

Derivation of Humidity and NOx Humidity Correction Factors

(This document follows the presentations in

"Vapor Pressure Equation for Water in the Range 0 to 100 °C", by Arnold Wexler and Lewis Greenspan, February 19, 1971
 JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 75A, No. 3, May-June 1971

"Vapor Pressure Formulation for Water in Range 0 to 100 °C. A Revision", by Arnold Wexler, July 15, 1976
 JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 80A, Nos. 5 and 6, September-December 1976

"Vapor Pressure Formulation for Ice", by Arnold Wexler, September 23, 1976
 JOURNAL OF RESEARCH of the National Bureau of Standards Vol. 81A, No. 1, September 23, 1976

"New Equations for Computing Vapor Pressure and Enhancement Factor", 11 May 1981 and 15 August 1981
 by Arden L. Buck, 1982 Journal of Applied Meteorology, 11 May 1981 and 15 August 1981.)

Input of Ambient Conditions

$$\text{Baro mmHg} := 750.0612$$

$$\text{Drybulb degC} := 20$$

$$\text{Dewpoint degC} := 10$$

$$\text{Baro inHg} := \frac{\text{Baro mmHg}}{25.4}$$

$$\text{Baro inHg} = 29.53$$

$$\text{Baro mb} := \text{Baro inHg} \cdot 33.86389$$

$$\text{Baro mb} = 1000$$

$$\text{Drybulb degF} := \text{Drybulb degC} \cdot \frac{9}{5} + 32$$

$$\text{Drybulb degF} = 68$$

$$\text{Drybulb degK} := 273.15 + \left(\frac{5}{9}\right) \cdot (\text{Drybulb degF} - 32)$$

$$\text{Drybulb degK} = 293.15$$

$$\text{Dewpoint degF} := \text{Dewpoint degC} \cdot \frac{9}{5} + 32$$

$$\text{Dewpoint degF} = 50$$

$$\text{Dewpoint degK} := 273.15 + \left(\frac{5}{9}\right) \cdot (\text{Dewpoint degF} - 32)$$

$$\text{Dewpoint degK} = 283.15$$

Derivation of Humidity and NOx Humidity Correction Factors

Compute Corrected Saturation Vapor Pressure for Dry Bulb Temperature above and below ice point. This value will be used to compute Relative Humidity.

The following are Wexler's most accurate formulations: Eq. (15) (Wexler, 1976) for vapor pressure over water and Eq. (5) (Wexler, 1977) for vapor pressure over ice. Wexler's formulations are given below. These calculations are most clearly presented in Buck's Eqs. (5a) and (5b) (Buck, 1981).

Absolute Temperature in K

$$\theta := \text{Drybulb}_{\text{degK}} \qquad \theta = 293.15$$

Vapor pressure over water

$$e_w := 0.01 \cdot \exp \left[\begin{array}{l} 2.858487 \cdot \ln(\theta) \dots \\ + \frac{-2991.2729}{\theta^2} + \frac{-6017.0128}{\theta} + 18.87643854 + 0.028354721 \cdot \theta \dots \\ + 1.7838301 \cdot 10^{-5} \cdot \theta^2 + -8.4150417 \cdot 10^{-10} \cdot \theta^3 \dots \\ + 4.4412543 \cdot 10^{-13} \cdot \theta^4 \end{array} \right] \qquad e_w = 23.3854$$

Vapor pressure over ice

$$e_i := 0.01 \cdot \exp \left[\begin{array}{l} 0.69186510 \cdot \ln(\theta) \dots \\ + \frac{-5865.3696}{\theta} + 22.24103300 + 1.3749042 \cdot 10^{-2} \cdot \theta \dots \\ + -3.4031775 \cdot 10^{-5} \cdot \theta^2 + 2.6967687 \cdot 10^{-8} \cdot \theta^3 \end{array} \right] \qquad e_i = 28.3145$$

The following will select the appropriate derived vapor pressure for the temperature range.

$$\text{PsatDry}_{\text{mb}} := \text{if}(\text{Drybulb}_{\text{degC}} \leq 0, e_i, e_w) \qquad \text{PsatDry}_{\text{mb}} = 23.3854$$

Wexler's formulations relate the saturation vapor pressure of pure water to temperature; a slight correction must be made when one is dealing with moist air rather than pure water vapor. This correction, called the enhancement factor, is a weak function of temperature and pressure. These calculations are in Buck's Eq. (6) and Table 3 (Buck, 1981).

