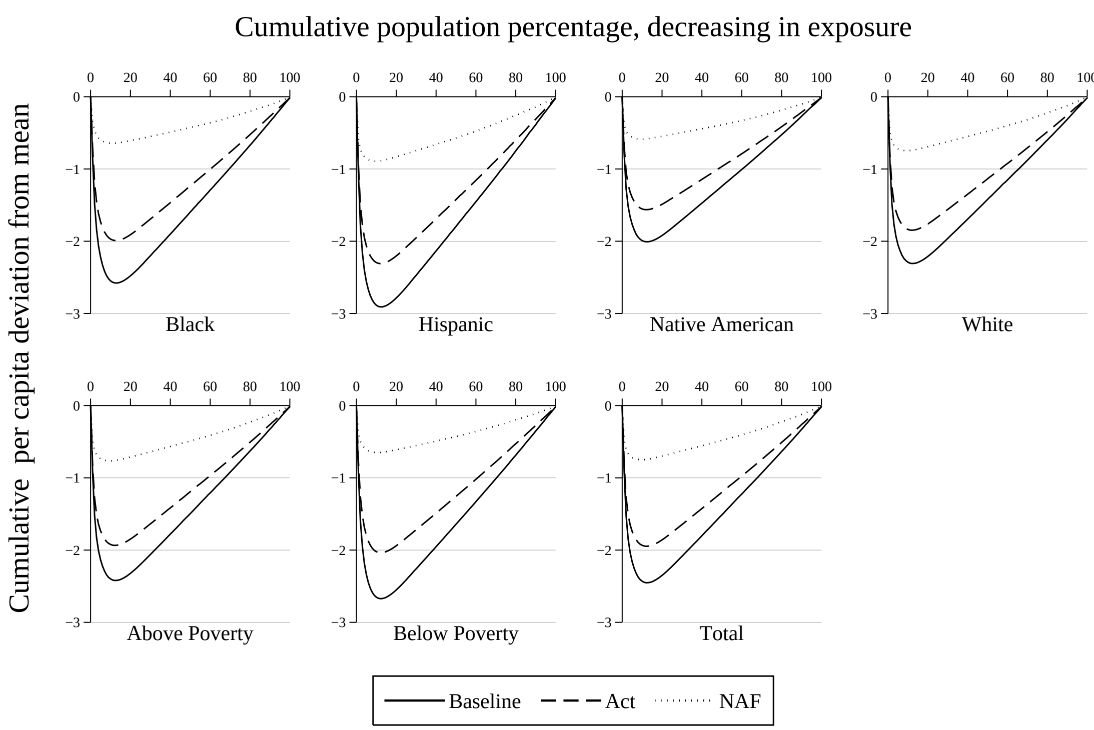


Figure 11: Generalized Lorenz curves for formaldehyde-induced cancer risk in cases/10^6

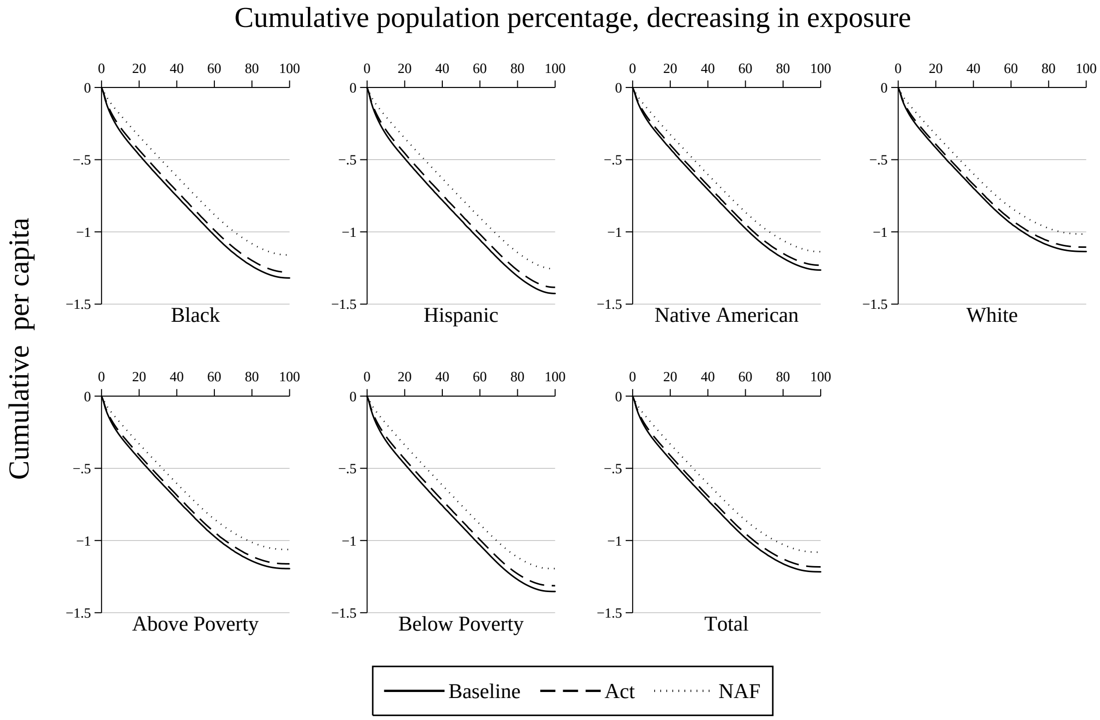


Figure 12: Relative Lorenz curves for formaldehyde-induced cancer risk

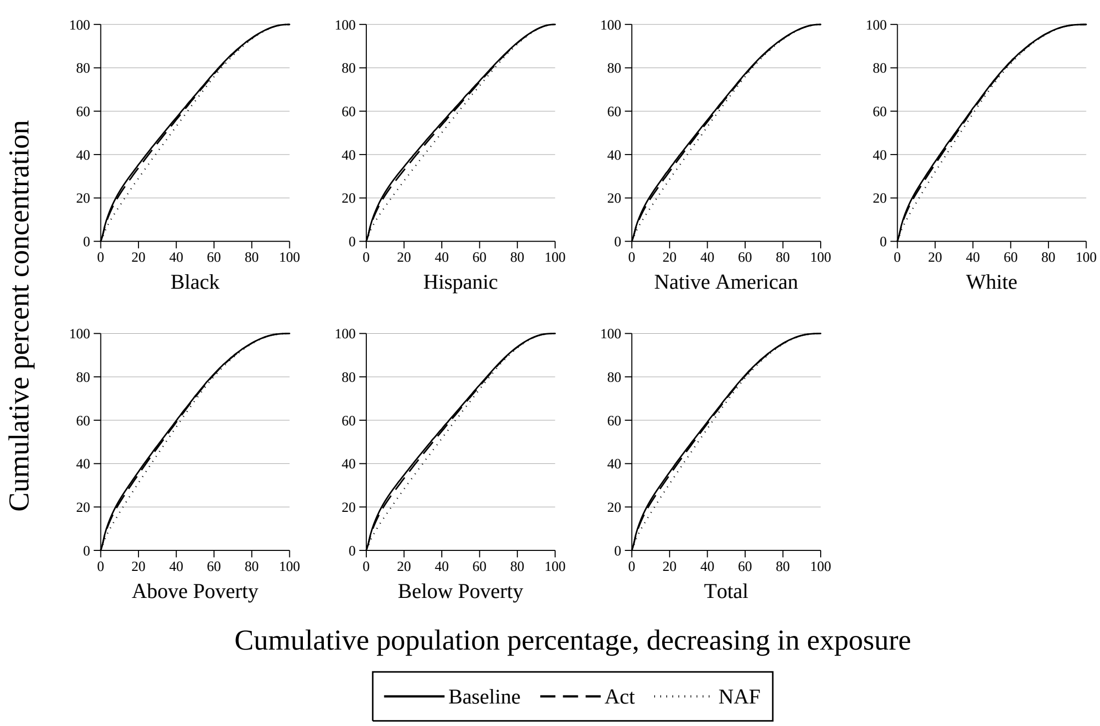


Figure 13: Absolute Lorenz curves for formaldehyde-induced cancer risk in cases/10^6

