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Working Paper # 10-07
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*Abstract*: The "social cost of carbon" (SCC) is the present value of the future damages from one additional unit of carbon emissions in a particular year. This paper develops a simple model for calculating the social cost of carbon. The model includes the essential ingredients for calculating the SCC at the global scale, and is designed to be transparent and easy to use by decision-makers and non-specialists. We use the model to compare estimates of the SCC under certainty and uncertainty in a Monte Carlo analysis. We find that, due to the combined effects of uncertainty and risk aversion, the certainty-equivalent SCC can be substantially larger than the expected value of the SCC. In our Monte Carlo simulation, the certainty-equivalent SCC corresponds to the 97th percentile of the simulated probability distribution of the deterministic SCC. We also compare the approximate present value of benefits estimated using the SCC to the exact value of compensating variation in the initial period for a wide range of hypothetical emission reduction policies.

*Keywords*: climate change, social cost of carbon, integrated assessment model

*Subject areas*: climate change, benefit-cost analysis

*JEL classification*: Q54

*Disclaimer*: The views expressed in this paper are those of the authors and do not necessarily represent those of the U.S. EPA. No Agency endorsement should be inferred.

# INTRODUCTION

The "social cost of carbon" (SCC) is a commonly estimated measure of the economic benefits of greenhouse gas (GHG) emission reductions (e.g., Tol 2005, 2008; Nordhaus 2008; Hope 2006, 2008; Anthoff *et al*. 2009a,b). The SCC represents the present value of the marginal social damages of increased GHG emissions in a particular year—including the impacts of global warming on agricultural productivity and human health, loss of property and infrastructure to sea level rise and extreme weather events, diminished biodiversity and ecosystem services, etc.—and therefore it also represents the marginal social benefits of emissions reductions. Properly defined, the SCC is the correct "shadow price" to place on GHG emissions in a benefit-cost or social welfare analysis of climate change policies. Abatement or mitigation policies that reduce or sequester GHGs at a per-unit cost lower than the SCC would pass a benefit-cost test; those that cost more would not. Furthermore, at the economically efficient level of emissions the SCC and the marginal cost of emissions abatement would be equal.[1]

The SCC typically is estimated using relatively complex dynamic optimization or simulation models that combine climate processes, economic growth, and feedbacks between the two in a single modeling framework, often referred to as "integrated assessment models" (IAMs). Many IAMs are designed mainly to estimate the economic costs of meeting pre-specified GHG concentration or surface temperature targets in some future year, and are therefore suitable for cost-effectiveness. Only a subset of IAMs explicitly model the economic damages of climate change impacts. These include early work by Cline (1992) and Fankhauser (1995), as well as more recent models such as the "Dynamic Integrated Climate and Economy" (DICE) model (Nordhaus and Boyer 2000, Nordhaus 2008), the "Climate Framework for Uncertainty, Negotiation, and Distribution" (FUND) model (Tol 2002a,b; Anthoff *et al*. 2009a,b; Tol 2009), the "Policy Analysis of the Greenhouse Effect" (PAGE) model (Hope 2006, 2008), and the "World Induced Technical Change Hybrid" (WITCH) model (Bosetti *et al*. 2007).[2]

While several IAMs are made freely available by their developers, some of these require a steep learning curve to understand sufficiently well to be put to use on practical problems and others require substantial modification to estimate the SCC under uncertainty. In discussing the apparent lack of influence of integrated assessment models on climate policy debates, Kelly and Kolstad (2000) suggested that "Perhaps policy makers are… unable to trust the 'black-box' nature of IAM results (that is, in many IAMs it is not clear what assumptions drive the model)."

[1] For more background on the economics of climate change and the social cost of carbon, including discussion of the role of benefit-cost analysis for informing climate policy, see Shogren and Toman (2001) and Pearce (2005).
[2] For more background on IAMs, see Kelly and Kolstad (2000) and Mastrandrea (2009).

In this paper we make four main contributions to the climate change economics literature. First, we provide a formal definition of the SCC based on first principles of welfare economics and expected utility theory. This definition clarifies the interpretation of the SCC and clearly identifies the theoretical quantity that we aim to estimate. Second, we develop an integrated assessment model that is more parsimonious and transparent than many of the existing models that are used to estimate the SCC. Our model includes the essential elements for calculating the SCC at the global scale, but by keeping it as simple as possible we hope to avoid creating a "black box." This simplified model is not meant as a substitute for more sophisticated IAMs in official policy evaluations, but rather a complement for understanding the ways in which SCC is likely to respond to various assumptions. By providing such an entry-level IAM, we hope to help demystify some of the other more complicated IAMs for a wider audience. We use our model to conduct a series of sensitivity analyses to build intuition regarding the key factors that influence the magnitude of the SCC. Our benchmark parameter values produce a deterministic SCC estimate that is close to the recent central estimates from other IAMs, and our sensitivity analyses complement those from previous studies. Third, we conduct a formal uncertainty analysis using Monte Carlo simulation to estimate the certainty-equivalent SCC. We contrast these estimates to the expected value of the SCC, which is the quantity that has been estimated in most previous studies that examine the SCC under uncertainty. We show that the certainty-equivalent SCC can be substantially larger than the expected SCC, and we trace the divergence between these quantities to the combined effects of uncertainty and risk aversion. Fourth, because the SCC is most appropriate for valuing marginal changes in GHG emissions, we compare the present value of benefits calculated using the SCC to the exact first-period compensating variation over a wide range of emission reduction scenarios. Taken together, these contributions should provide readers with a better understanding of the conceptual basis of the SCC, as well as the means to interpret and scrutinize the quantitative estimates of the benefits of climate change policies that appear elsewhere in the literature, including reports by government agencies and other organizations.

Recent central estimates of the SCC from three prominent IAMs include \$7.7 (DICE; Nordhaus 2008), \$5.2 (FUND; Anthoff *et al*. 2009), and \$5.1 per ton $CO_2$ in 2005 (PAGE; Hope 2008).[3] However, the narrow range of these point estimates is in stark contrast to the sensitivity of the underlying models and the much wider range of estimates reported in these and other studies. For example, in addition to the mean estimate cited above, Hope (2008) reported a 90-percent confidence interval for the SCC of \$1.1 to \$15. Tol (2005, 2008)

[3] All SCC values in this paper refer to global marginal damages and are reported using two significant figures and in units of 2005 U.S. dollars per ton of $CO_2$. These and other estimates cited from the literature were converted from the base year as reported in the original source using an assumed growth rate of 2% per year to account for real growth in the SCC and the consumer price index to account for inflation, and from units of carbon to carbon dioxide assuming 3.66 tons of carbon dioxide = 1 ton of carbon, as needed.

conducted a meta-analysis and found that the distribution of published SCC estimates spans several orders of magnitude and is heavily right-skewed: for the full sample, the median was \$12, the mean was \$43, and the 95th percentile was \$150; see Figure 1. The National Research Council concluded that "[g]iven the uncertainties and the still preliminary nature of the climate damage literature… the range of estimates of marginal global damages [social cost of carbon] can vary by two orders of magnitude, from a negligible value of about \$1 per ton to \$100 per ton of $CO_2$-eq" (NRC 2009 p 219). In light of the wide range of SCC estimates in the literature, it is important to understand the factors that can lead to systematic variations in the SCC. This should help inform decision makers' judgments about the competing estimates of the benefits and costs of new climate change policies.

### *1.1 The SCC defined*

In this paper we define the social cost of carbon using the concept of a "social welfare function" (SWF), which formalizes the normative judgments required to rank the desirability of all possible allocations of "consumption"—broadly construed, including all market and non-market goods and services that may contribute to people's well-being—among the individuals who comprise society (Bergson 1938, Graaff 1967, Samuelson 1977, Kaplow 2008). This allows us to derive, and later to calculate, both the SCC and the associated consumption discount rate under certainty and uncertainty based on first principles of welfare economics and expected utility theory. From this social welfare perspective, we define the SCC as follows:

> *The "social cost of carbon" is the decrease in aggregate consumption that would change the current expected value of social welfare by the same amount as a one unit increase in carbon emissions in a particular year*.

To make this definition precise, let $W_0$ be social welfare in the current period, let $C_t$ be aggregate consumption, and let $x_t$ be greenhouse gas emissions in period $t(\geq 0)$. Note that social welfare in the current period, $W_0$, generally will depend not only on the well-being of individuals alive in the current period, but also on (projections of) the well-being of individuals who will live in all future periods. The social cost of carbon can be determined by setting the total derivative of the expected value of the social welfare function equal to zero, $d\mathbb{E}[W_0]=(\partial\mathbb{E}[W_0]/\partial x_t)dx_t+(\partial\mathbb{E}[W_0]/\partial C_t)dC_t=0$, and solving for $dC_t/dx_t$:

$$SCC_t \equiv \frac{dC_t}{dx_t} = -\frac{\partial\mathbb{E}[W_0]/\partial x_t}{\partial\mathbb{E}[W_0]/\partial C_t}. \tag{1}$$

In other words, the SCC is the marginal rate of substitution in the expected social welfare function between greenhouse gas emissions and aggregate consumption in period $t$.[4] This is the amount of consumption in some future year $t$ that a benevolent social planner would be willing to sacrifice today to reduce greenhouse gas emissions by one unit (by convention, one metric ton) in year $t$. Expression (1) applies to scenarios under certainty or uncertainty, where in the former case the expectation operators are unnecessary.[5] Also note that this definition of the SCC does not depend on the specific operational definition of social welfare. That is, conditional on a particular chosen form of the social welfare function, expression (1) gives the blueprint for calculating the social cost of carbon, which can be used as a convenient summary measure of the benefits of any projected schedule of (sufficiently small) changes in GHG emissions over time.[6] Specifically, the present value of the social benefits of a policy that

[4] This definition of the SCC is closely related to the notion of "accounting prices" (or "shadow prices") as used by Dasgupta and Mäler (2000), Dasgupta (2001a,b), and Arrow *et al*. (2003). The two key differences are that, first, as defined in this paper the SCC is the shadow price of a *flow* variable, greenhouse gas emissions, whereas Dasgupta *et al*.'s accounting prices are attached to the *stocks* that comprise the resource base of the economy, and second, we use current consumption rather than welfare as the numeraire. These differences notwithstanding, most of the key welfare economic concepts underlying Dasgupta *et al*.'s accounting prices, including their applicability to "imperfect economies," apply equally well to the definition of the SCC used here.

[5] Note that the social welfare function plays the same role for the benevolent social planner—or, in a more practical vein, for the real-world decision-maker who wants to evaluate alternative public policies under uncertainty in a systematic and consistent way—as an individual's utility function in expected utility theory (e.g., Kreps 1988 Ch 1; Gilboa 2009 Section 6.3.3). Binmore (2007 p 5) gives a good plain English explanation: "Anyone who chooses consistently in risky situations will look to an observer as though he or she were trying to maximize the expected value of something. This abstract 'something' is what is called utility in the modern theory." A (somewhat) more rigorous explanation of what is meant by "choosing consistently in risky situations" is as follows. If the decision-maker's preferences are complete (all possible states of the world can be ranked), transitive (if state $X$ is preferred to state $Y$ and $Y$ is preferred to $Z$ then $X$ is preferred to $Z$), independent of irrelevant alternatives (if $X$ is preferred to $Y$ and $Z$ then $X$ is preferred to $Y$ when $Z$ is not available), and continuous in probabilities (if the sure outcome $X$ is preferred to the sure outcome $Y$, then the uncertain outcome $(p+\Delta)X+[1-(p+\Delta)]Y$ is preferred to the uncertain outcome $pX+[1-p]Y$, for $\Delta>0$, where $p$ is the probability that $X$ obtains in the second case), then there exists a function such that the decision-maker will behave as if she aims to maximize the expected value of that function.