Barometric Pressure in millibars

$$P := \text{Baro}_{\text{mb}} \qquad P = 1000$$

Enhancement for vapor pressure over water.

$$f_w := 1 + 0.00041 + P \cdot \left[3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot [(\theta - 273.15) + 30.6 - 0.038 \cdot P]^2 \right] \qquad f_w = 1.004$$

Enhancement for vapor pressure over ice.

$$f_i := 1 + 0.00048 + P \cdot \left[3.47 \cdot 10^{-6} + 5.9 \cdot 10^{-10} \cdot [(\theta - 273.15) + 23.8 - 0.031 \cdot P]^2 \right] \qquad f_i = 1.004$$

The following will select the appropriate vapor pressure enhancement for the temperature range.

$$\text{EnhanceDry} := \text{if}(\text{Drybulb}_{\text{degC}} \leq 0, f_i, f_w) \qquad \text{EnhanceDry} = 1.004$$

Apply the correction to finally obtain the enhanced partial saturation vapor pressure.

$$\text{PsatDryCorrected}_{\text{mb}} := \text{PsatDry}_{\text{mb}} \cdot \text{EnhanceDry} \qquad \text{PsatDryCorrected}_{\text{mb}} = 23.4792$$

$$\text{PsatDryCorrected}_{\text{inHg}} := \frac{\text{PsatDryCorrected}_{\text{mb}}}{33.86389} \qquad \text{PsatDryCorrected}_{\text{inHg}} = 0.693$$

Derivation of Humidity and NOx Humidity Correction Factors

Compute Corrected Saturation Vapor Pressure for Dew Point Temperature above and below ice point. This value will be used in Relative and Specific Humidity derivations.

The following are Wexler's most accurate formulations: Eq. (15) (Wexler, 1976) for vapor pressure over water and Eq. (5) (Wexler, 1977) for vapor pressure over ice. Wexler's formulations are given below. These calculations are most clearly presented in Buck's Eqs. (5a) and (5b) (Buck, 1981).

Absolute Temperature in K

$$\theta := \text{Dewpoint}_{\text{degK}}$$

$$\theta = 283.15$$

Vapor pressure over water

$$e_w := 0.01 \cdot \exp \left[\begin{array}{l} 2.858487 \cdot \ln(\theta) \dots \\ + \frac{-2991.2729}{\theta^2} + \frac{-6017.0128}{\theta} + 18.87643854 + 0.028354721 \cdot \theta \dots \\ + 1.7838301 \cdot 10^{-5} \cdot \theta^2 + -8.4150417 \cdot 10^{-10} \cdot \theta^3 \dots \\ + 4.4412543 \cdot 10^{-13} \cdot \theta^4 \end{array} \right] \quad e_w = 12.2794$$

Vapor pressure over ice

$$e_i := 0.01 \cdot \exp \left[\begin{array}{l} 0.69186510 \cdot \ln(\theta) \dots \\ + \frac{-5865.3696}{\theta} + 22.24103300 + 1.3749042 \cdot 10^{-2} \cdot \theta \dots \\ + -3.4031775 \cdot 10^{-5} \cdot \theta^2 + 2.6967687 \cdot 10^{-8} \cdot \theta^3 \end{array} \right] \quad e_i = 13.5203$$

The following will select the appropriate derived vapor pressure for the temperature range.

$$\text{PsatDew}_{\text{mb}} := \text{if}(\text{Dewpoint}_{\text{degC}} \leq 0, e_i, e_w)$$

$$\text{PsatDew}_{\text{mb}} = 12.2794$$

Wexler's formulations relate the saturation vapor pressure of pure water to temperature; a slight correction must be made when one is dealing with moist air rather than pure water vapor. This correction, called the enhancement factor, is a weak function of temperature and pressure. These calculations are in Buck's Eq. (6) and Table 3 (Buck, 1981).

Barometric Pressure in millibars

$$P := \text{Baro}_{\text{mb}}$$

$$P = 1000$$

Enhancement for vapor pressure over water.

$$f_w := 1 + 0.00041 + P \cdot \left[3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot [(\theta - 273.15) + 30.6 - 0.038 \cdot P]^2 \right] \quad f_w = 1.004$$

Enhancement for vapor pressure over ice.

$$f_i := 1 + 0.00048 + P \cdot \left[3.47 \cdot 10^{-6} + 5.9 \cdot 10^{-10} \cdot [(\theta - 273.15) + 23.8 - 0.031 \cdot P]^2 \right] \quad f_i = 1.004$$

The following will select the appropriate vapor pressure enhancement for the temperature range.

$$\text{EnhanceDew} := \text{if}(\text{Dewpoint}_{\text{degC}} \leq 0, f_i, f_w)$$

$$\text{EnhanceDew} = 1.0039$$

Apply the correction to finally obtain the enhanced partial saturation vapor pressure.