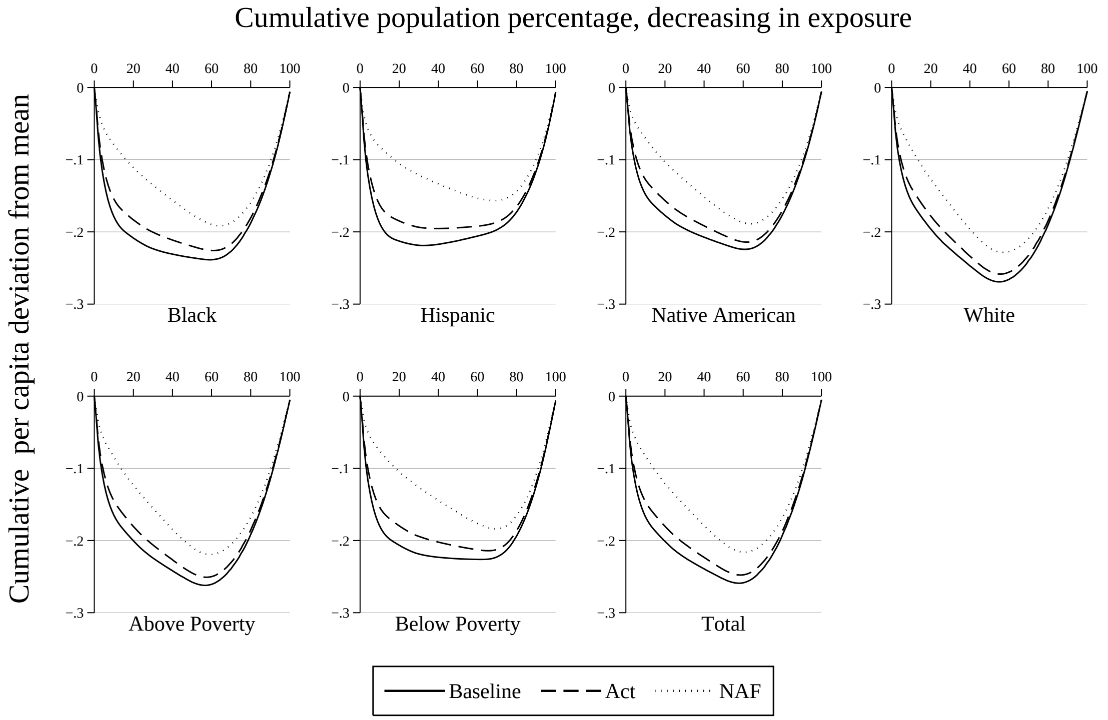


Figure 14: Generalized Lorenz curves for monetized health impact from formaldehyde in dollars