[6] To keep our model simple, in this paper we use a representative agent framework to specify the social welfare function—that is, average well-being (utility) in each period will be represented by a single function of the global average per capita consumption. To explicitly account for the differences in consumption and climate change impacts among individuals within time periods, the model could be expanded using multiple representative agents. In such a model, there may be many levels of aggregate consumption changes that would make social welfare equal to that with a one unit increase in emissions, depending on how the changes in aggregate consumption were distributed among the representative agents. Therefore, in a model with multiple representative agents it may be necessary to further elaborate the definition of the SCC to make it uniquely defined. For example, the SCC could be defined as the decrease in aggregate consumption that would change the welfare of *each* representative agent by the same amount as a one unit increase in carbon emissions in a particular year. (This would be a "distribution-neutral" change in aggregate consumption, in the sense of Kaplow [2008]).

would reduce greenhouse gas emissions by the amounts $\Delta x_t$ for $t = 0, 1, 2, \ldots, H$ can be estimated as

$$PVB = \sum_{t=0}^{H} SCC_t \Delta x_t \delta_t \,, \tag{2}$$

where $\delta_t$ is the consumption discount factor, i.e., the marginal rate of substitution in the expected social welfare function between aggregate consumption in periods 0 and $t$.[7] The present value of social benefits calculated in this way would be compared to the present value of social costs calculated separately. If the sum of these is positive (negative), then the policy would increase (decrease) social welfare on net.[8]

It is important to note that the SCC is based on a first-order approximation of the social welfare effect of emission changes, so equation (2) is an approximation as well. In a later section we will compare estimates of the present value of benefits calculated using the SCC to exact values calculated directly by comparing aggregate consumption along baseline and policy paths for a wide range of very small to very large emissions reduction policies.

# 2 AN SCC "RAPID ASSESSMENT MODEL"

The model is comprised of four main components:

1) exogenous projections of per capita income, population, and greenhouse gas emissions;
2) a climate function that transforms the projection of emissions into a projection of average global surface temperatures;
3) an economic growth function and a loss function that transforms changes in average global surface temperatures into consumption-equivalent losses in all future periods; and
4) a social welfare function that defines the trade-offs between present and future gains and losses in consumption from a social perspective.

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[7] To derive equation (2), first note that the change in expected social welfare due to a sufficiently small change in emissions in each future period can be approximated by $\Delta\mathbb{E}[W_0] = \sum_{t=0}^{H}\left(\partial\mathbb{E}[W_0]/\partial x_t\right)\Delta x_t$. Divide both sides of this equation by $\partial\mathbb{E}[W_0]/\partial C_0$ to convert to current consumption units. Then multiply and divide each term in the sum on the right hand side by $\partial\mathbb{E}[W_0]/\partial C_t$. Then rearrange each term in the sum to get $\sum_{t=0}^{H}\left(\partial\mathbb{E}[W_0]/\partial x_t\right)/\left(\partial\mathbb{E}[W_0]/\partial C_t\right)\times\left(\partial\mathbb{E}[W_0]/\partial C_t\right)/\left(\partial\mathbb{E}[W_0]/\partial C_0\right)\times\Delta x_t$, where the first marginal rate of substitution is the SCC and the second is the consumption discount factor.

[8] Referring back to the qualifier in footnote 5, if a model with multiple representative-agents were used to calculate the distribution-neutral SCC, then comparing the present value of benefits to the present value of costs would indicate whether the policy passes a Kaldor-Hicks potential compensation test among the representative agents. Determining whether the policy would increase or decrease social welfare—assuming that the "potential compensation" of the Kaldor-Hicks test will not in fact be paid—would require direct calculation and summation of the net welfare effects, including both benefits and costs, on each representative agent.

The model is probabilistic by design. Some input parameters are treated as certain and assigned fixed point values, but many parameters are treated as uncertain and represented by probability distributions. To specify the point values for the fixed parameters and the probability distributions for the uncertain parameters used in the calculations reported in this paper, we reviewed a variety of relevant studies in the climate economics literature and other related areas and formed a subjective judgment about the best central value, and, for the uncertain parameters, the range of plausible values and the relative probabilities for two or more values within each range. To keep the model simple, we use piecewise linear probability distributions to describe all uncertain parameters. Table 1 assembles the functional forms used to represent each model component, Table 2 defines all key parameters and lists the parameter values used for the calculations reported in Section 3. Figure 2 gives examples of the probability density functions used to represent the uncertain input parameters. To streamline the description of the model in the main text, the principal sources for the numerical values we assign to each parameter and notes on parameter calibrations are given in Table 3. While our review of the literature was fairly extensive, it was neither exhaustive nor highly systematic. The model has been designed to be easily modified so parameter values can be adjusted with more up-to-date empirical estimates as they become available.

In the sub-sections below we describe each of the four main components of the model in turn, including the key simplifications involved and some comparisons to other IAMs.

## 2.1 Exogenous projections

The main inputs to the model include projections over time of three key state variables: global per capita income, population, and greenhouse gas emissions. The model does not account for interactions among these variables, and to keep the model simple we specify these inputs exogenously and independently. Ideally these state variables would be modeled endogenously within a single comprehensive framework, or at least based on a general equilibrium model of the global economy that itself incorporates the key feedback effects or interdependencies among them.

### 2.1.1 Per-capita income

First, we assume that, ignoring climate damages, global per capita income, $y_t$, is projected to grow at a possibly diminishing rate, $g_t$:

$$y_{t+1} = y_t e^{g_t}; \quad g_t = g_\infty + (g_0 - g_\infty) e^{-\omega t}, \tag{3}$$

where $\omega$ is the rate at which $g_t$ converges from its starting level, $g_0$, to its long-run steady-state level, $g_\infty$. If $g_\infty = g_0$, then equation (3) reduces to simple exponential growth of per capita income. Exponential increase is a common starting point for introductory textbook models of economic growth, and it is roughly consistent with the last one or two hundred years of historic experience (e.g., Blanchard and Fisher 1996, Valdes 1999, Barro and Sala-i-Martin 2001). However, there are good reasons to expect global per capita income growth to diminish over time. For example, the rapid growth in developing countries may converge to that of developed countries due to technology transfers and the diffusion of innovations (e.g., Lucas 2000, Helpman 2004). Furthermore, many question the feasibility of perpetual economic growth due to the ultimately finite supply of natural resources (e.g., Meadows *et al*. 2004), so we allow for the possibility of $g_t$ shrinking over time.

As noted in Section 1, we use a representative agent framework, so we make no attempt to forecast the change in the distribution of income over time. This means that we cannot account for intra-generational equity concerns. The globally aggregated nature of the economic growth function—as well as the economic loss function in equation (8) below—also means that we cannot calculate the SCC at a national scale. It may be important to consider benefits at a national scale for some purposes, such as designing domestic policies and determining an equitable distribution of control efforts among countries, but these issues are beyond the scope of our model.

The modeling of economic growth is handled differently in other IAMs. For example, DICE is built on a traditional Ramsey-style optimal growth model, so economic growth is driven by exogenous technical change and the endogenous choice of the rate of savings and emissions reductions over time (Nordhaus 2008). WITCH extends the DICE model to allow economic growth to be partly driven by endogenous technical change (Bosetti *et al*. 2007). FUND and PAGE treat income growth as exogenous, but these models track income growth and climate damages in 16 and 8 world regions, respectively, and both models allow for equity weighting among regions (Hope 2006, Anthoff *et al*. 2009a).

### *2.1.2 Population*

We represent the dynamics of global population, $N_t$, as the outcome of average annual per capita birth rates, *b*, and death rates, *d*, that converge exponentially to their respective asymptotic values, $b_\infty$ and $d_\infty$:

$$N_{t+1} = N_t\left[1 + b_\infty + \left(b_0 - b_\infty\right)e^{-\theta_b t} - d_\infty - \left(d_0 - d_\infty\right)e^{-\theta_d t}\right]. \tag{4}$$

As described in Table 3, we calibrated the six parameters of equation (4) to match the low, central, and high population growth scenarios in the most recent long run population projections by the United Nations, which ranged between 7.4 and 10.6 billion in 2050 and 2.3 and 36.4 billion in 2300.

By way of comparison to other IAMs, DICE and WITCH use a deterministic population growth function that rapidly converges to a steady-state population size of 9 billion. PAGE uses a set of deterministic population growth trajectories for each region, which may or may not reach a stable population size before the end of the model's 200-year time horizon, depending on the scenario. FUND also models population growth deterministically, but FUND is alone among IAMs in allowing for a feedback effect between population growth and the climate system through the effects of changes in mortality risks from extreme hot and cold weather events (i.e., population is partly endogenous) (Anthoff and Tol 2008). For an expansive discussion of long run population projections, see Cohen (1995).

### *2.1.3 Greenhouse gas emissions*

We assume that annual GHG emissions, $x_t$, will increase at a diminishing rate, $h_t$, until they reach a peak in some future year, $t_p$, after which time emissions will decline due to the rising costs of discovery and extraction relative to the prices of non-fossil fuel sources of energy. To avoid the complications of modeling carbon releases due to land use changes we consider fossil fuel emissions only, so we also require that total emissions do not exceed the economically recoverable reserves of fossilized carbon, $R$. Specifically, our emissions projection is specified as follows:

$$x_{t+1} = x_t e^{h_t}\,;\; h_t = h_0\left(1 - t/t_p\right);\; \sum_{t=0}^{H} x_t \le R\,. \qquad (5)$$

For comparison to other IAMs, DICE, WITCH, and FUND specify $CO_2$ emissions as a function of GDP, the carbon intensity of energy use, and the energy intensity of production. In DICE and WITCH, industrial $CO_2$ emissions are determined endogenously by optimizing the time path of abatement given an exogenous rate of change of marginal abatement costs, while all other climate forcings are exogenous. In FUND, $CH_4$, $N_2O$, and emissions from land use changes and deforestation are exogenous while $SF_6$ and $SO_2$ are functions of GDP and population, respectively. PAGE treats $CO_2$, $SF_6$, and $CH_4$ emissions as exogenous inputs.

Tables 2 contains the values we assign to the parameters of equation (5), and Table 3 contains our sources and rationale for those values. Figure 3 shows the resulting projections of global $CO_2$ emissions, along with per capita income and population, for the first 300 years of the planning horizon using the 25th, 50th, and 75th percentile values for the associated parameters. Our associated projections of $CO_2$ emissions in 2100 are 27, 36, and 47 GtC, respectively. These figures are near the high end of the range of other recent projections. For example, the Stanford Energy Modeling Forum reference scenario projections of $CO_2$ emissions in 2100 range from 12 to 36 GtC (Stanford Energy Modeling Forum 2009), the highest projections among the IPCC SRES scenarios are around 35 GtC (Intergovernmental Panel on Climate Change 2000 Fig 3), and the $\pm$ one standard deviation range of projections in the DICE2007 model is 10-35 GtC (Nordhaus 2008 p 49). The steep drop in emissions between 2150 and 2200 in our projections represents a rapid transition to alternative (non-carbon) energy sources once the cost of alternative fuels drops below the cost of further discovery and extraction of fossil fuels.

## *2.2 Climate dynamics*

We use a simple 2-box model to represent the dynamics of the atmospheric carbon stock. The total stock is partitioned into two compartments, one subject to relatively fast removal and the other to slow removal.[9] We denote the total stock of carbon in the atmosphere in period *t* as $X_t$, and the fraction of the stock in the fast compartment as $f_t$. We assume that in each period a fraction $\alpha_F$ ($\alpha_S$) of natural background emissions, $n$, and anthropogenic emissions, $x_t$, enter the fast (slow) compartment, and a fraction $\beta_F$ ($\beta_S$) of the stock is removed from the fast (slow) compartment via ocean sequestration and other natural processes. With these assumptions, the equations of motion describing the dynamics of the atmospheric carbon stock are:

$$X_{t+1} = \left[1 - \beta_F f_t - \beta_S \left(1 - f_t\right)\right] X_t + \left(\alpha_F + \alpha_S\right)\left(n + x_t\right)$$

and (6)

$$f_{t+1} = \left[\left(1 - \beta_F\right) X_t f_t + \alpha_F \left(n + x_t\right)\right] \big/ X_{t+1}.$$

We also assume that the system was in equilibrium prior to the onset of industrial emissions, so $n = X_{PI}\beta_F\beta_S \big/ \left(\alpha_F\beta_S + \alpha_S\beta_F\right)$, where $X_{PI}$ is the pre-industrial atmospheric carbon stock.