$$\text{PsatDewCorrected}_{\text{mb}} := \text{PsatDew}_{\text{mb}} \cdot \text{EnhanceDew}$$

$$\text{PsatDewCorrected}_{\text{mb}} = 12.3272$$

$$\text{PsatDewCorrected}_{\text{inHg}} := \frac{\text{PsatDewCorrected}_{\text{mb}}}{33.86389}$$

$$\text{PsatDewCorrected}_{\text{inHg}} = 0.364$$

Derivation of Humidity and NOx Humidity Correction Factors

Compute Relative Humidity, Specific Humidity and NOx Humidity Corrections Factors per 40 CFR 86.144, 86.344, 86.544, and 86.1342

$$\text{RelativeHumidity}_{\text{pct}} := 100 \cdot \frac{\text{PsatDewCorrected}_{\text{inHg}}}{\text{PsatDryCorrected}_{\text{inHg}}} \qquad \text{RelativeHumidity}_{\text{pct}} = 52.503$$

The following equations have the Dry Bulb Vapor Pressure factored out and are presented in English Units..

$$\text{SpecificHumidity}_{\text{grperlb}} := \frac{4347.8 \cdot \text{PsatDewCorrected}_{\text{inHg}}}{\text{Baro}_{\text{inHg}} - \text{PsatDewCorrected}_{\text{inHg}}} \qquad \text{SpecificHumidity}_{\text{grperlb}} = 54.265$$

$$\text{NOxCorrection}_{\text{gasoline}} := \frac{1}{\left[1 - 0.0047 \cdot (\text{SpecificHumidity}_{\text{grperlb}} - 75) \right]} \qquad \text{NOxCorrection}_{\text{gasoline}} = 0.9112$$

86.1342 for Diesel Engines

$$\text{NOxCorrection}_{\text{diesel}} := \frac{1}{\left[1 - 0.0026 \cdot (\text{SpecificHumidity}_{\text{grperlb}} - 75) \right]} \qquad \text{NOxCorrection}_{\text{diesel}} = 0.9488$$

86.344 Calculation of Specific Humidity - SI Units

$R_a := \text{RelativeHumidity}_{\text{pct}}$

Grams of H2O per gram of dry air

$K := 0.6220$

$P_a := \text{Baro}_{\text{mb}} \cdot 100$

$P_v := \text{PsatDewCorrected}_{\text{mb}} \cdot 100$

Specific Humidity, Grams H2) per gram of dry air

$$H := \frac{K \cdot P_v}{P_a - P_v} \qquad H = 0.00776$$

86.144 and 86.1342 SI Unts

$$\text{SpecificHumidity}_{\text{grperkg}} := \frac{6.211 \cdot \text{RelativeHumidity}_{\text{pct}} \cdot \text{PsatDryCorrected}_{\text{mb}}}{\text{Baro}_{\text{mb}} - \text{PsatDryCorrected}_{\text{mb}}} \cdot \frac{\text{RelativeHumidity}_{\text{pct}}}{100} \qquad \text{SpecificHumidity}_{\text{grperkg}} = 7.752$$

$$\text{NOxCorrection}_{\text{gasoline}} := \frac{1}{\left[1 - 0.0329 \cdot (\text{SpecificHumidity}_{\text{grperkg}} - 10.71) \right]} \qquad \text{NOxCorrection}_{\text{gasoline}} = 0.9113$$

86.1342 for Diesel Engines

$$\text{NOxCorrection}_{\text{diesel}} := \frac{1}{\left[1 - 0.0182 \cdot (\text{SpecificHumidity}_{\text{grperkg}} - 10.71) \right]} \qquad \text{NOxCorrection}_{\text{diesel}} = 0.9489$$

Derivation of Humidity and NOx Humidity Correction Factors

The following derivations support plotting of vapor pressure, humidity and NOx correction factors.

$i := 0..90$

$$t_i := (i - 40) + 273.15$$

$$e_i := \text{if } [t_i > 273.15, \left[\begin{array}{l} 2.858487 \cdot \ln(t_i) \dots \\ + \frac{-2991.2729}{(t_i)^2} + \frac{-6017.0128}{t_i} + 18.87643854 + -0.028354721 \cdot t_i \dots \\ + 1.7838301 \cdot 10^{-5} \cdot (t_i)^2 + -8.4150417 \cdot 10^{-10} \cdot (t_i)^3 \dots \\ + 4.4412543 \cdot 10^{-13} \cdot (t_i)^4 \end{array} \right], 0]$$

$$e_i := \text{if } [t_i > 273.15, e_i, \left[\begin{array}{l} 0.69186510 \cdot \ln(t_i) \dots \\ + \frac{-5865.3696}{t_i} + 22.24103300 + 1.3749042 \cdot 10^{-2} \cdot t_i \dots \\ + -3.4031775 \cdot 10^{-5} \cdot (t_i)^2 + 2.6967687 \cdot 10^{-8} \cdot (t_i)^3 \end{array} \right]]$$