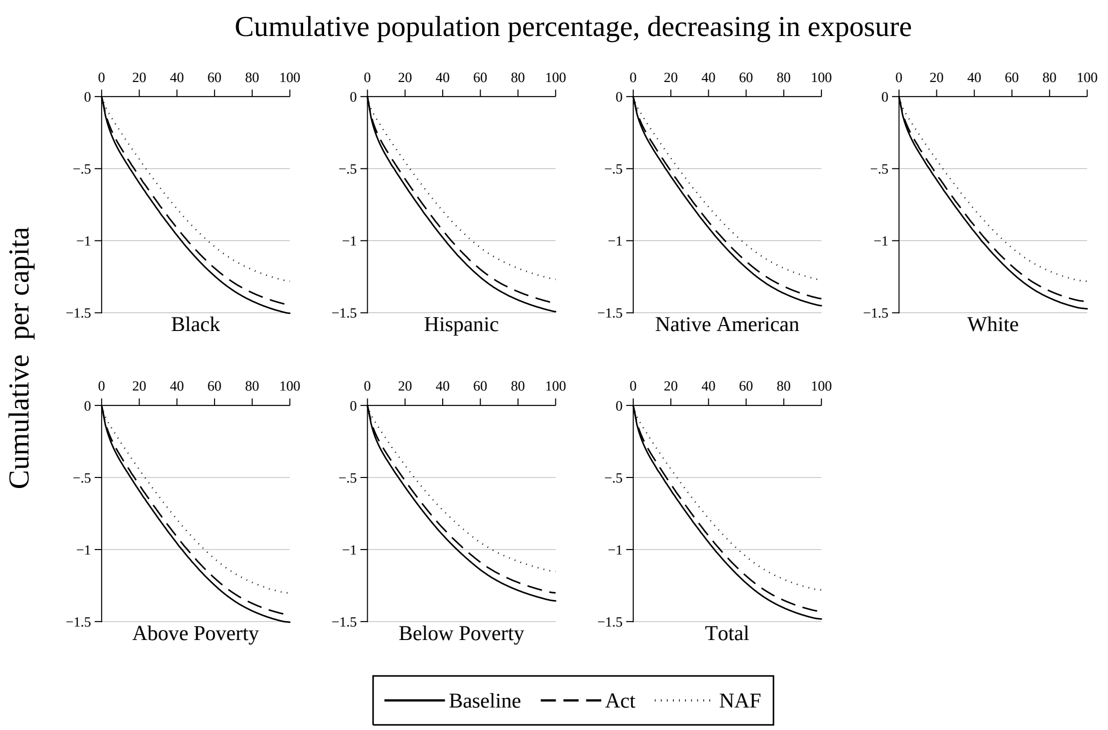


Figure 15: Relative Lorenz curves for monetized health impact from formaldehyde

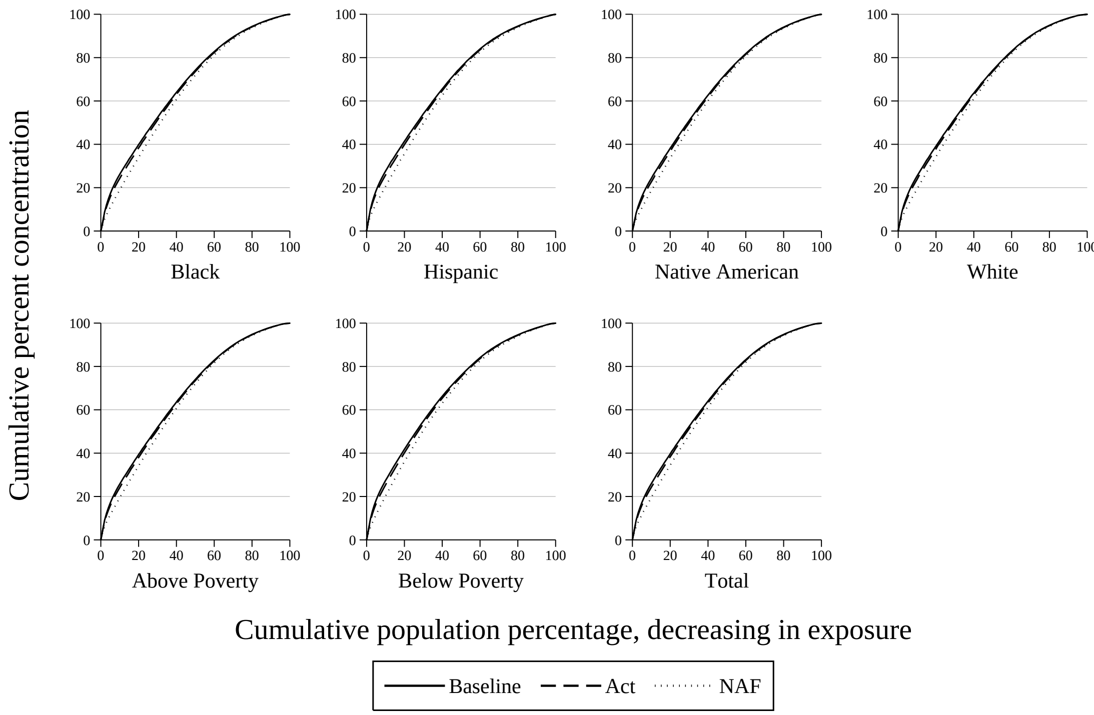
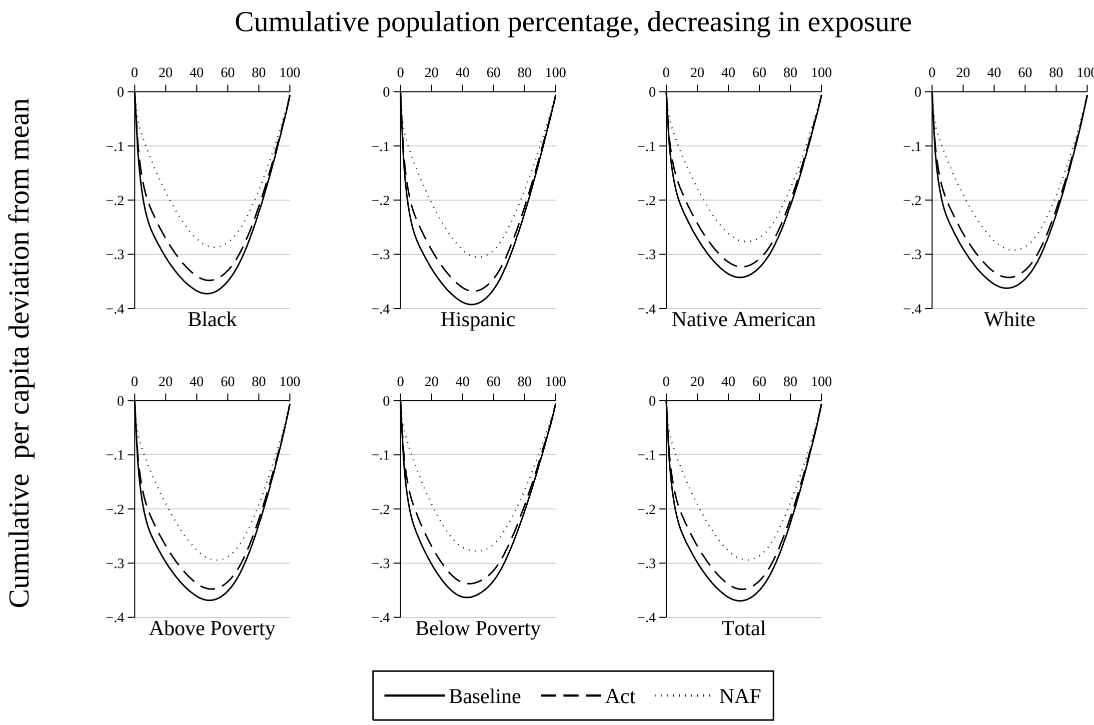


Figure 16: Absolute Lorenz curves for monetized health impact from formaldehyde in dollars



That is, current formaldehyde exposure is assumed to affect the distribution of future cancer risk conditional on an individual's current age. This analysis of cancer risk does not make a distinction between fatal and non-fatal cancers nor expected date of diagnosis.

Figure 11 shows that the Act causes an unambiguous improvement in cancer risk for each population group relative to baseline and that the NAF scenario represents further improvement. Unlike the case with irritation, the distribution for the White population group clearly dominates that of the other racial groups. Similarly, the Above Poverty income group dominates the Below Poverty group for cancer risk. Little discernible gain regarding the relative distribution of cancer risk is apparent from Figure 12, however. In contrast, Figure 13 shows that all race and income groups experience a drop in absolute inequality. Interestingly, the AL curves for the White population group intersect those of the other racial groups, precluding a ranking of intra-group inequality by AL curve dominance. The AL curves for the two income groups similarly intersect.

Finally, we analyze the distribution of monetized damages. For each health outcome (irritation, nonfatal cancer, and fatal cancer) the EPA calculates a willingness to pay to reduce risk. The present value of monetized risk of future cancer cases is calculated using a 7 percent discount rate. These monetized values can be thought of as a means of weighting the various health outcomes to generate an aggregate index. As a welfare measure, they impose the strong assumption that marginal willingness to pay is constant for each outcome regardless of the magnitude of the risk, and thus that the marginal rate of substitution across outcomes is also constant (i.e., all outcomes are perfect substitutes).[27]

Given the differences in health outcomes, the GL curves for monetized damages are remarkably similar across racial groups. Interestingly, in spite of having a worse distribution for both irritation and cancer risk, the Below Poverty income group dominates Above Poverty for monetized damages. Such a reversal is possible due to differences in age distribution between the two groups and the effects of discounting and the timing of future cancer cases.

While Lorenz curves require relatively few restrictions on social preferences, it is difficult to

[27]Since such assumptions are commonly used in benefit-cost analysis, however, we comment no further upon them here.

draw definitive conclusions from this purely graphical information. We now examine inequality index analysis as an alternative.

4.2 Inequality index analysis

Tables 2 and 3 present results for the I_{A-} and I_K evaluated at $\alpha = 0.25$, a value commonly used in the income inequality literature, and κ calculated using Eq. (18).[28] Similar to the Lorenz curve analysis, we present results for formaldehyde concentrations, health outcomes, and monetized benefits across demographic groups. By construction, rankings generated by the EDEs of these measures are equivalent, and consistent with GL dominance, if GL curves do not intersect. Similarly, I_{A-} generates the same ranking of pure inequality as RL dominance, and I_K generates the same ranking as AL dominance for cases in which those curves do not cross. In addition to the advantage of facilitating a complete, rather than a partial ordering of distributions, presenting distributional information in tables such as these is much more compact than the Lorenz figures.