To model the response of the long-run equilibrium surface temperature to changes in the atmospheric carbon concentration, we adopt the standard assumption (going back to Arrhenius [1896]) that the downward radiative flux anomaly (i.e., "forcing") at the top of the atmosphere increases with the logarithm of the carbon concentration, i.e., $F_t = F_{2X} \ln\left(X_t / X_{PI}\right) \big/ \ln 2$, where $X_{PI}$ is the pre-industrial carbon concentration and $F_{2X}$ is the forcing from a sustained doubling of atmospheric carbon.[10]

To represent the inertial lags in the climate system, we use a simple one-dimensional diffusive ocean heat transfer model, largely following Baker and Roe (2009) and Marten (2011). The model is based on a set of energy balance equations for the surface land and ocean layers and multiple deep ocean layers. As the radiative forcing at the top of the atmosphere

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[9] A 1-box model would be simpler since it would require only one equation, but we chose a 2-box representation to ensure that the model can reproduce the important feature of results from general circulation models that predict a very long residence time of carbon in the atmosphere if a large fraction of the geological carbon stock is released (e.g., Archer 2005, Archer 2009, Solomon *et al*. 2009).

[10] The phenomenon that gives rise to this diminishing marginal effect of $CO_2$ concentrations on the radiative forcing is known as the "band saturation effect," whereby further additions of a gas have a weaker impact on the net radiative forcing because "at high concentrations, much of the light most easily absorbed by the gas will have already been absorbed even before you add the new slug of gas" (Archer and Rahmstorf 2010 p 22.)

increases, additional heat is trapped at the earth's surface, which increases the transfer of heat to the deep ocean layers. The rate of transfer among the ocean layers is controlled by the heat capacity of water, the vertical temperature gradient in the ocean, and the ocean upwelling velocity. The model equations for the temperature dynamics—shown in Table 1—are comprised of ordinary partial differential equations for the ocean surface temperature, land surface temperature, and multiple layers of the deep ocean. The average global surface temperature is an area-weighted average of the ocean and land surface temperatures.[11]

A key simplifying assumption in this model is that all GHGs are treated as $CO_2$ emissions. While $CO_2$ makes up the majority of anthropogenic greenhouse gas emissions, other greenhouse gases also are important, including $CH_4$, $N_2O$, various CFCs, aerosols, and more. These can be converted to "$CO_2$-equivalents," but only at the loss of some realism since different gases are removed from the atmosphere at different rates and have different radiative efficiencies (Forster *et al*. 2007, Marten and Newbold 2011). Some of these gases are represented more accurately in other IAMs. For example, PAGE treats the three main categories of greenhouse gases separately (Hope 2006), and FUND distinguishes five different GHGs (Anthoff and Tol 2008).

## *2.3 Economic growth and losses due to climate change impacts*

To model economic growth, we use a traditional Solow-Swan growth model with a Cobb-Douglas production function (e.g., Valdes 1999 Ch 2) adjusted by a Hicks-neutral economic loss function (following Nordhaus 2008). Specifically,

$$\begin{aligned} &K_{t+1} = K_t\left(1-\delta_K\right)+sY_t, \\ &Y_t = A_t K_t^{\gamma} N_t^{1-\gamma}\left(1-L_t\right), \text{ and} \\ &C_t = \left(1-s\right)Y_t, \end{aligned} \tag{7}$$

where $K_t$ is the physical capital stock, $s$ is the fixed rate of saving, $Y_t$ is global economic output, $N_t$ is population (labor supply is assumed always proportional to population), $L_t$ is the fraction of output lost due to climate damages, and $C_t$ is aggregate consumption in year *t*. This growth model allows us use the forecast of per capita income growth ignoring climate damages, $y_t$, to solve for a projected path of total factor productivity, $A_t$. We then can project changes in income and consumption under different climate change scenarios net of climate

[11] Marten (2011) conducted a detailed set of simulation experiments and showed that the more simplified temperature response functions in DICE, FUND, and PAGE can lead to first-period SCC estimates that are between 25% lower and 50% higher than the estimates produced using essentially the same temperature response model we use here.

damages, holding the path of $A_t$ fixed. That is, climate change is assumed to directly affect economic output, not the rate of technical change or the physical capital stock.

The fraction of global economic output lost in each year due to climate change impacts is represented by the following function of the global average surface temperature anomaly, $T_t$, which depends on emissions, $x_t$, through the carbon cycle and temperature response functions described above and shown in Table 1:

$$L_t = L(T_t) = a\left(T_t - T_{neg}\right)^b, \tag{8}$$

for $T_t \geq T_{neg}$ and $L_t = 0$ otherwise, where $T_{neg}$ is the temperature anomaly below which losses are negligible. This damage functional form is borrowed directly from the PAGE model, except PAGE distinguishes between market and non-market damages (Hope 2006, 2008).

Ideally the loss function should incorporate all impacts due to climate change—negative and positive, market and non-market, tangible and intangible—expressed as a fraction of global economic output. Potential climate damages can be estimated in a relatively rigorous fashion for some sectors in some regions, for example agriculture in the United States (e.g., Mendelsohn and Neumann 1999, Schlenker *et al*. 2005, Deschenes and Greenstone 2007, Schlenker *et al*. 2007). However, monetizing the full range of potential damages to all market and non-market sectors in all regions of the world requires large doses of extrapolation and expert judgment (e.g., Nordhaus and Boyer 2000 Ch 4, Stern 2006, Cline 2007). It seems safe to say that the representation of aggregate climate damages—and, crucially, their extrapolation to temperatures beyond the range of historical experience (Weitzman 2009, 2010a)—is one of the weakest links in the chain of relationships that comprise any integrated assessment model. To improve this component of IAMs, an updated review and synthesis of the economic studies of climate change damages should be a high priority for research in the near term.

Some additional limitations of our economic loss function also should be highlighted. First, as described in Table 3, it is calibrated using the results of an expert elicitation survey that is more than ten years old. Some recent studies in the climate science literature suggest that damages may appear sooner and be more severe than previously thought (e.g., Smith *et al*. 2009, Horowitz 2009), in which case our SCC estimates will be biased downward. Second, the functional form in equation (8) excludes the possibility that climate change could lead to net benefits at low temperature changes (e.g., Mendelsohn and Neumann 1999 Ch 12, Deschenes and Greenstone 2007, Anthoff and Tol 2009a,b). Third, it ignores the possibility that economic losses may also depend on the speed of temperature changes and the level of income (Tol 2002a,b). Insofar as vulnerability to climate change will decrease as incomes grow over time, this omission would bias our SCC estimates upward. Fourth, a globally aggregated loss function such as equation (8) necessarily masks what may be considerable geographic and demographic variation in the impacts.

### *2.4 Social welfare function*

We assume that utility (well-being) increases with consumption at a diminishing rate. Specifically, we use a utility function with a lower bound and a constant elasticity of marginal utility (Hall 2010), so utility in period *t* is:

$$u_t = \left( \left( C_t / N_t \right)^{1-\eta} - c_{sub}^{1-\eta} \right) \Big/ \left( 1-\eta \right) \tag{9}$$

for $C_t / N_t \geq c_{sub}$ and $u_t = 0$ otherwise, where $C_t / N_t$ is per capita consumption, $\eta$ is the elasticity of marginal utility (also known as the coefficient of relative risk aversion),[12] and $c_{sub}$ is a subsistence level of consumption, which places a lower bound on utility and an upper bound on marginal utility. The intuition behind this approach to bounding the utility function is that it effectively treats all outcomes that would drive consumption below the subsistence level as equally bad worst-case scenarios. Bounding the utility function in some way is necessary to guarantee that marginal utility is finite, so that the SCC is always defined. This will be important when estimating the SCC under uncertainty in Section 3.2 below.[13]

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[12] For some intuition about the elasticity of marginal utility, note that if $\eta$ = 0, then a one *dollar* increase in consumption is equally valuable to a person no matter their level of income. If $\eta$ = 1, then a one *percent* increase in consumption is equally valuable no matter the level of income (and the utility function becomes $u_t = \ln c_t$); and if $\eta$ > 1, then a one percent increase in consumption is less valuable the higher is the person's level of income. Also, if $\eta$ = 0 the individual is risk-neutral; if $\eta$ > 0 the individual is risk-averse. That is, for an uncertain change in consumption, $\Delta c$, there is some sure ("certainty-equivalent") change in consumption $\Delta c_{CE}$ that is just as valuable to the individual as the uncertain prospect, i.e., $u(c + \Delta c_{CE}) = \mathbb{E}\left[ u(c + \Delta c) \right]$. If $\eta$ = 0 then $\Delta c_{CE} = \mathbb{E}\left[ \Delta c \right]$; if $\eta$ > 0 then $\Delta c_{CE} < \mathbb{E}\left[ \Delta c \right]$. In other words, if the individual is risk-averse then she would prefer some sure amount smaller than $\mathbb{E}\left[ \Delta c \right]$ to the uncertain prospect represented by the probability distribution over $\Delta c$.

[13] It is instructive to consider the view of one of the founders of expected utility theory on the general concept of unbounded utility functions: "…it is often said, in effect, that the utility to a person of immediate death is a consequence of minus infinite utility, but casual observation shows that this is not true of anyone—at least not of anyone who would cross the street to greet a friend… My personal feeling is that, theological questions aside, there are no acts of infinite or minus infinite utility, and that one might reasonably so postulate, which would amount to assuming utility to be bounded. Justifiable though it might be, that assumption would entail a certain mathematical awkwardness in many practical contexts… I propose, therefore, not to assume bounded utility formally, but to remember that problems involving unbounded utility are to be handled cautiously" (Savage 1972 p 81-82). In our case the subsistence consumption parameter introduces a "certain mathematical awkwardness," but some device such as this is necessary to ensure bounded SCC estimates, at least if the damages could reach one hundred percent of income in any period. An alternative but closely related functional form is $u = (c+d)^{1-\eta} \big/ (1-\eta) - d^{1-\eta} \big/ (1-\eta)$. This form is advocated by Millner (2011) because it is differentiable for all levels of consumption equal to or greater than zero. We use the modified CRRA form because its extra parameter has the (arguably) more straight-forward and intuitive interpretation of a subsistence level of consumption, all levels below which are equally bad.

Finally, we define social welfare as the sum of discounted future utilities of all individuals who will live between the current date and the end of the planning horizon, $H$ (e.g., Dasgupta 2001b p 98-101):

$$W_t = \sum_{\tau=t}^{H} N_\tau u_\tau e^{-\rho(\tau-t)} . \qquad (10)$$

The pure rate of time preference, $\rho$, which discounts future utilities, has been the subject of much discussion in the economics literature and beyond. Many economists argue that the value used for $\rho$ should be based on people's revealed time preferences, while others argue that to use any value other than $\rho = 0$ unjustly discriminates against future generations (Arrow *et al*. 1996). In this paper we use $H$ = 400 years and a probability density function for $\rho$ that spans the range of values most commonly used in the climate economics literature (see Table 3).[14]

# 3 RESULTS

With the model laid out in the previous section and parameterized as indicated in Table 2, we are now in a position to estimate the SCC under certainty and uncertainty and put it to use in some illustrative policy scenarios. First, we use fixed values for all input parameters to

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See Hall (2010 p 28-30) for a discussion of this functional form applied to the valuation of health risk reductions, and see Pindyck (2010) for an examination of bounded marginal utility functions for climate change economics.