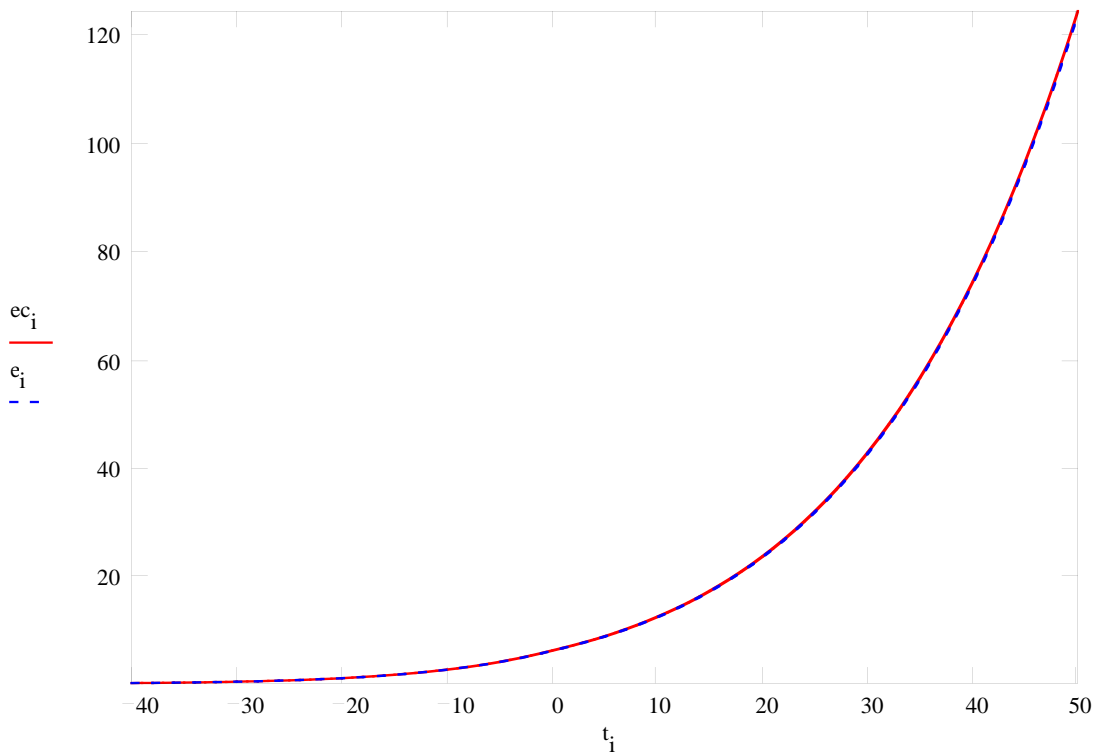
$$e_i := 0.01 \exp(e_i)$$

$$f_i := \text{if } [t_i > 273.15, 1 + 0.00041 + P \cdot [3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot [(t_i - 273.15) + 30.6 - 0.038 \cdot P]^2], 0]$$

$$f_i := \text{if } [t_i > 273.15, f_i, 1 + 0.00048 + P \cdot [3.47 \cdot 10^{-6} + 5.9 \cdot 10^{-10} \cdot [(t_i - 273.15) + 23.8 - 0.031 \cdot P]^2]]$$