By glancing at the first three columns in Tables 2 and 3, the reader can immediately determine the average outcome for each group and determine how this changes across policy scenarios, demographic groups, and outcomes. This information is also present in the GL curves, the average outcome is the height of the GL curve at the 100th percentile, but is more difficult to discern and compare. In addition, the change in mean monetized benefits between policy scenarios for the Total demographic group correspond to the gross per capita monetized benefits used in a conventional benefit-cost analysis.[29]

The second three columns present the EDEs for I_{A-} and I_K, which can be interpreted as adjusting the average outcomes in the first three columns for the social welfare loss caused by any inequality in the distributions. The rankings implicit in several pages of GL curves is condensed to these columns. The change in monetized EDE values across policy scenarios for the Total demographic group has the interpretation of equity adjusted gross benefits.

[28] The appendix presents tables for α=0.1 and 0.5.

[29] Our results are not equal to the monetized benefits presented in the Economic Analysis of the EPA's proposed rule since we constructed a different monetized estimate as detailed in footnote 24.

Table 2: Formaldehyde outcome distributions, I_{A-}, $\alpha = 0.25$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-11.13	-10.76	-9.68	0.02	0.02	0.01
Hispanic	-10.96	-10.63	-9.66	-11.21	-10.84	-9.74	0.02	0.02	0.01
Native American	-10.54	-10.27	-9.49	-10.72	-10.40	-9.55	0.02	0.01	0.01
White	-10.73	-10.45	-9.62	-10.93	-10.61	-9.68	0.02	0.02	0.01
Other	-10.96	-10.64	-9.65	-11.19	-10.82	-9.71	0.02	0.02	0.01
Between race							0.0254	0.0210	0.0090
Above Poverty	-10.81	-10.52	-9.65	-11.03	-10.69	-9.72	0.02	0.02	0.01
Below Poverty	-10.72	-10.40	-9.48	-10.95	-10.58	-9.53	0.02	0.02	0.01
Income unknown	-10.07	-9.79	-9.00	-10.24	-9.92	-9.03	0.02	0.01	0.00
Between income							0.0000	0.0000	0.0000
Total	-10.80	-10.50	-9.63	-11.01	-10.68	-9.69	0.02	0.02	0.01
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-7.21	-6.08	-3.59	0.29	0.25	0.14
Hispanic	-5.96	-5.21	-3.42	-7.93	-6.79	-4.18	0.33	0.30	0.22
Native American	-4.87	-4.31	-3.07	-6.19	-5.34	-3.55	0.27	0.24	0.16
White	-5.25	-4.66	-3.26	-6.82	-5.94	-3.89	0.30	0.27	0.19
Other	-5.67	-4.93	-3.17	-7.25	-6.13	-3.57	0.28	0.24	0.12
Between race							0.3506	0.3165	0.2059
Above Poverty	-5.39	-4.77	-3.28	-7.02	-6.09	-3.92	0.30	0.28	0.19
Below Poverty	-5.65	-4.87	-3.15	-7.40	-6.18	-3.62	0.31	0.27	0.15
Income unknown	-4.71	-4.09	-2.73	-5.80	-4.85	-2.85	0.23	0.19	0.04
Between income							0.0001	0.0000	0.0001
Total	-5.42	-4.78	-3.26	-7.06	-6.10	-3.88	0.30	0.28	0.19
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.38	-1.33	-1.19	0.04	0.04	0.03
Hispanic	-1.43	-1.38	-1.26	-1.48	-1.43	-1.28	0.04	0.03	0.02
Native American	-1.26	-1.23	-1.14	-1.31	-1.28	-1.17	0.04	0.04	0.03
White	-1.14	-1.11	-1.02	-1.20	-1.17	-1.06	0.06	0.05	0.04
Other	-1.34	-1.30	-1.18	-1.40	-1.35	-1.21	0.04	0.04	0.03
Between race							0.0825	0.0780	0.0654
Above Poverty	-1.19	-1.16	-1.06	-1.26	-1.22	-1.10	0.05	0.05	0.04
Below Poverty	-1.35	-1.31	-1.19	-1.41	-1.36	-1.23	0.04	0.04	0.03
Income unknown	-1.59	-1.55	-1.42	-1.62	-1.57	-1.43	0.02	0.01	0.00
Between income							0.0002	0.0002	0.0002
Total	-1.22	-1.18	-1.08	-1.28	-1.24	-1.12	0.05	0.05	0.04
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.60	-1.53	-1.34	0.06	0.06	0.04
Hispanic	-1.49	-1.43	-1.27	-1.61	-1.53	-1.34	0.08	0.07	0.05
Native American	-1.45	-1.40	-1.27	-1.54	-1.48	-1.33	0.06	0.05	0.04
White	-1.47	-1.42	-1.28	-1.57	-1.51	-1.34	0.07	0.06	0.05
Other	-1.52	-1.46	-1.29	-1.62	-1.55	-1.35	0.07	0.06	0.04
Between race							0.0944	0.0878	0.0704
Above Poverty	-1.50	-1.45	-1.30	-1.60	-1.54	-1.36	0.07	0.06	0.05
Below Poverty	-1.36	-1.30	-1.15	-1.46	-1.39	-1.21	0.07	0.07	0.05
Income unknown	-0.64	-0.61	-0.53	-0.67	-0.63	-0.54	0.05	0.04	0.01
Between income							0.0003	0.0003	0.0004
Total	-1.48	-1.43	-1.28	-1.58	-1.52	-1.34	0.07	0.06	0.05