[14] With all functional forms specified, the reader can now confirm that the consumption discount factor in equation (2) is $\delta_t = \mathbb{E}\left[c_0^{-\eta} e^{-[\eta \hat{g}_{0t}+\rho]t}\right] \big/ \mathbb{E}\left[c_0^{-\eta}\right]$ under uncertainty and $\delta_t = e^{-[\eta \hat{g}_t + \rho]t}$ under certainty, where $\hat{g}_t$ is the growth rate of per capita consumption up to period $t$ (which generally will be lower than $g_t$, the growth rate of per capita income in equation (3), due to losses from climate change). This is the familiar Ramsey discount factor (Ramsey 1928). Also note that the SCC is sometimes directly defined as the present value of the stream of future damages from a unit of emissions today (e.g., NRC 2009 Equation 5-2, Tol 2009 Equation 1). In our nomenclature, this would be written as $SCC_t = \sum_{\tau=t}^{H} \left(-\partial C_\tau / \partial x_t\right) \delta_{t\tau}$, where $-\partial C_\tau / \partial x_t$ is the marginal damage (lost consumption) in year $\tau$ from an extra unit of emissions in year $t$. This convenient expression for the deterministic SCC—which will show up again in Section 3.2 below —can be derived in a few short steps from our definition in equation (1) and our social welfare function in equation (10), where again $\delta_{t\tau}$ is the Ramsey discount factor between periods $t$ and $\tau$. First, recall that $SCC_t = -\left(\partial W_0 / \partial x_t\right) / \left(\partial W_0 / \partial C_t\right)$. If we write current social welfare as $W_0 = \sum_{\tau=0}^{t-1} U\left(C_\tau\right) e^{-\rho\tau}$ $+ \sum_{\tau=t}^{H} U\left(C_\tau\right) e^{-\rho\tau}$, we see that $\partial W_0 / \partial x_t = \sum_{\tau=t}^{H} \left(\partial U / \partial C_\tau\right) \times \left(\partial C_\tau / \partial x_t\right) e^{-\rho\tau}$ and $\partial W_0 / \partial C_t = e^{-\rho t}\, \partial U / \partial C_t$. And writing social welfare at an arbitrary future period $t$ as $W_t = \sum_{\tau=t}^{H} U\left(C_\tau\right) e^{-\rho(\tau-t)}$, we see that $\delta_{t\tau} = \left(\partial W_t / \partial C_\tau\right) / \left(\partial W_t / \partial C_t\right)$ $= e^{-\rho(\tau-t)} \left(\partial U / \partial C_\tau\right) / \left(\partial U / \partial C_t\right)$. Putting these pieces together gives the simplified expression for SCC that we set out to derive: $SCC_t = \sum_{\tau=t}^{H} \left(-\partial C_\tau / \partial x_t\right) \delta_{t\tau}$.

calculate the SCC under certainty. These deterministic estimates are useful for comparison to central estimates from other IAMs, and as a starting point for sensitivity analyses that change one or a few parameters at a time. We then estimate the SCC under uncertainty, using Monte Carlo analysis drawing randomly from the probability distributions for all uncertain input parameters. This produces a "certainty- equivalent" SCC path and associated path of consumption discount rates. This is the conceptually correct approach under expected utility theory when one or more input parameters are uncertain. We compare these estimates to the deterministic estimates and to the path of the expected value of the SCC. To preview our results, we find that the certainty-equivalent SCC path is substantially higher, and the certainty-equivalent discount rate path is substantially lower, than the paths of the deterministic estimates and expected values of these quantities. This result with respect to the discount rate is by now well-known (e.g., Weitzman 2001, Newell and Pizer 2003, Gollier and Weitzman 2009), but here we show that an analogous result also applies to the SCC itself.

### *3.1 The SCC under certainty*

The first six columns of numbers in Table 4 report the deterministic SCC and associated consumption discount rates for the first 50 years of the planning horizon, using the modes, medians, and means of all input parameter distributions summarized in Table 2. Figure 3 shows the projections of several key state variables for the first 300 years of the planning horizon using the 25th, 50th, and 75th percentile values for the associated parameters. The estimated SCC values in 2005 using the modes, medians, and means of all input parameters are $6.6, $10, and $11 per metric ton of $CO_2$ per year, with average growth rates over the first 50 years of 2.4%, 2.3%, and 2.3% per year, respectively. These deterministic estimates are slightly higher but reasonably close to the range of recent central estimates from DICE ($7.7), FUND ($5.2), and PAGE ($5.1) cited earlier.

We emphasize that these deterministic estimates should not be viewed as definitive. Our central parameter values may be only rough approximations of the best current estimates of these quantities, and we fully expect the best estimates of (at least some of) these parameters and projections to be adjusted over time as more scientific and economic studies are completed and as the state of the economy evolves. More importantly, these deterministic estimates ignore uncertainty in the model parameters. In the next sub-section we will formally account for uncertainty in calculating the SCC, but first we use sensitivity analysis to examine the effect of several key input parameters on the deterministic SCC. Specifically, we use sensitivity analysis to isolate the influence of all three components of the consumption discount rate (the elasticity of marginal utility, the growth rate of forecast per capita income, and the pure rate of time preference), the population scenarios, the projected path of emissions, equilibrium climate sensitivity, and the parameters of the economic loss function. We conduct our sensitivity analyses by changing one or a few parameters from their median values to their

25th and 75th percentile values in turn, while holding all other parameters at their median values. Our aim is to make the variations in each parameter comparable in the sense that values below the low and above the high values we examine for each parameter would be viewed by most readers familiar with the climate economics literature as roughly equally (un)likely. A more systematic review of the literature, perhaps including some form of quantitative meta-analysis, would be highly useful but is beyond the scope of this paper.

The results of the sensitivity analysis are shown as a horizontal bar plot in Figure 4. The elasticity of marginal utility, $\eta$, is the single most influential uncertain parameter as measured by the corresponding variation in the deterministic SCC, holding all other parameters fixed at their median values. The next most influential parameters are those of the damage function, and roughly tied for third are equilibrium climate sensitivity, pure time preference, and population growth. The growth rate of per capita income has a non-negligible but smaller effect on the SCC, and the effect of variation in the forecast GHG emissions paths is negligible.

By way of comparison, the major influences on the first-period SCC in PAGE as reported by Hope (2008) were, in decreasing order, the equilibrium climate sensitivity, the pure rate of time preference, the elasticity of marginal utility, and the damage function parameters. Nordhaus (2008 Ch 7) conducted a sensitivity analysis of several key parameters (but not including the pure rate of time preference or the elasticity of marginal utility) and found that the four most influential parameters were the damage function coefficient, equilibrium climate sensitivity, the growth rate of total factor productivity, and the long run population size (Table 7-2).

## *3.2 The SCC under uncertainty*

It is by now routine to point out that the benefits of climate change policies are highly uncertain. This was the motivation for the sensitivity analysis above. However, while sensitivity analysis is useful for examining the individual *ceteris paribus* influence of each uncertain parameter, it does not reveal the aggregate effect of uncertainty over all parameters. We can account for uncertainty in all parameters simultaneously using Monte Carlo simulation, taking many random draws from the probability distributions of all input parameters, to estimate the SCC as defined in equation (1).

Note that we use a four-point piecewise linear probability density function for the equilibrium climate sensitivity distribution, as described in Table 3 and Figure 2. The purpose of this elaboration is to stretch out the upper tail of the climate sensitivity probability density function to account for the low-probability but possibly high-impact scenarios that could occur if $T_{2X}$ is even larger than the "likely" range commonly cited in the literature. This is important in light of the scientific uncertainty surrounding this key physical parameter (Roe and Baker

2007) and the economic uncertainty associated with the effects of such Earth system changes on human well-being (Weitzman 2009).

We estimated the SCC under uncertainty using 100,000 Monte Carlo iterations with random parameter values drawn from the distributions summarized in Table 2.[15] To confirm that we used a sufficient number of iterations to achieve a stable estimate, Figure 5 plots the running estimate of the SCC, calculated using all Monte Carlo draws up to that point, against the iteration number. After roughly 30,000 iterations the first-period SCC estimate has settled down to a relatively narrow range. The time path of the SCC and the associated discount rate under uncertainty are shown in the final two columns of Table 4. Under uncertainty, the SCC in 2005 is $81, which is approximately an order of magnitude larger than the deterministic estimates based on the central parameter values, and the associated consumption discount rate starts at 2.7% per year (declining slightly to 2.6% per year over the first 50 years of the planning horizon), which is roughly one half of the deterministic discount rate estimates based on the central parameter values.[16]

It is important to note that the SCC calculated under uncertainty in the penultimate column of Table 4 is not the same as the expected value of the deterministic SCC. The former can be thought of as the certainty-equivalent SCC, since by our definition in expression (1) it is

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[15] We treat all probability distributions for the input parameters as independent, with two exceptions. First, we treat the parameters of the economic loss function— $T_{neg}$ , $L(3)$, and $L(6)$ —as perfectly correlated. That is, if the $x$ percentile value of $T_{neg}$ is drawn in a particular iteration, then it is paired with the $(1-x)$ percentile values of $L(3)$ and $L(6)$. Second, we model uncertainty in the global population forecasts by using the low, central, and high UN forecasts as the lower bound, mode, and upper bound of a triangular probability distribution. That is, the simulated global population in all years of the forecast will be $100 \cdot x$ percent of the distance between the low and high UN forecasts, where $x$ is determined by a random draw from a triangular probability density function. This ensures that none of the global population forecasts cross each other.

[16] Weitzman (2009) emphasized the potentially high sensitivity of Monte Carlo IAM results to the truncation point of the climate sensitivity distribution and the upper bound placed on marginal utility. Weitzman concluded that any uncertainty analysis that truncates the climate sensitivity distribution in an ad hoc manner may "give a very misleading picture of the expected utility consequences of alternative GHG-mitigation policies." (The results of Newbold and Daigneault [2009] partially corroborated this concern, but Costello *et al*. [2010] were more circumspect.) In our simple SCC model the two key parameters that determine the severity of the worst-case outcomes are $T_{2X}^{\max}$ and $c_{sub}$. We examined the influence of these parameters on the certainty-equivalent SCC with additional sensitivity analyses, holding all other parameter distributions constant. This exercise indicated that the upper bound of the climate sensitivity distribution has a small effect on the SCC. Specifically, using 5000 Monte Carlo draws, at $T_{2X}^{\max}$ = 10, 15, and 25 deg C, the estimated SCC in 2005 was $110, $111, and $112, respectively. Furthermore, the SCC in 2005 was completely insensitive to three orders of magnitude variation in $c_{sub}$, from $3.65 yr$^{-1}$ to $3650 yr$^{-1}$. The insensitivity to these parameters is due largely to the long time lags in the climate system, which prevent the worst case scenarios from being realized within the 400 year time horizon in any of the Monte Carlo draws used in this sensitivity analysis.

the sure amount of consumption required to compensate society for a sure change in emissions. We can isolate the difference between these quantities as follows: the expected SCC is the expected value of the ratio of the marginal effects of emissions and consumption on social welfare, i.e.,

$$\mathbb{E}[SCC_t] = \mathbb{E}\left[-\frac{\partial W_0/\partial x_t}{\partial W_0/\partial C_t}\right] = \mathbb{E}\left[\sum_{\tau=t}^{H}\left(-\partial C_\tau/\partial x_t\right)\delta_{t\tau}\right], \tag{11}$$

while the certainty-equivalent SCC is the ratio of the marginal effects of emissions and consumption on the expected value of social welfare, i.e.,

$$SCC_t = -\frac{\partial\mathbb{E}[W_0]/\partial x_t}{\partial\mathbb{E}[W_0]/\partial C_t} = \frac{\mathbb{E}\left[c_t^{-\eta}e^{-\rho t}\sum_{\tau=t}^{H}\left(-\partial C_\tau/\partial x_t\right)\delta_{t\tau}\right]}{\mathbb{E}\left[c_t^{-\eta}e^{-\rho t}\right]}, \tag{12}$$

where here $\delta_{t\tau} = \left(\partial W_t/\partial C_\tau\right)/\left(\partial W_t/\partial C_t\right)$ is the deterministic consumption discount factor between periods *t* and $\tau$. Therefore, if the path of per capita consumption, $c_t$, the elasticity of marginal utility, $\eta$, and the pure rate of time preference, $\rho$—that is, all of the ingredients of the consumption discount rate—are (assumed to be) known with certainty, then the expected and certainty-equivalent SCCs will be equal; otherwise they will diverge. Furthermore, the certainty-equivalent SCC generally will be larger than the expected SCC. To see this, we can denote $\sum_{\tau=t}^{H}\left(-\partial C_\tau/\partial x_t\right)\delta_{t\tau}$ as $\Omega_t(\eta,\rho)$, so we can write the expected SCC as $\mathbb{E}\left[\Omega_t(\eta,\rho)\right]$ and the certainty-equivalent SCC as $\mathbb{E}\left[c_t^{-\eta}e^{-\rho t}\Omega_t(\eta)\right]/\mathbb{E}\left[c_t^{-\eta}e^{-\rho t}\right]$. Now using the definition of the covariance between two random variables, the certainty-equivalent SCC can be re-written as $\mathbb{E}\left[\Omega_t(\eta,\rho)\right] + \operatorname{cov}\left(c_t^{-\eta}e^{-\rho t}, \Omega_t(\eta,\rho)\right)/\mathbb{E}\left[c_t^{-\eta}e^{-\rho t}\right]$. Since $c_t^{-\eta}e^{-\rho t}$ and $\Omega_t(\eta,\rho)$ both are decreasing in $\eta$ and $\rho$, the covariance between these terms will be positive, which means that the certainty-equivalent SCC will be larger than the expected SCC. Therefore, the divergence between the expected SCC and the certainty-equivalent SCC arises largely due to the combined effects of uncertainty and risk aversion, including uncertainty *about* risk aversion.[17] Note that the same factors that cause the certainty-equivalent SCC to be larger than the expected SCC will

[17] Rather than uncertainty about the preference parameters $\eta$ and $\rho$ for a representative agent, the probability density functions associated with these parameters instead could represent heterogeneity in preferences among individuals. To examine the influence of $\eta$, we conducted an additional sensitivity analysis over a range of fixed values for $\eta$ from 0.5 to 3, with all other parameters represented by the probability distributions summarized in Table 2. In this case, $SCC_{2005}$ was \$150 at $\eta = 0.5$ and declined monotonically to \$4 at $\eta = 3$. So here we do not see the U-shaped pattern in the SCC with increasing $\eta$, as has been found in some previous studies (e.g., Newbold and Daigneault 2009, Anthoff *et al.* 2009b, Pindyck 2009). This occurs because none of our Monte Carlo draws produced scenarios with decreasing per capita consumption.

cause the certainty-equivalent discount rate to be lower than the expected discount rate, so this result is closely related to the results of Weitzman (1998, 2001) and Newell and Pizer (2003), among others, regarding discounting under uncertainty.