$$ec_i := e_i \cdot f_i$$

$$t_i := t_i - 273.15$$

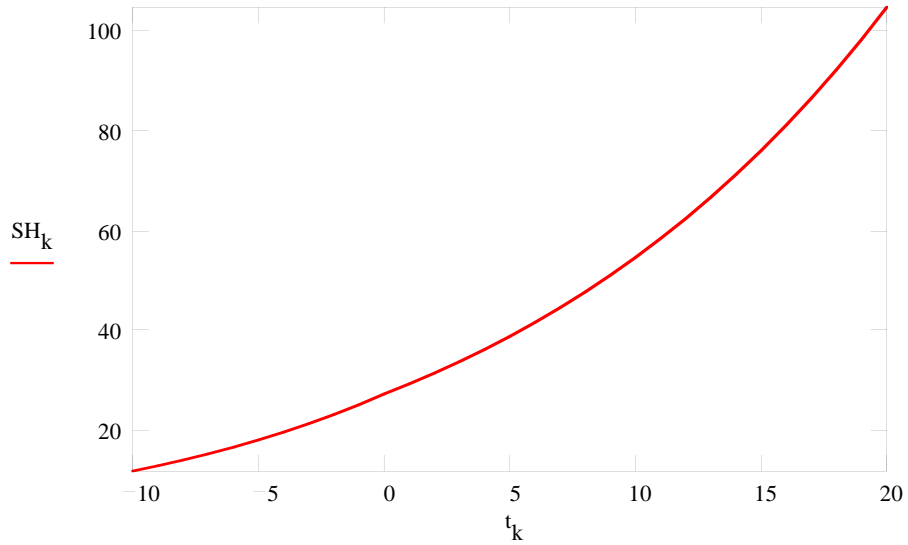


Corrected and Uncorrected Partial Saturation Vapor Pressure -40 to 50 °C

Derivation of Humidity and NOx Humidity Correction Factors

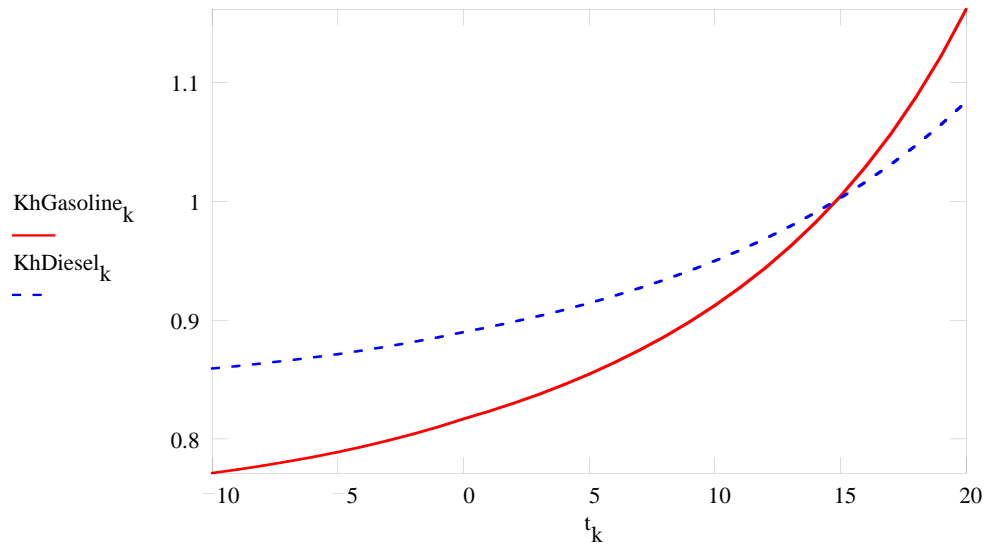
$$SH_i := \frac{4347.8 \cdot \frac{ec_i}{33.86389}}{\text{Baro inHg} - \frac{ec_i}{33.86389}}$$

k := 30..60



$$KhGasoline_i := \frac{1}{[1 - 0.0047 \cdot (SH_i - 75)]}$$

$$KhDiesel_i := \frac{1}{[1 - 0.0026 \cdot (SH_i - 75)]}$$



Derivation of Humidity and NOx Humidity Correction Factors

The following may be compared to Buck, 1981, Table 1 to verify agreement.
(40 is added to each subscript to bring it into a useable range of values.)

Vapor pressure pure water from -40 through 50 °C

$$e_{40+40} = 0.128486$$

$$e_{30+40} = 0.380239$$

$$e_{20+40} = 1.032761$$

$$e_{10+40} = 2.599229$$

$$e_{0+40} = 6.111536$$

$$e_{10+40} = 12.279396$$

$$e_{20+40} = 23.385445$$

$$e_{30+40} = 42.45202$$

$$e_{40+40} = 73.812731$$

$$e_{50+40} = 123.447791$$

Enhancement factor @ 1000 mb from -40 through 50 °C

$$f_{40+40} = 1.005264$$

$$f_{30+40} = 1.004766$$

$$f_{20+40} = 1.004387$$

$$f_{10+40} = 1.004125$$

$$f_{0+40} = 1.003981$$

$$f_{10+40} = 1.003895$$

$$f_{20+40} = 1.004007$$

$$f_{30+40} = 1.004268$$

$$f_{40+40} = 1.004676$$

$$f_{50+40} = 1.005233$$

Vapor pressure of moist air at 1000 mb from -40 through 50 °C

$$ec_{40+40} = 0.129163$$

$$ec_{30+40} = 0.382051$$

$$ec_{20+40} = 1.037291$$

$$ec_{10+40} = 2.60995$$

$$ec_{0+40} = 6.135863$$

$$ec_{10+40} = 12.327225$$

$$ec_{20+40} = 23.479161$$

$$ec_{30+40} = 42.633204$$

$$ec_{40+40} = 74.157912$$

$$ec_{50+40} = 124.093784$$

Derivation of Humidity and NOx Humidity Correction Factors

The following derivations support plotting of vapor pressure, humidity and NOx correction factors.
 $i := 0..90$