Table 3: Formaldehyde outcome distributions, I_K, $\alpha = 0.25$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-11.24	-10.84	-9.70	0.34	0.26	0.08
Hispanic	-10.96	-10.63	-9.66	-11.37	-10.96	-9.80	0.41	0.32	0.13
Native American	-10.54	-10.27	-9.49	-10.81	-10.47	-9.58	0.27	0.21	0.08
White	-10.73	-10.45	-9.62	-11.05	-10.70	-9.73	0.32	0.26	0.11
Other	-10.96	-10.64	-9.65	-11.31	-10.90	-9.73	0.34	0.26	0.08
Between race							0.0002	0.0001	0.0000
Above Poverty	-10.81	-10.52	-9.65	-11.15	-10.79	-9.76	0.34	0.27	0.11
Below Poverty	-10.72	-10.40	-9.48	-11.07	-10.66	-9.56	0.35	0.26	0.09
Income unknown	-10.07	-9.79	-9.00	-10.29	-9.94	-9.03	0.22	0.16	0.03
Between income							0.0000	0.0000	0.0001
Total	-10.80	-10.50	-9.63	-11.14	-10.77	-9.73	0.34	0.27	0.11
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-6.55	-5.49	-3.40	0.96	0.64	0.24
Hispanic	-5.96	-5.21	-3.42	-7.36	-6.23	-3.92	1.40	1.02	0.50
Native American	-4.87	-4.31	-3.07	-5.68	-4.89	-3.37	0.82	0.58	0.30
White	-5.25	-4.66	-3.26	-6.30	-5.45	-3.67	1.05	0.79	0.41
Other	-5.67	-4.93	-3.17	-6.55	-5.52	-3.38	0.88	0.58	0.20
Between race							0.0004	0.0002	0.0001
Above Poverty	-5.39	-4.77	-3.28	-6.47	-5.58	-3.69	1.08	0.81	0.41
Below Poverty	-5.65	-4.87	-3.15	-6.77	-5.58	-3.43	1.12	0.72	0.28
Income unknown	-4.71	-4.09	-2.73	-5.08	-4.29	-2.74	0.37	0.20	0.01
Between income							0.0000	0.0000	0.0000
Total	-5.42	-4.78	-3.26	-6.50	-5.58	-3.65	1.08	0.79	0.39
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.38	-1.33	-1.18	0.06	0.05	0.02
Hispanic	-1.43	-1.38	-1.26	-1.50	-1.44	-1.28	0.07	0.06	0.03
Native American	-1.26	-1.23	-1.14	-1.32	-1.27	-1.16	0.05	0.04	0.02
White	-1.14	-1.11	-1.02	-1.20	-1.16	-1.05	0.06	0.05	0.03
Other	-1.34	-1.30	-1.18	-1.40	-1.35	-1.20	0.06	0.05	0.02
Between race							0.0011	0.0010	0.0007
Above Poverty	-1.19	-1.16	-1.06	-1.26	-1.21	-1.09	0.06	0.05	0.03
Below Poverty	-1.35	-1.31	-1.19	-1.42	-1.37	-1.22	0.07	0.05	0.03
Income unknown	-1.59	-1.55	-1.42	-1.64	-1.58	-1.43	0.04	0.03	0.01
Between income							0.0003	0.0002	0.0002
Total	-1.22	-1.18	-1.08	-1.28	-1.24	-1.11	0.06	0.05	0.03
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.62	-1.54	-1.33	0.12	0.09	0.05
Hispanic	-1.49	-1.43	-1.27	-1.65	-1.57	-1.35	0.16	0.13	0.08
Native American	-1.45	-1.40	-1.27	-1.56	-1.49	-1.33	0.11	0.09	0.05
White	-1.47	-1.42	-1.28	-1.60	-1.53	-1.35	0.13	0.11	0.06
Other	-1.52	-1.46	-1.29	-1.64	-1.55	-1.34	0.12	0.09	0.05
Between race							0.0000	0.0000	0.0000
Above Poverty	-1.50	-1.45	-1.30	-1.64	-1.56	-1.37	0.13	0.11	0.06
Below Poverty	-1.36	-1.30	-1.15	-1.48	-1.40	-1.20	0.12	0.09	0.05
Income unknown	-0.64	-0.61	-0.53	-0.65	-0.62	-0.53	0.01	0.01	0.00
Between income							0.0003	0.0003	0.0003
Total	-1.48	-1.43	-1.28	-1.61	-1.54	-1.34	0.13	0.11	0.06

Consider the outcome of eye irritation risk for example. Using I_{A-}, under the statutory standards, society would be willing to accept at most 6 additional cases per thousand to have a perfectly equal distribution across the population. The stricter NAF standard is even more equitable; society would only be willing to accept 4 more cases per thousand. Both options are more equitable than the baseline. Looking across race and income, we find similar results. Equity improves most for Black, Hispanic, and Below poverty.

In addition, the last line in Table 3, for example, shows that a society that does not care about equity would be willing to pay 5 cents per capita to move from baseline to the standards required by the Act, and an additional 15 cents to achieve the NAF outcome. In contrast, a society with Kolm-Pollak preferences parameterized at $\alpha = 0.25$ would be willing to charge each person at most 7 cents and an additional 20 cents, respectively to attain the Act and NAF standards.[30] For I_{A-}, the social willingness to pay is respectively 6 and 24 cents per person.[31]

The final three columns allow one to analyze the equity of the distribution independent of the mean. This information is useful for cases in which RL or AL curves intersect. For example, it is not clear from Figure 13 whether the Hispanic or White group has a more equitable distribution of cancer risk using I_K. Table 3 shows that the equity of the White distribution dominates the other racial groups for this outcome.[32] Similarly, although the AL curves intersect in Figure 13 Above Poverty dominates Below Poverty using I_K. For I_K, using Eq. (10), the total willingness to pay, in units of the outcome variable, to move from one policy scenario to another for any population group can easily be decomposed into $\Delta\Xi_K^m = \Delta\bar{x}_m - \Delta I_K^m$.[33]

Inequality indexes allow one to gauge not only the relative inequality of a distribution of outcomes within each group, but also to use the Blackorby, Donaldson, and Auersperg (1981) approach to evaluate inequality across demographic groups. The Between Race and Between

[30]The fact that I_K does not change by adding a constant to each individual outcome allows this comparison. That is, $\Xi_K(\mathbf{x} - c\mathbf{1}) = \Xi_K(\mathbf{x}) - c$.

[31]For social preferences consistent with I_{A-}, the finance mechanism that would not alter the current income distribution would be to charge everyone a fixed proportion of their income such that the average charge is equal to the per capita willingness to pay figures.

[32]Appendix Tables A.3 and A.4 show that this result is not sensitive to α.

[33]For I_{A-}, this decomposition is less intuitive, the change in EDE from policy 1 to policy 2 is $\Delta\Xi_{A-}^m = \Delta\bar{x}_m + \bar{x}_{m2}I_{2A-}^m - \bar{x}_{m1}I_{1A-}^m$.

Income lines identify the remaining inequality for the total population if each member of each population group were to receive that group's EDE outcome rather than her actual outcome. For I_K, we see that *inter*-group inequality has small declines as the standards become more stringent. For I_{A-} the pattern is similar, with declines in inter-race inequality being more pronounced than the the declines in inter-income inequality.

5 Conclusion

Inequality indexes and their associated social evaluation functions are useful tools for characterizing distributions of adverse environmental or health outcomes. Grounded on an internally consistent set of social preferences that satisfy reasonable properties, they provide a transparent method for ranking outcomes of alternative policy options that takes entire distributions into account, not just means. They also facilitate easy comparisons of distributions of outcomes both within and across population groupings of EJ concern.

Their ability to provide normatively meaningful rankings provides clear advantages over other statistics (variance, correlations, regression coefficients, etc.) for benefit-cost analysis. However, inequality indexes may also have value as descriptive statistics presented in other analytical contexts. Empirical studies (e.g., Banzhaf and Walsh, 2008; Gamper-Rabindran and Timmins, 2011) have begun to examine the effects of environmental quality on residential sorting, for example. Presentation of EDEs and inter-group inequality measures over time for the analyzed outcomes (e.g., toxic releases, or proximity to Superfund sites) could provide the reader with more context as to the overall degree to which the distributions within and across groups changed during the period of study.

Similarly, other econometric research (for a recent example, see Fowlie, Holland, and Mansur, 2012) has examined the degree to which race or income is a predictor of increased exposure to pollution under alternative policy scenarios. In such studies, inclusion of EDEs could provide a sense of which population groups have the least desirable distributions under each policy before controlling for other factors. Inequality indexes for different demographic groups could also provide clues as to the whether exposure becomes more concentrated under different policies. It may be of little solace to community leaders to learn that their ethnicity was not a statistically significant predictor of changes in exposure from one policy to another, for example, if it were due to the fact that the second policy caused a regressive reallocation of exposure within the community.