For comparison, we calculated $\mathbb{E}[SCC_t]$ using the same random parameter draws used to calculate the certainty-equivalent SCC shown in Table 4. This gave $\mathbb{E}[SCC_{2005}]$ = \$18, increasing by about 2.1% per year to \$50 in 2055. This value in 2005 is roughly double the deterministic estimates using the central parameter values, but is less than one fourth of the certainty-equivalent estimate. In fact, the certainty-equivalent SCC corresponds to the 97th percentile of the simulated probability distribution of the deterministic SCC in 2005, which is shown in Figure 6. We highlight this comparison because the standard approach for modeling the SCC under uncertainty has been to estimate the probability distribution of the deterministic SCC, from which the mean or median might be highlighted as a central estimate (e.g., Stern 2006, Hope 2006, 2008; Nordhaus 2008 Ch 7, Anthoff *et al*. 2009b). However, under expected utility theory the proper shadow price for analyzing climate policies under uncertainty is the certainty-equivalent SCC, for the same reason that an individual's willingness to pay for an uncertain prospect is the certainty-equivalent value, not the expected monetary outcome of the prospect. Our results show that the expected SCC and the certainty-equivalent SCC can be very different.

### *3.3 Valuing marginal and non-marginal changes*

Because the SCC is based on a first-order approximation of the effect of a change in emissions on the current expected value of social welfare, using the path of the SCC to estimate the present value of benefits from a policy of emission reductions over time as in equation (2) also gives a first-order approximation. In general, the larger are the emission reductions being analyzed the less accurate will be the approximation.[18] In this section we compare estimates of the present value of benefits calculated using the SCC to exact values of the compensating variation of consumption in the first period, which we denote $CV$. This is the maximum amount of first period consumption that the hypothetical social planner would be willing to sacrifice for the policy. This first period equivalent value of consumption is defined by:

$$\mathbb{E}[W_0] = \mathbb{E}\left[N_0 u\left(c_0' - CV/N_0\right)\right] + \mathbb{E}\left[e^{-\rho} W_1'\right], \qquad (13)$$

where $c$, $u$, and $W$ represent per capita consumption, utility, and social welfare under the baseline scenario, and $c'$, $u'$, and $W'$ represent these quantities under a policy scenario with

[18] Stern (2008) emphasized the dangers of careless application of marginal shadow values—in particular the social discount rate, but the same concepts also apply to the SCC—for analyzing large policies aimed at addressing climate change.

discrete—not necessarily "marginal"—changes in emissions in one or more years, and subscripts denote time periods with 0 representing the initial period.[19]

Because it is based on a first-order Taylor series approximation, we know that the SCC should give a close approximation to the exact first period equivalent value of consumption for sufficiently small changes in emissions. However, it is not immediately clear how closely the SCC will approximate the exact value for emissions reductions in the range of magnitudes we might see in proposed climate policies in the real world. To examine this question, we calculated the approximate and exact present values for a wide range of progressively larger emission reduction policies, using equations (2) and (13). To keep things simple, we examined hypothetical policies where emissions were reduced by a linearly increasing amount, $\Delta x_t = \psi t$ [GtC yr$^{-1}$], for the first 40 years of the planning horizon, where $\psi$ ranged from 0.001 to 0.05. The high end of this range is close to the U.S. government policy targets as reported to the IPCC.[20]

Figure 7 shows the results of these calculations in a deterministic scenario based on the mean values of all input parameters (top graph) and under uncertainty using the certainty-equivalent SCC based on 5,000 Monte Carlo draws (bottom graph). In the deterministic case, the present value of benefits based on the SCC gives a very close approximation to the first period compensating variation for nearly the full range of emission reductions. In the uncertain case, the approximation begins to diverge from the true value around $\psi = 0.01$ GtC/yr$^2$, which is roughly one fifth of the rate of GHG emissions reductions that the U.S. Government has cited as a policy goal. At $\psi = 0.05$ GtC/yr$^2$, the first order approximation based on the SCC is 18% larger than the true first period compensating variation. These results confirm that the SCC should provide a reasonably good approximation of the present value of "small" policies such as individual regulations that cover a limited range of GHG emitting activities or sectors of the economy, but for "large" policies such as an economy-wide carbon tax or cap-and-trade system the second-order effects may be important and complete simulations of both the baseline and policy paths may be needed. Also note that this experiment captures only the "income effect" of large changes. This is because our model treats the paths of technological change and emissions as exogenous. A general equilibrium

[19] If costs were included in our representative-agent model, then $CV > 0$ would be a necessary and sufficient condition for the policy to increase social welfare over the status quo. Adding the first period compensating variation for benefits to costs calculated in an independent analysis would introduce a separate error, increasing in the size of the policy. Thus, for very large policies a fully integrated assessment model that can handle benefits and costs simultaneously would be needed.

[20] The stated goal of the Obama administration is to reduce U.S. GHG emissions by 2050 by approximately 83 percent from 2005 levels. Given the U.S. baseline GHG projections, this amounts to increasing emission reductions by nearly 0.05 GtC each year starting from 0 GtC yr$^{-1}$ in 2010 and growing to 1.9 GtC yr$^{-1}$ in 2050 (United States Department of State 2010 Figure 5-1).

model, possibly with endogenous technical change, would be required to capture the additional feedback effects between these and the other state variables of the model and thereby give a more complete answer to the broader question of marginal vs. non-marginal changes. For these reasons, a more extensive set of simulation experiments with more sophisticated integrated assessment models would be needed to expand on these results and test their generality.

# 4 SUMMARY AND CONCLUSIONS

The model presented in this paper is mainly intended to generate rough estimates of the SCC under certainty and uncertainty, and as a learning tool to help build intuition about the factors that will most strongly influence the value of the SCC. To this end, we began with a formal definition of the SCC based on first principles of welfare economics and expected utility theory. Next we developed a simple integrated assessment model for calculating the SCC under uncertainty. We demonstrated how this model can be used to estimate the SCC and evaluate greenhouse gas emission reduction policies with a series of sensitivity analyses and other numerical experiments. We showed that the way in which uncertainty is treated can have a substantial impact on the resulting SCC estimates. In particular, we demonstrated that the certainty-equivalent SCC can be significantly larger than the expected SCC. The former is the appropriate shadow price for valuing small sure changes in greenhouse gas emissions, but the latter is what has been estimated in most previous studies of the SCC under uncertainty. We also examined the range of emission reductions for which the SCC can provide an accurate estimate of the equivalent present value of consumption. By comparing the present value estimated using the SCC to an exact measure of the first period compensating variation, we confirmed that the SCC can give a good approximation of welfare changes for reasonably small emission reduction policies. However, for large policy changes that would be required to approach the targets reported by the U.S. government by 2050, the approximation based on the SCC is much less accurate.

As with any model, the many simplifying assumptions underlying our SCC rapid assessment model should be kept in mind when interpreting its results. For example, all of the exogenous inputs to the model—namely, the per capita income, population, and emissions forecasts—will in reality be determined endogenously. Ideally, a more complex modeling approach based on a general equilibrium framework would be used to properly account for the causal linkages and thereby propagate the feedbacks among these state variables and the impacts of climate change. A related limitation of our model, and most other IAMs developed to date, is that only a single undifferentiated form of "consumption" is included, which implicitly assumes that all varieties of market and non-market goods and services are perfectly substitutable in the utility function (Hoel and Sterner 2007, Sterner and Persson 2008, Weitzman 2010a).

Other limitations of the model point to important areas for further research. For example, the economic loss function includes average surface temperature change as its only argument and is based on an expert elicitation survey conducted more than ten years ago. We can accept these limitations for our present purposes, but an up-to-date review and synthesis of empirical climate damage estimates based on a more detailed accounting of climate impacts—including sea level rise, ocean acidification, damages from extreme weather events, and more—and in different sectors and geographic regions would be highly useful. Such a review also would aid in identifying important gaps in the literature where additional empirical research is needed. Furthermore, most of the probability distributions for the uncertain parameters in our model could be refined with a more systematic review of the literature and more empirical study. In the meantime, our model could be used to help prioritize such research efforts by conducting more thorough sensitivity and uncertainty analyses to identify those parameters for which better information would have the largest impact on the resulting SCC estimates.

Finally, note that the certainty-equivalent social cost of carbon is suitable for estimating the benefits of small *sure* (i.e., certain) changes in emissions over time. If the time path of certainty-equivalent costs can be estimated in a companion analysis, then discounting these projected costs using the path of certainty-equivalent consumption discount rates will give an estimate of the present value of costs that can be subtracted from the present value of benefits to estimate the net present value of the policy (again, assuming that the policy is sufficiently small). Accounting for uncertainty in the projected changes in emissions and costs over time adds an extra layer of complexity, since in this case the analyst would need to account for the correlations between the projected emission changes and cost outcomes. An ideal analysis would employ a fully integrated dynamic stochastic general equilibrium model that could measure both benefits and costs and account for uncertainty in all key parameters and forecasts simultaneously, as well as the dynamics of learning about the uncertain state variables over time. Some preliminary work in these areas has already been done (e.g., Kelly and Kolstad 1999, Karp and Zhang 2001, Webster *et al*. 2008, Anda *et al*. 2009, Lemoine and Traeger 2010), but this brings us to the frontier of integrated assessment modeling. Much work remains to incorporate all of these complicating factors into a quantitative model that is suitable for realistic policy evaluations, so we highlight these as additional areas for future research. However, until such a "model of everything" is developed, the social cost of carbon estimated in a manner similar to that illustrated in this paper may still serve as a useful measure of the economic value of greenhouse gas emissions under uncertainty.