$$t_i := (i - 40) + 273.15$$

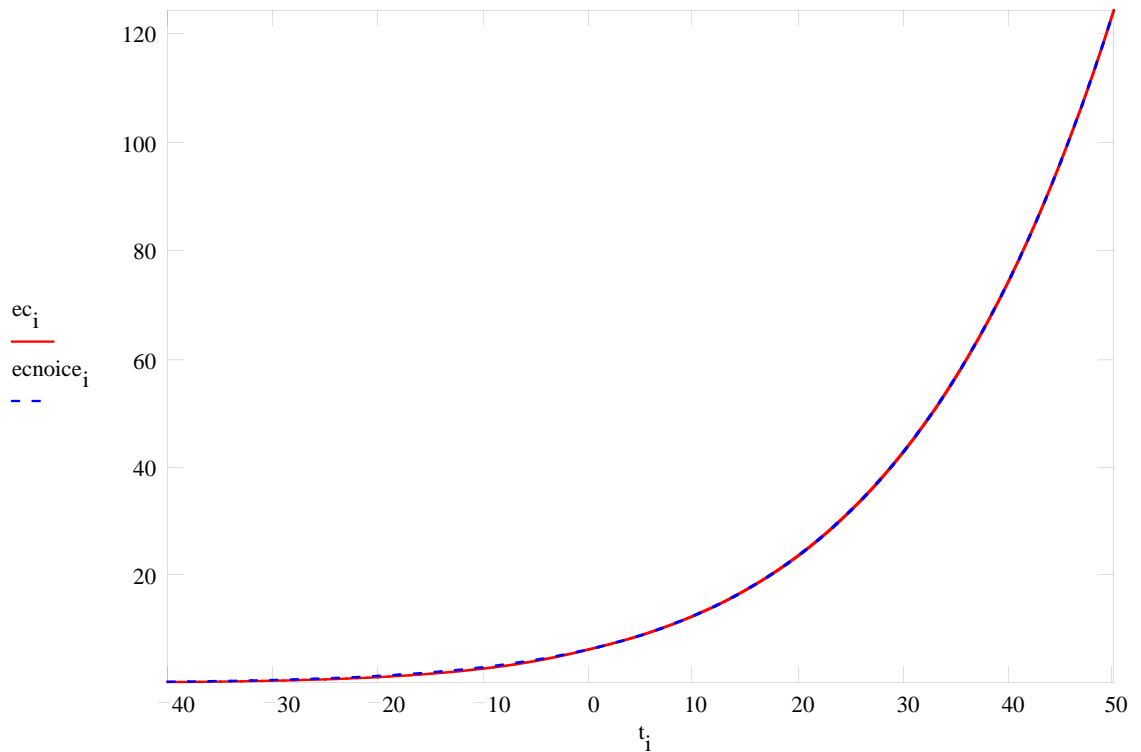
$$\text{enoice}_i := \text{if } [t_i > 0, \left[\begin{array}{l} 2.858487 \cdot \ln(t_i) \dots \\ + \frac{-2991.2729}{(t_i)^2} + \frac{-6017.0128}{t_i} + 18.87643854 + -0.028354721 \cdot t_i \dots \\ + 1.7838301 \cdot 10^{-5} \cdot (t_i)^2 + -8.4150417 \cdot 10^{-10} \cdot (t_i)^3 \dots \\ + 4.4412543 \cdot 10^{-13} \cdot (t_i)^4 \end{array} \right], 0]$$

$$\text{enoice}_i := 0.01 \exp(\text{enoice}_i)$$

$$\text{fnoice}_i := \text{if } [t_i > 0, 1 + 0.00041 + P \cdot [3.48 \cdot 10^{-6} + 7.4 \cdot 10^{-10} \cdot [(t_i - 273.15) + 30.6 - 0.038 \cdot P]^2], 0]$$