Caution should be exercised, however, when using inequality indexes designed for analyzing the distribution of income to examine environmental or health outcomes. In particular, the standard Atkinson index loses its desirable properties if used to analyze distributions of bads rather than goods. We have shown how to transform this index in a manner that permits analysis of distributions consisting only of bads. It is important to note that this transformation does not allow the analysis of outcome vectors that contain both goods and bads. Although the example considered here focused exclusively on bad outcomes, in principle one may want to analyze distributions of an outcome vector that could take positive or negative values (e.g., a money metric welfare indicator that subtracts environmental harm from monetary income). The Kolm-Pollak index, although rarely used even in the context of income distribution, would be particularly useful in such contexts as it easily accommodates all real-valued outcome vectors.

As an illustrative example, we apply the inequality index approach, alongside Lorenz curve analysis, to evaluate the distributional consequences of recent legislation setting emission standards for formaldehyde released from composite wood products. This statute provides an interesting case study since it was partly motivated by concern for poor and minority communities exposed to formaldehyde in emergency trailers. Since indoor formaldehyde levels tend to be highest in new and newly remodeled homes, however, it may have been the case that less economically advantaged groups would not benefit from the standards.

Using data generated from a hybrid engineering/epidemiological/valuation model developed by the EPA, we analyzed the baseline distribution of formaldehyde emissions, two health outcomes, and the monetized value of health outcomes under three policy scenarios. Results suggest that at baseline, individuals above poverty indeed had worse distributions of formaldehyde concentrations and monetized health losses, but that people below poverty had worse distributions of undiscounted health outcomes (cases of eye irritation and cancer). In terms of race, Hispanics generally fared the worst for all outcomes, and Native Americans fared the best.

We found no tradeoffs between gross benefits and equity; stricter standards resulted in more equal distributions for each outcome and demographic group. Moreover, stricter standards increased equity both within and between population groups.

If one is willing to make the assumption (common in applied benefit-cost analysis) that marginal willingness-to-pay to avoid an adverse health outcome is constant, one can compare the change in EDE with the per capita change in cost to determine whether the incremental benefits of a policy alternative, adjusted for equity, are worth the incremental cost. Due to the increased equity of the distribution, the legislated standards pass a benefit-cost test if per-capita costs are less than 7 cents per person using Kolm-Pollak preferences with an inequality aversion parameter of 0.25 versus 5 cents using preferences that do not value equity (i.e., with an inequality aversion parameter of zero).

EDE values depend upon the choice of inequality aversion parameter. As such it is standard practice in the income distribution literature to present results for a range of values. Since the parameter value can theoretically range from zero to infinity, a useful path for future research would be to undertake stated and revealed preference studies to better identify a reasonable range for this value in the context of social preferences over bads.[34]

Finally, here we focused on the point estimate of the EDE. Another path for future research would be to develop a better understanding of the statistical properties of the Kolm-Pollak index and EDE to enable analysts to develop confidence intervals and hypothesis tests using modeled data.[35]

[34]For experimental and survey evidence regarding inequality aversion parameter values for income see Amiel, Creedy, and Hurn (1999) and Pirtillä and Uusitalo (2010), among others.

[35]See Cowell (1989) and Thistle (1990) for derivation of sampling variance for the Atkinson index.

Appendix

Table A.1: Formaldehyde outcome distributions, I_{A-}, $\alpha = 0.10$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-10.99	-10.65	-9.64	0.01	0.01	0.00
Hispanic	-10.96	-10.63	-9.66	-11.05	-10.71	-9.69	0.01	0.01	0.00
Native American	-10.54	-10.27	-9.49	-10.61	-10.32	-9.51	0.01	0.01	0.00
White	-10.73	-10.45	-9.62	-10.81	-10.51	-9.64	0.01	0.01	0.00
Other	-10.96	-10.64	-9.65	-11.05	-10.71	-9.67	0.01	0.01	0.00
Between race							0.0101	0.0083	0.0036
Above Poverty	-10.81	-10.52	-9.65	-10.90	-10.59	-9.68	0.01	0.01	0.00
Below Poverty	-10.72	-10.40	-9.48	-10.80	-10.47	-9.50	0.01	0.01	0.00
Income unknown	-10.07	-9.79	-9.00	-10.13	-9.84	-9.01	0.01	0.01	0.00
Between income							0.0000	0.0000	0.0000
Total	-10.80	-10.50	-9.63	-10.88	-10.57	-9.65	0.01	0.01	0.00
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-6.16	-5.28	-3.30	0.10	0.09	0.04
Hispanic	-5.96	-5.21	-3.42	-6.65	-5.75	-3.67	0.12	0.10	0.07
Native American	-4.87	-4.31	-3.07	-5.32	-4.66	-3.22	0.09	0.08	0.05
White	-5.25	-4.66	-3.26	-5.79	-5.10	-3.46	0.10	0.09	0.06
Other	-5.67	-4.93	-3.17	-6.23	-5.35	-3.30	0.10	0.09	0.04
Between race							0.1355	0.1218	0.0769
Above Poverty	-5.39	-4.77	-3.28	-5.95	-5.22	-3.49	0.10	0.09	0.06
Below Poverty	-5.65	-4.87	-3.15	-6.26	-5.32	-3.30	0.11	0.09	0.05
Income unknown	-4.71	-4.09	-2.73	-5.10	-4.36	-2.77	0.08	0.07	0.02
Between income							0.0000	0.0000	0.0000
Total	-5.42	-4.78	-3.26	-5.99	-5.23	-3.46	0.10	0.09	0.06
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.34	-1.30	-1.17	0.02	0.02	0.01
Hispanic	-1.43	-1.38	-1.26	-1.45	-1.40	-1.27	0.02	0.01	0.01
Native American	-1.26	-1.23	-1.14	-1.28	-1.25	-1.15	0.02	0.01	0.01
White	-1.14	-1.11	-1.02	-1.16	-1.13	-1.03	0.02	0.02	0.02
Other	-1.34	-1.30	-1.18	-1.36	-1.32	-1.19	0.02	0.02	0.01
Between race							0.0322	0.0304	0.0255
Above Poverty	-1.19	-1.16	-1.06	-1.22	-1.19	-1.08	0.02	0.02	0.02
Below Poverty	-1.35	-1.31	-1.19	-1.38	-1.33	-1.21	0.02	0.02	0.01
Income unknown	-1.59	-1.55	-1.42	-1.60	-1.56	-1.43	0.01	0.01	0.00
Between income							0.0001	0.0001	0.0001
Total	-1.22	-1.18	-1.08	-1.24	-1.21	-1.10	0.02	0.02	0.02
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.54	-1.48	-1.30	0.03	0.02	0.02
Hispanic	-1.49	-1.43	-1.27	-1.54	-1.47	-1.30	0.03	0.03	0.02
Native American	-1.45	-1.40	-1.27	-1.48	-1.43	-1.29	0.02	0.02	0.02
White	-1.47	-1.42	-1.28	-1.51	-1.46	-1.31	0.03	0.02	0.02
Other	-1.52	-1.46	-1.29	-1.56	-1.49	-1.31	0.03	0.02	0.02
Between race							0.0370	0.0344	0.0276
Above Poverty	-1.50	-1.45	-1.30	-1.54	-1.49	-1.33	0.03	0.02	0.02
Below Poverty	-1.36	-1.30	-1.15	-1.40	-1.33	-1.17	0.03	0.03	0.02
Income unknown	-0.64	-0.61	-0.53	-0.65	-0.61	-0.53	0.02	0.01	0.01
Between income							0.0001	0.0001	0.0002
Total	-1.48	-1.43	-1.28	-1.52	-1.46	-1.30	0.03	0.02	0.02