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# TABLES AND FIGURES

**Table 1.** Functional forms for all components of the Social Cost of Carbon Rapid Assessment Model (SCCRAM).

| Model component | Functional forms |
|---|---|
| Per capita income | $y_{t+1} = y_t e^{g_t}$; $g_t = g_\infty + (g_0 - g_\infty) e^{-\omega t}$ |
| Population | $N_{t+1} = N_t \left[1 + b_\infty + (b_0 - b_\infty) e^{-\theta_b t} - d_\infty - (d_0 - d_\infty) e^{-\theta_d t}\right]$ |
| Emissions | $x_{t+1} = x_t e^{h_t}$; $h_t = h_0 (1 - t/t_p)$; $\sum_{t=0}^{H} x_t \le R$ |
| Atmospheric carbon stock | $X_{t+1} = [1 - \beta_F f_t - \beta_S (1 - f_t)] X_t + (\alpha_F + \alpha_S)(n + x_t)$ |
| Fraction of carbon in "fast" compartment | $f_{t+1} = [(1 - \beta_F) X_t f_t + \alpha_F (n + x_t)] / X_{t+1}$ |
| Background emissions | $n = X_{PI} \beta_F \beta_S / (\alpha_F \beta_S + \alpha_S \beta_F)$ |
| Radiative forcing | $F_t = F_{2X} \ln(X_t / X_{PI}) / \ln 2$ |
| Ocean surface temperature | $\rho_O C_O h \frac{dT_t^O}{dt} = F_t - \frac{F_{2X}}{T_{2X}} T_t^O + \kappa \frac{\partial T_{t,0}^D}{\partial z} + \nu \frac{a_L}{1 - a_L} \left[T_t^L - T_t^O\right] + \rho_O C_O w \left[T_t^O - T_{t,Z}^D\right]$ |
| Land surface temperature | $\mu \frac{dT_t^L}{dt} = F_t - \frac{F_{2X}}{T_{2X}} T_t^L - \nu \left[T_t^L - T_t^O\right]$ |
| Deep ocean layers temperatures | $\frac{\partial T_{t,z}^D}{\partial t} = \chi \frac{\partial^2 T_{t,z}^D}{\partial z^2} - w \frac{\partial T_{t,z}^D}{\partial z}$ |
| Global average surface temperature | $T_t = a_L T_t^L + (1 - a_L) T_t^O$ |
| Physical capital, output, and consumption | $K_{t+1} = K_t (1 - \delta_K) + s Y_t$; $Y_t = A_t K_t^\gamma N_t^{1-\gamma} (1 - L_t)$; $C_t = (1 - s) Y_t$ |
| Economic loss (damage function) | $L_t = a \left(T_t - T_{neg}\right)^b$ |
| Utility of representative agent | $u_t = \left((C_t / N_t)^{1-\eta} - c_{sub}^{1-\eta}\right) / (1 - \eta)$ |
| Social welfare | $W_t = \sum_{\tau=t}^{H} N_\tau u_\tau e^{-\rho(\tau - t)}$ |
| Social cost of carbon | $SCC_t = -\frac{\partial \mathbb{E}[W_0] / \partial x_t}{\partial \mathbb{E}[W_0] / \partial C_t}$ |

**Table 2.** Input parameter descriptions and assigned or calibrated central values and probability density functions. Initial values, with subscript "0," refer to the year 2005. See Figure 2 for examples of the piecewise linear probability density functions represented in the "Uncertain" column.

| Symbol | Description [units] | Certain | | | Uncertain |
|---|---|---|---|---|---|
| | | Mode | Median | Mean | [Nodes] [Relative prob.] |
| $y_0$ | Initial per cap. income [ 2005$US ] | 7004 | 7004 | 7004 | [7004] [1] |
| $g_0$ | Initial per cap. income growth [ $yr^{-1}$ ] | 0.022 | 0.02 | 0.0197 | [0.013, 0.022, 0.024] [0, 1, 0] |
| $g_\infty$ | Long-run per cap. income growth [ $yr^{-1}$ ] | 0.01 | 0.01 | 0.01 | [0, 0.01, 0.02] [0, 1, 0] |
| $\omega$ | Convergence rate of per capita income growth [ $yr^{-1}$ ] | 0.0036 | 0.0036 | 0.0036 | [0.0036] [1] |
| $N_t$ | Global population | Exogenously specified to fall within range of U.N. scenarios. See Table 3. | | | |
| $x_0$ | Initial emissions [ GtC $yr^{-1}$ ] | 8.5 | 8.5 | 8.5 | [ 8.5 ] [ 1 ] |
| $h_0$ | Initial rate of emissions growth [ $yr^{-1}$ ] | 0.0205 | 0.0205 | 0.0205 | [0.011, 0.0205, 0.03] [0, 1, 0] |
| $t_p$ | Time to emissions peak [ yr ] | 200 | 194 | 192 | [75, 200, 300] [0, 1, 0] |
| $R$ | Total fossil fuel reserves [ GtC ] | 5000 | 5524 | 5659 | [2000, 5000, 10000] [0, 1, 0] |
| $X_0$ | Initial (2005) atmospheric $CO_2$ stock [ GtC ] | 804 | 804 | 804 | [804] [1] |
| $\beta_F$ | Removal rate of carbon from fast compartment [ $yr^{-1}$ ] | 0.01 | 0.01 | 0.01 | [0.01] [1] |
| $\beta_S$ | Removal rate of carbon from slow compartment [ $yr^{-1}$ ] | 8.6E-4 | 8.6E-4 | 8.6E-4 | [8.6E-4] [1] |
| $\alpha_F$ | Fraction of emissions entering fast compartment | 0.29 | 0.29 | 0.29 | [0.245, 0.335] [1, 1] |
| $\alpha_S$ | Fraction of emissions entering slow compartment | 0.29 | 0.29 | 0.29 | [0.245, 0.335] [1, 1] |
| $F_{2X}$ | Forcing if $X_t = 2X_{PI}$ [ W $m^{-2}$ ] | 3.7 | 3.7 | 3.7 | [3.7] [1] |
| $T_{2X}$ | Temperature change from a sustained doubling of $CO_2$ [ K ] | 3 | 3.45 | 3.74 | [1.2 3 6 10] [0 1 .0929 0] |
| $X_{PI}$ | Pre-industrial atmospheric $CO_2$ stock [ GtC ] | 594 | 594 | 594 | [594] [1] |
| $\rho_O$ | Density of ocean water [ kg $m^{-3}$ ] | 1000 | 1000 | 1000 | [1000] [1] |
| $c_O$ | Specific heat of ocean water [ J $kg^{-1}$ $K^{-1}$ ] | 4218 | 4218 | 4218 | [4218] [1] |
| $h$ | Depth of surface ocean layer [ m ] | 75 | 75 | 75 | [75] [1] |
| $\kappa$ | Ocean thermal conductivity [ J $s^{-1}$ $m^{-2}$ $K^{-1}$ ] | 632.7 | 632.7 | 632.7 | [632.7] [1] |

| | | | | | |
|---|---|---|---|---|---|
| $\nu$ | Land-ocean coupling [ J $s^{-1}$ $m^{-2}$ $K^{-1}$ ] | 2.83 | 2.83 | 2.83 | [2.83] [1] |
| $a_L$ | Fraction of Earth's surface covered by land | 0.3 | 0.3 | 0.3 | [0.3] [1] |
| $w$ | Ocean upwelling rate [ m $s^{-1}$ ] | -1.3E-7 | -1.3E-7 | -1.3E-7 | [-1.3E-7] [1] |
| $\mu$ | Thermal capacity of land [ J $K^{-1}$ $m^{-2}$ ] | 1E7 | 1E7 | 1E7 | [1E7] [1] |
| $\chi$ | Ocean diffusivity [ $m^2$ $s^{-1}$ ] | 1.5E-4 | 1.5E-4 | 1.5E-4 | [1.54E-4] [1] |
| $T_0$ | Initial (2005) temperature anomaly [ K ] | 0.7 | 0.7 | 0.7 | [0.7] [1] |
| $s$ | Rate of saving | 0.22 | 0.22 | 0.22 | [0.22] [1] |
| $\gamma$ | Capital share | 0.33 | 0.33 | 0.33 | [0.33] [1] |
| $\delta_K$ | Capital depreciation rate [ $yr^{-1}$ ] | 0.1 | 0.1 | 0.1 | [0.1] [1] |
| $MPK$ | Marginal product of physical capital | 0.084 | 0.084 | 0.084 | [0.084] [1] |
| $L(3)$ | Loss if $T = 3\,^{\circ}\mathrm{C}$ [ fraction of income ] | 0.036 | 0.040 | 0.041 | [0.007, 0.036, 0.08] [0, 1, 0] |
| $L(6)$ | Loss if $T = 6\,^{\circ}\mathrm{C}$ [ fraction of income ] | 0.104 | 0.115 | 0.118 | [0.033, 0.104, 0.217] [0, 1, 0] |
| $T_{neg}$ | Temperature anomaly below which economic losses are negligible [ K ] | 0 | 0.59 | 0.67 | [0, 2] [1, 0] |
| $c_{sub}$ | Subsistence per capita consumption [ 2005$US ] | 365 | 365 | 365 | [365] [1] |
| $\eta$ | Elasticity of marginal utility / coefficient of relative risk aversion | 2 | 1.71 | 1.70 | [0.5, 1, 2, 3] [0, 0.75, 1, 0] |
| $\rho$ | Pure rate of time preference / utility discount rate [ $yr^{-1}$ ] | 0.01 | 0.0134 | 0.0140 | [0, 0.01, 0.03] [0.25, 1, 0.25] |

**Table 3.** Information sources and notes on assignments or calibrations of point values and probability density functions for all parameters.

| Symbol | Information sources and calibration notes |
|---|---|
| $y_0$ | World per capita GDP in 2005 current US$ (World Bank). |
| $g_0$ | Our modal value is based on Maddison (2007 Table 7.10), who projected the growth of per capita world GDP to 2030 to be 2.23% per year, and Duval and de la Maisonneuve (2009 Table 2), who projected the growth of per capita world GDP over the next two decades to be 2.2% per year. Lower and upper bound values are the smallest and largest of the ten most recent 5-year running average world GDP per capita growth rates (World Bank). |
| $g_\infty$ | Lucas (2000) noted that "the per capita income growth in the leading economies is more like 0.015 in the postwar period, and even slower than that since 1970." We use a central value for the very long run of 1% per year. We use lower and upper bound values of minus and plus one hundred percent of our central value. |
| $\omega$ | Calibrated to match the decline in the per capita income growth rate between 2000 and 2100 by Lucas (2000 Figure 3), which was roughly one sixth. |
| $N_t$ | The six parameters of the population growth function— $b_0$, $d_0$, $b_\infty$, $d_\infty$, $\theta_b$, and $\theta_d$ —were calibrated to match the low, central, and high population growth scenarios in the United Nations' long run population projections (United Nations 2004 Figure 7). Specifically, the asymptotic average annual mortality rate, $d_\infty$, was constrained to lie between 0 and 0.01, and then the parameters were adjusted to minimize the sum of squared deviations between our model projections and the global population projections by the UN in 2050, 2100, 2150, 2200, 2250, and 2300. The U.N. global population projections for these years for the low scenario were 7.4, 5.5, 3.9, 3.2, 2.7, and 2.3 billion. Projections for the middle scenario were 8.9, 9.1, 8.5, 8.5, 8.8, and 9 billion. Projections for the high scenario were 10.6, 14, 16.7, 21.2, 27.8, and 36.4 billion. |
| $h_0$ | Raupach *et al*. (2007) reported that the global $CO_2$ emissions growth rate was around 1.1% per year between 1990 and 1999 and greater than 3% per year between 2000 and 2004. We use these estimates as our lower and upper bound values, and we use the midpoint of this range for our central value. |
| $t_p$ | DICE2007 (Nordhaus 2008) projected that under a business-as-usual scenario GHG emissions will peak in approximately 200 years. Some research suggests that the peak in fossil fuel use may occur in 100 years or less (Kharecha and Hansen 2008, Brecha 2008, Shafiee and Topal 2009). |
| $R$ | Archer (2009 p 103). |
| $X_0$, $X_{PI}$ | Solomon *et al*. (2007 p 25), converted from ppm to GtC using 1 ppm $CO_2$ = 2.12 GtC (Huggett 1993 p 114). |
| $\beta_F$ | The IPCC TAR cited a range of 5-200 years for the atmospheric lifetime of carbon, noting that "No single lifetime can be defined for $CO_2$ because of the different rates of uptake by different removal processes" (http://www.grida.no/publications/ other/ipcc_tar/?src=/CLIMATE/IPCC_TAR/WG1/016.htm). We assume an average atmospheric lifetime of 100 years for carbon in the "fast" compartment of our 2-box model. |
| $\beta_S$ | Using our central greenhouse gas emissions trajectory and central estimates for $\alpha_F$ and $\alpha_S$, $\beta_S$ was calibrated so that 25% of the cumulative anthropogenic emissions of carbon still resided in the atmosphere after 1000 years (Archer 2005 p 5). For this calibration we assumed that the cumulative anthropogenic emissions prior to 2005 was 5E11 metric tons of carbon. |
| $\alpha_F$ | The IPCC Third Assessment Report: "Fossil fuel burning… released on average 5.4 $\pm$ 0.3 PgC/yr during |