$$\text{ecnoice}_i := \text{enoice}_i \cdot \text{fnoice}_i$$

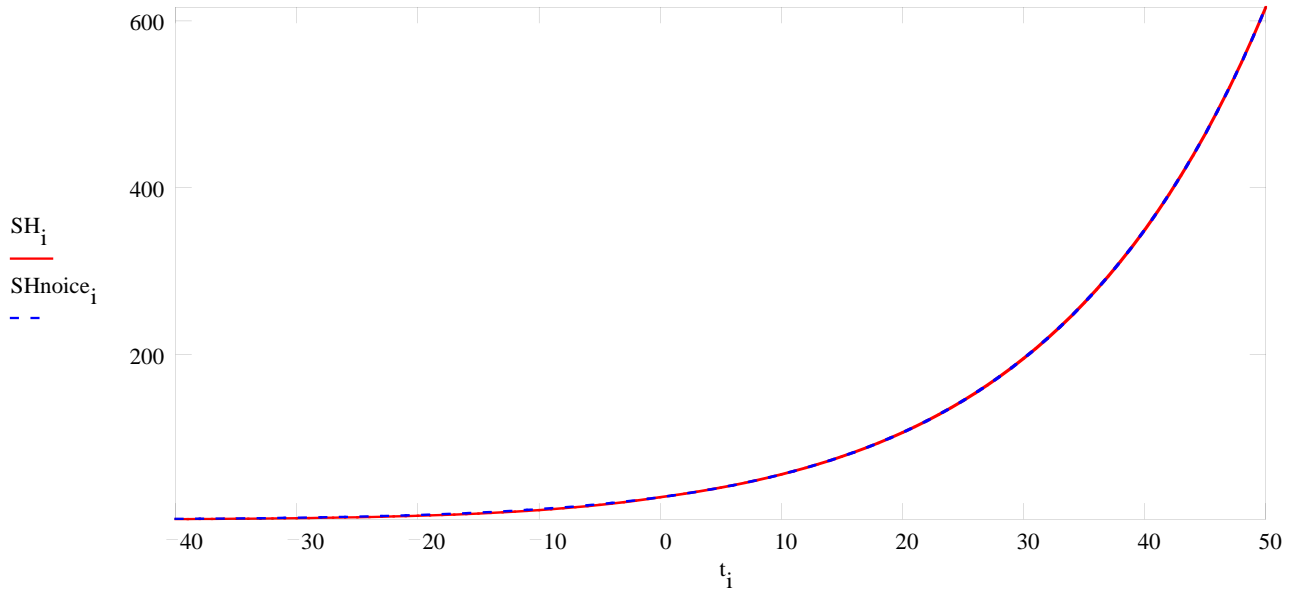
$$t_i := t_i - 273.15$$



Ice Point and No Ice Point Partial Saturation Vapor Pressure -40 to 50 °C

Derivation of Humidity and NOx Humidity Correction Factors

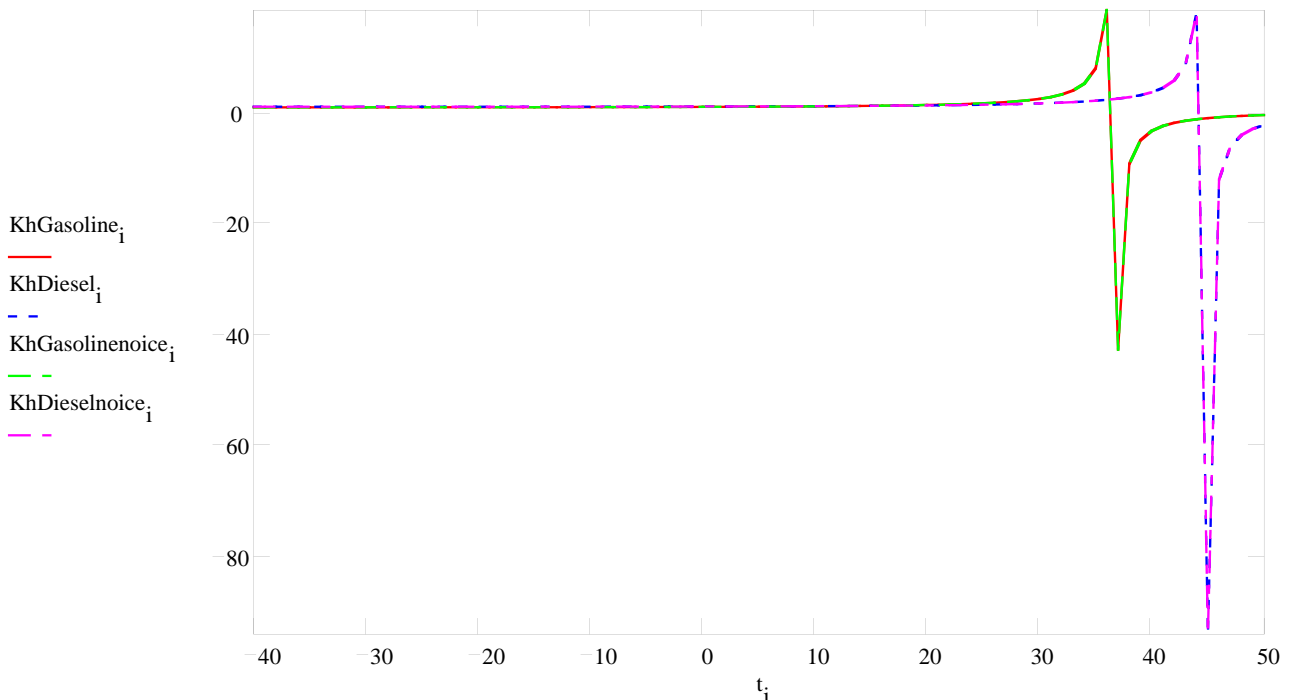
$$SH_{noice_i} := \frac{4347.8 \cdot \frac{ecnoice_i}{33.86389}}{Baro \text{ inHg} - \frac{ecnoice_i}{33.86389}}$$