Table A.2: Formaldehyde outcome distributions, I_{A-}, $\alpha = 0.50$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-11.40	-10.97	-9.75	0.05	0.04	0.01
Hispanic	-10.96	-10.63	-9.66	-11.52	-11.09	-9.85	0.05	0.04	0.02
Native American	-10.54	-10.27	-9.49	-10.93	-10.57	-9.61	0.04	0.03	0.01
White	-10.73	-10.45	-9.62	-11.18	-10.81	-9.77	0.04	0.03	0.02
Other	-10.96	-10.64	-9.65	-11.47	-11.04	-9.77	0.05	0.04	0.01
Between race							0.0519	0.0430	0.0186
Above Poverty	-10.81	-10.52	-9.65	-11.29	-10.90	-9.81	0.04	0.04	0.02
Below Poverty	-10.72	-10.40	-9.48	-11.22	-10.79	-9.60	0.05	0.04	0.01
Income unknown	-10.07	-9.79	-9.00	-10.44	-10.07	-9.06	0.04	0.03	0.01
Between income							0.0000	0.0000	0.0000
Total	-10.80	-10.50	-9.63	-11.27	-10.89	-9.78	0.04	0.04	0.02
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-9.54	-7.91	-4.42	0.71	0.63	0.40
Hispanic	-5.96	-5.21	-3.42	-10.78	-9.19	-5.60	0.81	0.76	0.64
Native American	-4.87	-4.31	-3.07	-8.22	-7.01	-4.51	0.69	0.63	0.47
White	-5.25	-4.66	-3.26	-9.20	-7.96	-5.12	0.75	0.71	0.57
Other	-5.67	-4.93	-3.17	-9.49	-7.88	-4.30	0.67	0.60	0.36
Between race							0.7139	0.6577	0.4772
Above Poverty	-5.39	-4.77	-3.28	-9.44	-8.15	-5.14	0.75	0.71	0.57
Below Poverty	-5.65	-4.87	-3.15	-9.93	-8.15	-4.54	0.76	0.67	0.44
Income unknown	-4.71	-4.09	-2.73	-7.32	-5.91	-3.01	0.55	0.45	0.10
Between income							0.0001	0.0001	0.0006
Total	-5.42	-4.78	-3.26	-9.50	-8.14	-5.06	0.75	0.70	0.55
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.44	-1.38	-1.22	0.09	0.08	0.05
Hispanic	-1.43	-1.38	-1.26	-1.54	-1.48	-1.31	0.08	0.07	0.05
Native American	-1.26	-1.23	-1.14	-1.37	-1.32	-1.20	0.08	0.07	0.05
White	-1.14	-1.11	-1.02	-1.27	-1.22	-1.10	0.11	0.11	0.08
Other	-1.34	-1.30	-1.18	-1.46	-1.40	-1.24	0.09	0.08	0.05
Between race							0.1753	0.1660	0.1397
Above Poverty	-1.19	-1.16	-1.06	-1.32	-1.28	-1.14	0.11	0.10	0.08
Below Poverty	-1.35	-1.31	-1.19	-1.47	-1.42	-1.26	0.09	0.08	0.05
Income unknown	-1.59	-1.55	-1.42	-1.65	-1.59	-1.44	0.04	0.03	0.01
Between income							0.0004	0.0004	0.0003
Total	-1.22	-1.18	-1.08	-1.34	-1.30	-1.16	0.10	0.10	0.07
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.71	-1.62	-1.39	0.14	0.12	0.09
Hispanic	-1.49	-1.43	-1.27	-1.74	-1.65	-1.41	0.16	0.15	0.11
Native American	-1.45	-1.40	-1.27	-1.63	-1.56	-1.38	0.13	0.11	0.09
White	-1.47	-1.42	-1.28	-1.68	-1.61	-1.41	0.14	0.13	0.10
Other	-1.52	-1.46	-1.29	-1.73	-1.64	-1.40	0.14	0.13	0.09
Between race							0.1969	0.1834	0.1472
Above Poverty	-1.50	-1.45	-1.30	-1.72	-1.64	-1.43	0.14	0.13	0.10
Below Poverty	-1.36	-1.30	-1.15	-1.57	-1.49	-1.27	0.16	0.14	0.10
Income unknown	-0.64	-0.61	-0.53	-0.70	-0.66	-0.55	0.11	0.08	0.03
Between income							0.0005	0.0006	0.0007
Total	-1.48	-1.43	-1.28	-1.70	-1.62	-1.41	0.14	0.13	0.10