| | |
|---|---|
| $\alpha_S$ | 1980 to 1989, and 6.3 $\pm$ 0.4 PgC/yr during 1990 to 1999…The rate of increase of atmospheric $CO_2$ content was 3.3 $\pm$ 0.1 PgC/yr during 1980 to 1989 and 3.2 $\pm$ 0.1 PgC/yr during 1990 to 1999. These rates are less than the emissions, because some of the emitted $CO_2$ dissolves in the oceans, and some is taken up by terrestrial ecosystems" (Prentice *et al*. 2001 p 185). We use the minimum and maximum of the ratios based on all eight possible combinations of the upper and lower ends of these ranges for each time period, which gives a range of 0.49 to 0.67. We assume that half of the carbon that enters the atmosphere enters the fast compartment and half enters the slow compartment. |
| $F_{2X}$ | Ramaswamy *et al*. (2001 p 357). |
| $\rho_0, C_O, h,$ $\kappa, w, \chi$ | Baker and Roe (2009). |
| $\nu$, $a_L$, $\mu$ | Lindzen and Giannitsis (1988). |
| $T_{2X}$ | The most commonly cited "likely" range for equilibrium climate sensitivity is 1.5 to 4.5 deg C (e.g., NAS 1979, Forster *et al*. 2007 p 788-799). In light of several recent studies that emphasize uncertainty on the high end of the climate sensitivity distribution (e.g., Roe and Baker 2007, Weitzman 2009, Zickfeld *et al*. 2010), in uncertainty analysis we use a default upper bound value of 10 deg C and in sensitivity analysis we examine an upper bound as high as 25 deg C. In all cases, to identify the height of the distribution for climate sensitivity at 3 deg C and 6 deg C, we constrained the cumulative probability that $T_{2X}$ is greater than 6 deg C to equal 0.073, as estimated by Newbold and Daigneault (2009) using a Bayesian model averaging approach combining climate sensitivity estimates from 21 prior studies. |
| $s$ | Average of world gross savings as percent of GDP between 1976-2007 (World Bank). |
| $\gamma$ | Caselli and Feyer (2005 p 17). |
| $\delta_K$ | Intermediate value among the estimates reported by Nadiri and Prucha (1996 Table II). |
| $MPK$ | Caselli and Feyer (2007 Table III). Used to calibrate the initial capital stock, i.e., $MPK \equiv \partial Y_0 / \partial K_0 = \gamma Y_0 / K_0$. |
| $L(3)$ $L(6)$ | Based on a survey of climate change experts by Nordhaus, as reported by Roughgarden and Schneider (1999). The survey asked respondents for their best professional judgments of the likely damages across all market and non-market sectors, as a percentage of global GDP, if the global average temperature increased by 3 or 6 deg C by the year 2100. Our central values are the means of the responses across all experts, and our lower and upper bound values are the 10th and 90th percentile responses, respectively. In each Monte Carlo draw, the parameters *a* and *b* in the loss function are calibrated using the draw of $T_{neg}$, $L(3)$, and $L(6)$. |
| $T_{neg}$ | Our modal and lower bound value ($T_{neg} = 0$ deg C) assumes that any additional warming will cause economic losses. Our upper bound value ($T_{neg} = 2$ deg C) is based on Hope (2006). |
| $c_{sub}$ | World Bank (1990). |
| $\eta$ | Our modal value of 2 is near the middle of the range of values estimated or used by Szpiro (1986), Hall and Jones (2007), Arrow (2007), Dasgupta (2006, 2007), Weitzman (2007, 2009), Nordhaus (2008), and Hall (2010). Our lower and upper bound values of 0.5 and 3 are based on the wider range of estimates in Shepard and Zeckhauser (1984), Eeckhoudt and Hammitt (2001), Kaplow (2005), Barro (2006), Chetty (2006), and Weitzman (2010a,b). Values closer to 1 also seem highly plausible (e.g., Feldstein and Ranguelova 2001, Layard *et al*. 2008), so we chose 1 as another node and assigned it a relative probability of 0.75. |
| $\rho$ | Our modal value is based most directly on Arrow (1995). Our lower and upper bound values were chosen to cover most of the range found in climate economics studies including, among others, Arrow *et al*. (1996) and Portney and Weyant (1999). |

**Table 4.** Estimated time paths of the SCC [2005$US] and associated consumption discount rates, $r_t = -\ln\left(\delta_{t+1}/\delta_t\right)$ [yr $^{-1}$] for 50 years. The first three sets of estimates were calculated using the mode, median, and mean parameter values shown in the middle columns of Table 2. The fourth set of estimates are certainty-equivalent values calculated using 100,000 Monte Carlo iterations using random draws from the probability distributions summarized in the final column of Table 2.

| | Certain | | | | | | Uncertain | |
|---|---|---|---|---|---|---|---|---|
| | modes | | medians | | means | | | |
| Year | $SCC_t$ | $r_t$ | $SCC_t$ | $r_t$ | $SCC_t$ | $r_t$ | $SCC_t$ | $r_t$ |
| 2005 | 6.6 | .053 | 10 | .047 | 11 | .047 | 81 | 0.027 |
| 2015 | 8.9 | .053 | 14 | .047 | 15 | .047 | 104 | 0.027 |
| 2025 | 12 | .052 | 18 | .047 | 20 | .047 | 130 | 0.027 |
| 2035 | 15 | .052 | 23 | .046 | 24 | .046 | 160 | 0.027 |
| 2045 | 18 | .052 | 27 | .046 | 29 | .046 | 200 | 0.026 |
| 2055 | 22 | .051 | 32 | .046 | 34 | .045 | 240 | 0.026 |

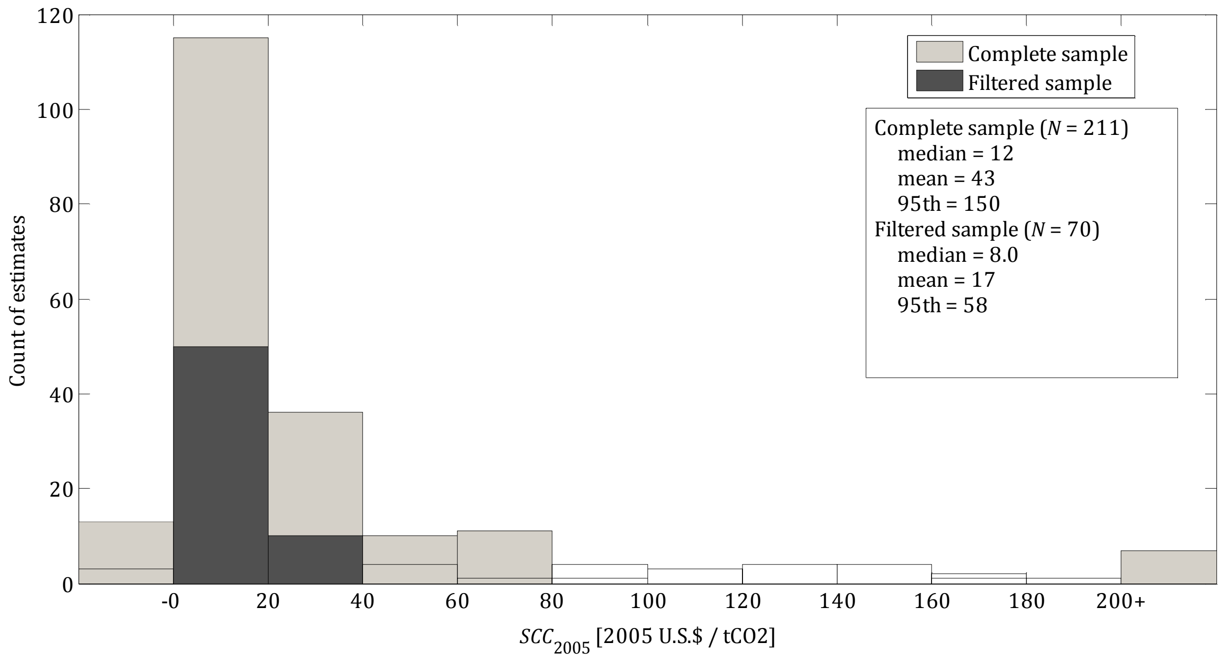
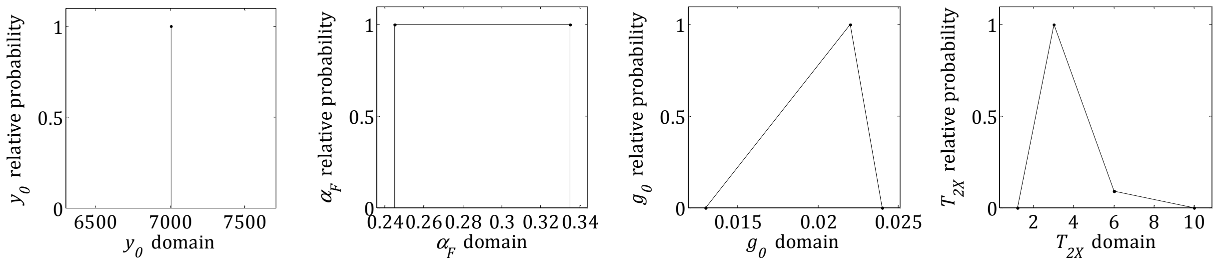



**Figure 1.** Histogram of SCC estimates assembled by Tol (2005, 2008). The complete sample includes all estimates gathered by Tol. The filtered sample excludes non-peer reviewed studies, those using equity weights, and those based on "unrealistic climate scenarios" (Tol 2005). All estimates assembled by Tol were assumed to be reported in 1995$US and were converted to 2005$US assuming a 2% real growth rate of the SCC and adjusted for inflation using the CPI.




These probability density functions are summarized in Table 2 as:

[7004] [1]    [0.245, 0.335] [1, 1]    [0.013, 0.022, 0.024] [0, 1, 0]    [1.2, 3, 6, 10] [0, 1, 0.0929, 0]

**Figure 2**. Illustrative piecewise linear probability density functions, as summarized in the "Uncertain" column of Table 2. Proper pdfs were constructed by dividing the range of each uncertain parameter into equal-sized increments and then dividing all relative probabilities by their sum to ensure that the discrete pdf integrates to one.