Specific Humidity from -40 through 50 °C

$$KhGasolinenoice_i := \frac{1}{1 - 0.0047 \cdot (SHnoice_i - 75)}$$

$$KhDieselnoice_i := \frac{1}{1 - 0.0026 \cdot (SHnoice_i - 75)}$$

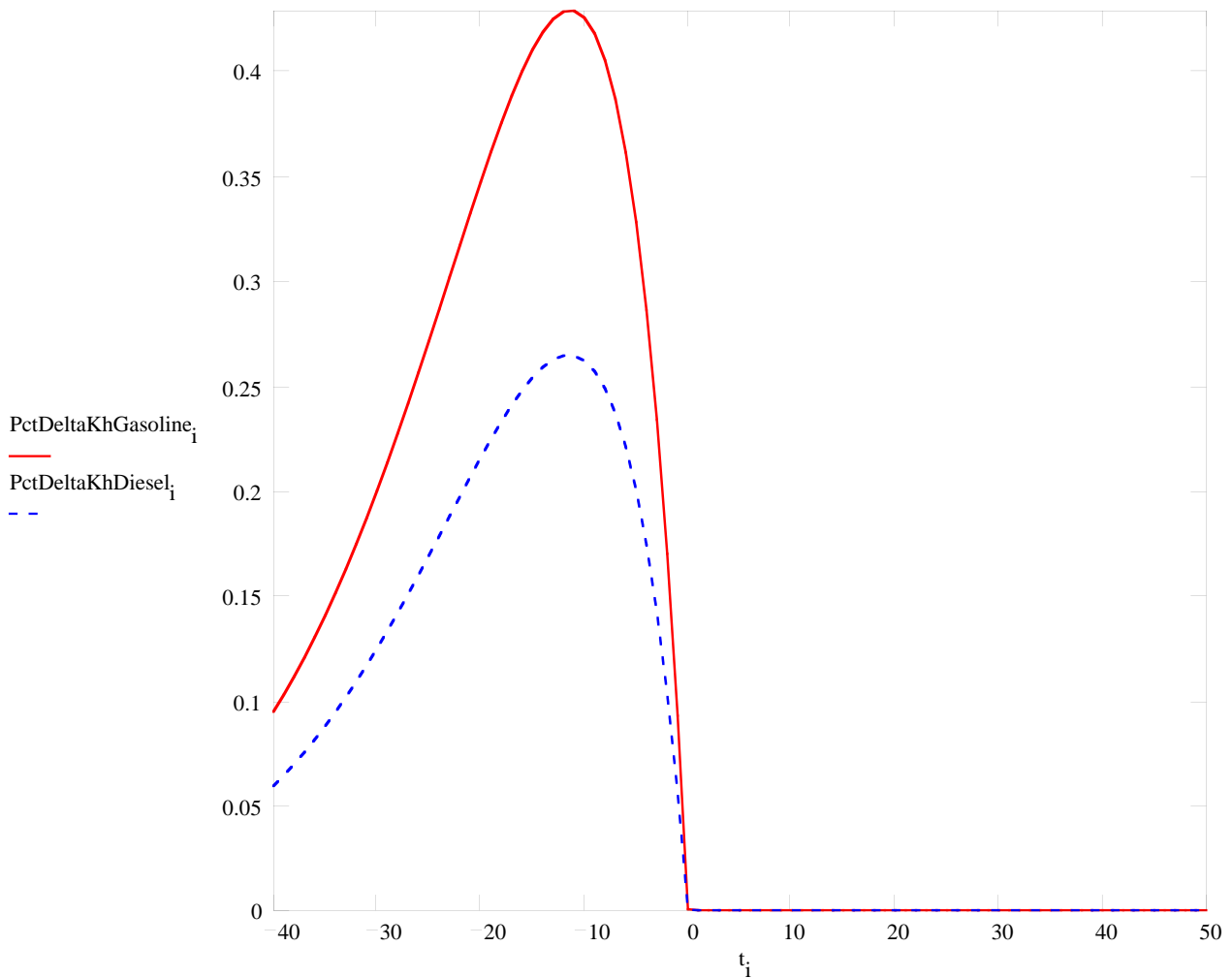


Derivation of Humidity and NOx Humidity Correction Factors

Oxides of Nitrogen Correction Factors from -40 through 50 °C

$$\text{PctDeltaKhGasoline}_i := 100 \cdot \frac{(\text{KhGasolinenoice}_i - \text{KhGasoline}_i)}{\text{KhGasoline}_i}$$

$$\text{PctDeltaKhDiesel}_i := 100 \cdot \frac{(\text{KhDieselnoice}_i - \text{KhDiesel}_i)}{\text{KhDiesel}_i}$$



Percent Difference Between NOx Factors Computed with and without Ice Point Saturation Vapor Preseure Equations