Table A.3: Formaldehyde outcome distributions, I_K, $\alpha = 0.10$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-11.02	-10.67	-9.65	0.12	0.09	0.03
Hispanic	-10.96	-10.63	-9.66	-11.10	-10.75	-9.71	0.14	0.11	0.04
Native American	-10.54	-10.27	-9.49	-10.63	-10.34	-9.52	0.09	0.07	0.03
White	-10.73	-10.45	-9.62	-10.84	-10.54	-9.65	0.11	0.09	0.04
Other	-10.96	-10.64	-9.65	-11.09	-10.73	-9.68	0.12	0.09	0.03
Between race							0.0001	0.0000	0.0000
Above Poverty	-10.81	-10.52	-9.65	-10.93	-10.61	-9.69	0.12	0.09	0.04
Below Poverty	-10.72	-10.40	-9.48	-10.84	-10.49	-9.51	0.12	0.09	0.03
Income unknown	-10.07	-9.79	-9.00	-10.15	-9.84	-9.01	0.08	0.06	0.01
Between income							0.0000	0.0000	0.0000
Total	-10.80	-10.50	-9.63	-10.92	-10.60	-9.66	0.12	0.09	0.04
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-5.89	-5.06	-3.23	0.30	0.21	0.07
Hispanic	-5.96	-5.21	-3.42	-6.39	-5.53	-3.57	0.43	0.32	0.15
Native American	-4.87	-4.31	-3.07	-5.11	-4.49	-3.16	0.25	0.18	0.09
White	-5.25	-4.66	-3.26	-5.57	-4.91	-3.38	0.32	0.25	0.12
Other	-5.67	-4.93	-3.17	-5.95	-5.12	-3.24	0.28	0.19	0.06
Between race							0.0001	0.0001	0.0000
Above Poverty	-5.39	-4.77	-3.28	-5.72	-5.02	-3.41	0.33	0.25	0.12
Below Poverty	-5.65	-4.87	-3.15	-6.00	-5.10	-3.24	0.35	0.23	0.08
Income unknown	-4.71	-4.09	-2.73	-4.84	-4.17	-2.73	0.13	0.08	0.01
Between income							0.0000	0.0000	0.0000
Total	-5.42	-4.78	-3.26	-5.75	-5.03	-3.38	0.33	0.25	0.12
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.34	-1.30	-1.17	0.02	0.02	0.01
Hispanic	-1.43	-1.38	-1.26	-1.45	-1.40	-1.27	0.03	0.02	0.01
Native American	-1.26	-1.23	-1.14	-1.28	-1.25	-1.15	0.02	0.02	0.01
White	-1.14	-1.11	-1.02	-1.16	-1.12	-1.03	0.02	0.02	0.01
Other	-1.34	-1.30	-1.18	-1.37	-1.32	-1.19	0.02	0.02	0.01
Between race							0.0004	0.0004	0.0003
Above Poverty	-1.19	-1.16	-1.06	-1.22	-1.18	-1.07	0.02	0.02	0.01
Below Poverty	-1.35	-1.31	-1.19	-1.38	-1.33	-1.20	0.02	0.02	0.01
Income unknown	-1.59	-1.55	-1.42	-1.61	-1.56	-1.43	0.01	0.01	0.00
Between income							0.0001	0.0001	0.0001
Total	-1.22	-1.18	-1.08	-1.24	-1.20	-1.09	0.02	0.02	0.01
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.54	-1.48	-1.30	0.04	0.03	0.02
Hispanic	-1.49	-1.43	-1.27	-1.54	-1.48	-1.29	0.05	0.04	0.02
Native American	-1.45	-1.40	-1.27	-1.49	-1.43	-1.29	0.03	0.03	0.02
White	-1.47	-1.42	-1.28	-1.51	-1.46	-1.30	0.04	0.03	0.02
Other	-1.52	-1.46	-1.29	-1.56	-1.49	-1.31	0.04	0.03	0.02
Between race							0.0000	0.0000	0.0000
Above Poverty	-1.50	-1.45	-1.30	-1.55	-1.49	-1.32	0.04	0.04	0.02
Below Poverty	-1.36	-1.30	-1.15	-1.40	-1.33	-1.17	0.04	0.03	0.02
Income unknown	-0.64	-0.61	-0.53	-0.64	-0.61	-0.53	0.01	0.00	0.00
Between income							0.0001	0.0001	0.0001
Total	-1.48	-1.43	-1.28	-1.52	-1.46	-1.30	0.04	0.03	0.02

Table A.4: Formaldehyde outcome distributions, I_K, $\alpha = 0.50$

	Mean			EDE			Index		
	Base	Act	NAF	Base	Act	NAF	Base	Act	NAF
Home formaldehyde concentration in $\mu g/m^3$									
Black	-10.90	-10.58	-9.62	-11.77	-11.23	-9.84	0.86	0.65	0.22
Hispanic	-10.96	-10.63	-9.66	-12.04	-11.49	-10.05	1.08	0.86	0.38
Native American	-10.54	-10.27	-9.49	-11.24	-10.80	-9.73	0.70	0.54	0.24
White	-10.73	-10.45	-9.62	-11.58	-11.13	-9.93	0.85	0.68	0.32
Other	-10.96	-10.64	-9.65	-11.81	-11.28	-9.85	0.85	0.64	0.20
Between race							0.0006	0.0004	0.0001
Above Poverty	-10.81	-10.52	-9.65	-11.70	-11.23	-9.97	0.88	0.71	0.32
Below Poverty	-10.72	-10.40	-9.48	-11.62	-11.07	-9.71	0.90	0.67	0.24
Income unknown	-10.07	-9.79	-9.00	-10.58	-10.14	-9.06	0.51	0.36	0.06
Between income							0.0001	0.0001	0.0002
Total	-10.80	-10.50	-9.63	-11.68	-11.20	-9.93	0.89	0.70	0.31
Eye irritation in cases per 1000 individuals									
Black	-5.59	-4.85	-3.16	-9.18	-6.95	-3.99	3.59	2.10	0.83
Hispanic	-5.96	-5.21	-3.42	-11.63	-8.78	-5.12	5.67	3.58	1.70
Native American	-4.87	-4.31	-3.07	-8.01	-6.23	-4.10	3.14	1.92	1.03
White	-5.25	-4.66	-3.26	-9.35	-7.38	-4.66	4.10	2.72	1.41
Other	-5.67	-4.93	-3.17	-8.76	-6.80	-3.86	3.09	1.87	0.69
Between race							0.0043	0.0019	0.0007
Above Poverty	-5.39	-4.77	-3.28	-9.59	-7.55	-4.68	4.20	2.77	1.40
Below Poverty	-5.65	-4.87	-3.15	-10.05	-7.25	-4.10	4.40	2.38	0.95
Income unknown	-4.71	-4.09	-2.73	-5.63	-4.56	-2.76	0.92	0.47	0.03
Between income							0.0004	0.0002	0.0003
Total	-5.42	-4.78	-3.26	-9.63	-7.50	-4.60	4.21	2.72	1.33
Cancer risk in cases per 1,000,000 individuals									
Black	-1.32	-1.28	-1.16	-1.48	-1.40	-1.21	0.16	0.12	0.05
Hispanic	-1.43	-1.38	-1.26	-1.61	-1.53	-1.33	0.18	0.15	0.07
Native American	-1.26	-1.23	-1.14	-1.39	-1.33	-1.19	0.13	0.10	0.06
White	-1.14	-1.11	-1.02	-1.28	-1.23	-1.09	0.15	0.12	0.07
Other	-1.34	-1.30	-1.18	-1.49	-1.42	-1.23	0.15	0.12	0.05
Between race							0.0025	0.0021	0.0013
Above Poverty	-1.19	-1.16	-1.06	-1.35	-1.29	-1.13	0.15	0.13	0.07
Below Poverty	-1.35	-1.31	-1.19	-1.52	-1.44	-1.25	0.17	0.13	0.06
Income unknown	-1.59	-1.55	-1.42	-1.69	-1.62	-1.44	0.10	0.07	0.01
Between income							0.0006	0.0005	0.0003
Total	-1.22	-1.18	-1.08	-1.37	-1.31	-1.15	0.15	0.13	0.07
Monetized damages in dollars per person									
Black	-1.50	-1.45	-1.28	-1.89	-1.74	-1.44	0.39	0.30	0.15
Hispanic	-1.49	-1.43	-1.27	-2.08	-1.90	-1.54	0.59	0.47	0.27
Native American	-1.45	-1.40	-1.27	-1.82	-1.69	-1.46	0.37	0.29	0.18
White	-1.47	-1.42	-1.28	-1.92	-1.79	-1.50	0.45	0.36	0.22
Other	-1.52	-1.46	-1.29	-1.89	-1.74	-1.43	0.37	0.28	0.14
Between race							0.0004	0.0003	0.0001
Above Poverty	-1.50	-1.45	-1.30	-1.97	-1.83	-1.53	0.46	0.38	0.22
Below Poverty	-1.36	-1.30	-1.15	-1.77	-1.61	-1.32	0.41	0.31	0.17
Income unknown	-0.64	-0.61	-0.53	-0.67	-0.63	-0.53	0.03	0.02	0.00
Between income							0.0010	0.0010	0.0008
Total	-1.48	-1.43	-1.28	-1.94	-1.80	-1.50	0.46	0.37	0.22

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