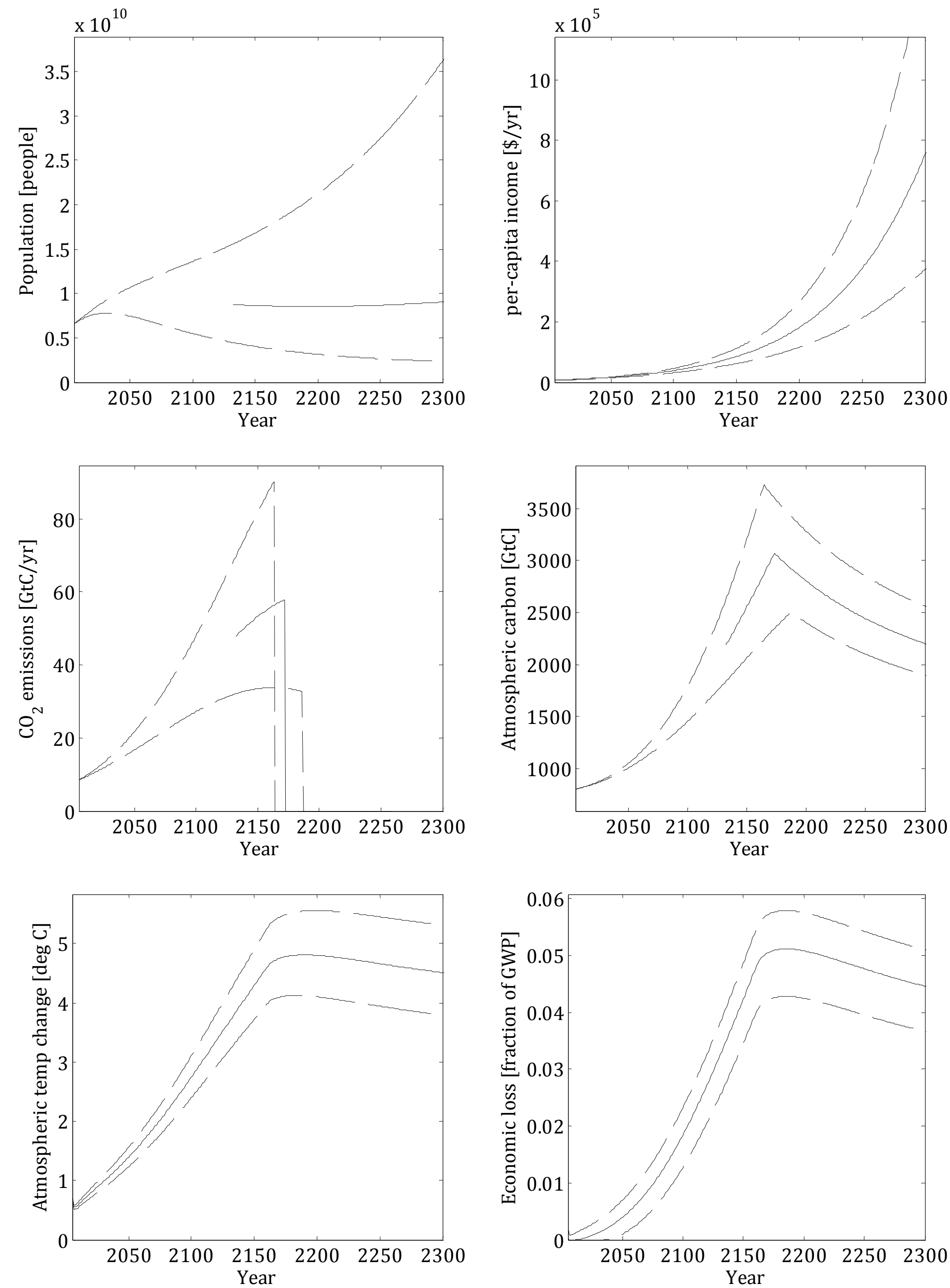



**Figure 3.** Projections of key state variables to 2300. Solid lines are based on the median parameter values, and dashed lines are based on the 25th and 75th percentile values of the associated parameter distributions listed in Table 2. For example, the bottom right graph shows the economic loss over time at the 25th, 50th, and 75th percentiles of the damage function parameters, holding all other parameters at their 50th percentile values.

| Parameter(s) | | A | B | C | Sensitivity | |
|---|---|---|---|---|---|---|
| Income growth | $g_0$<br>$g_\infty$ | .022<br>.013 | .020<br>.010 | .018<br>.007 | -8% | +11% |
| Population scenario | $N_t$ | $N_{low}$ | $N_{mid}$ | $N_{hi}$ | -16 | +23 |
| Emissions growth | $h_0$<br>$t_p$<br>$R$ | .023<br>225<br>6825 | .021<br>194<br>5524 | .018<br>159<br>4446 | -0.2% | +0.1% |
| Eq. climate sensitivity | $T_{2X}$ | 2.8 | 3.5 | 4.5 | -18% | +20% |
| Damages | $T_{neg}$<br>$L(3)$<br>$L(6)$ | 1.0<br>.030<br>.090 | 0.6<br>.040<br>.115 | 0.3<br>.052<br>.145 | -51% | +41% |
| Elasticity of m.u. | $\eta$ | 2.1 | 1.7 | 1.3 | -32% | +72% |
| Pure time preference | $\rho$ | .019 | .013 | .008 | -21% | +24% |

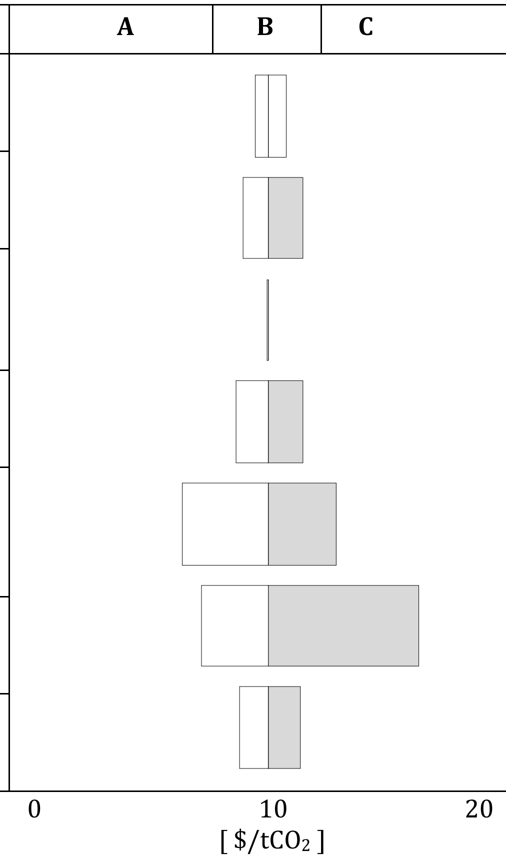



**Figure 4.** Deterministic sensitivity analysis. Each bar represents the change in the deterministic SCC in 2005 due to a change in one to three parameters from their median values to their 25th and 75th percentile values, holding all other parameters at their medians, relative to the benchmark estimate of $10 based on the median values of all parameters.

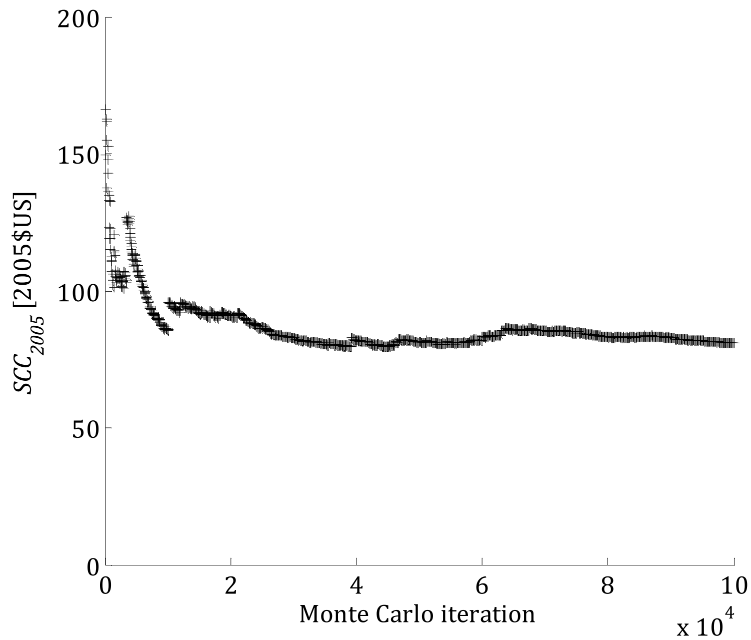



**Figure 5**. Running estimate of the SCC in 2005, in increments of 50 Monte Carlo iterations. Each point on the graph represents the certainty-equivalent SCC calculated using all Monte Carlo draws up to that iteration.  After roughly 30,000 iterations the SCC estimate has settled down to a narrow range.

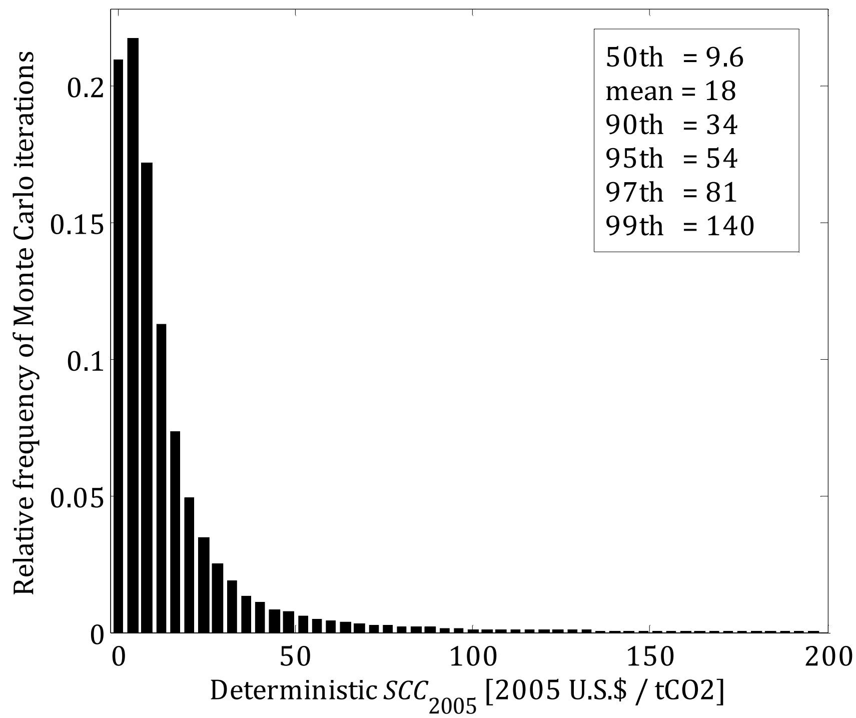



**Figure 6**. Frequency distribution of deterministic SCC values in 2005 from a Monte Carlo simulation with 100,000 iterations using the input parameter probability distributions shown in Table 2. The certainty-equivalent SCC estimate in 2005, $81/t$CO_2$, corresponds to the 97th percentile of this distribution.

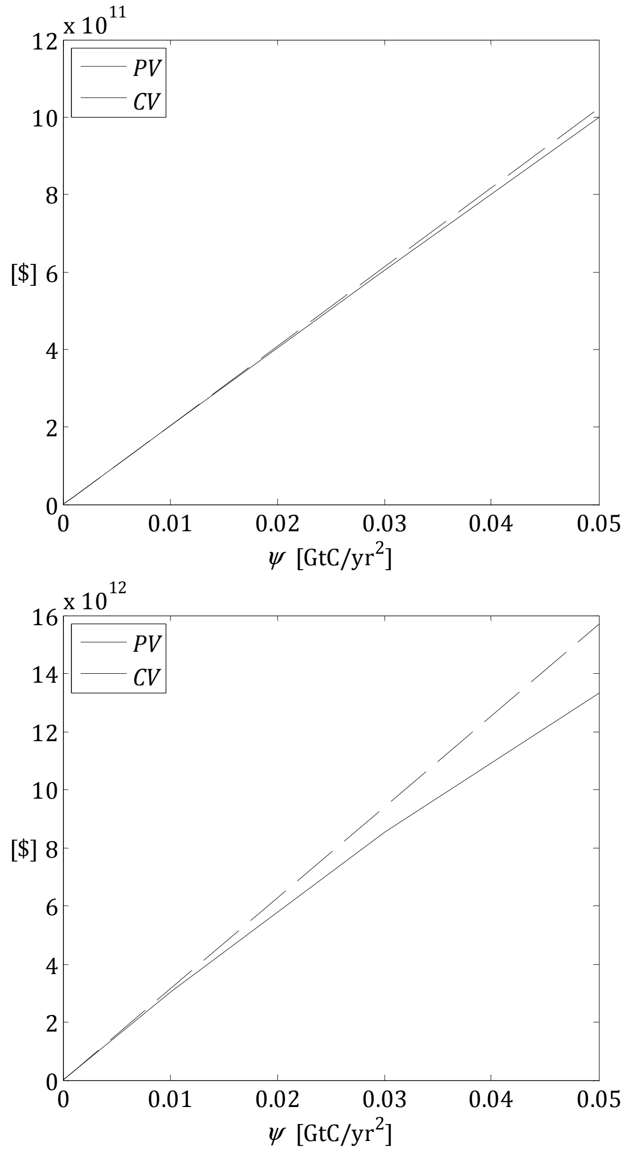



**Figure 7.** Exact and approximate values of 40-year emission reduction policies. The *x*-axis is the slope of the policy "ramp," i.e., the linear rate at which emissions reductions increase over time. The top graph is based on the deterministic model using the means of all input parameters, and the bottom graph is based on the Monte Carlo analysis using 1,000 draws from the probability distributions of all uncertain parameters. The dashed line in each graph shows the present value of benefits as approximated using the SCC, *PV*, and the solid line shows the aggregate first period compensating variation, *CV*.