NCEE Working Paper

Welfare Analysis in a Two-Stage Inverse Demand Model: An Application to Harvest Changes in the Chesapeake Bay

Chris Moore and Charles Griffiths

NCEE

NATIONAL CENTER FOR
ENVIRONMENTAL ECONOMICS

Welfare Analysis in a Two-Stage Inverse Demand Model: An Application to Harvest Changes in the Chesapeake Bay

Chris Moore and Charles Griffiths

Abstract

Like many agricultural commodities, fish and shellfish are highly perishable and producers cannot easily adjust supply in the short run to respond to changes in demand. In these cases, it is more appropriate to conduct welfare analysis using inverse demand models that take quantities as given and allow prices to adjust to clear the market. One challenge faced by economists conducting demand analysis is how to limit the number of commodities in the analysis while accounting for the relevant substitutability and complementarity among goods. A common approach in direct demand modeling is to assume weak separability of the utility function and apply a multi-stage budgeting approach. This approach has not, however, been applied to an inverse demand system or the associated welfare analysis. This paper develops a two-stage inverse demand model and derives the total quantity flexibilities which describe how market clearing prices respond to supply changes in other commodity groups. The model provides the means to estimate consumer welfare impacts of an increase in finfish and shellfish harvest from the Chesapeake Bay while recognizing that harvests from other regions are potential substitutes. Comparing the two-stage results with single-stage analysis of the same data shows that ignoring differentiation of harvests from different regions, or the availability of substitutes not affected by a supply shock, can bias welfare estimates.

Introduction

Casual observation that seafood menus list both the prices and the origin of their fare is evidence that patrons care about the source of their seafood. Empirical studies support the notion that location matters to consumers and that some will seek out and pay a premium for seafood from a specific region (Stiles et al. 2013). We also know that a consumer looking to purchase grouper could be convinced to buy seabass instead *if* the price is right. When analyzing the demand for seafood, inverse demand systems are often used to capture consumer preferences and substitution possibilities such as these. Barten and Bettendorf (1989) make the case that demands for perishable goods like produce, meat, and seafood should be examined using inverse demand equations in which supply is treated as exogenous and price adjusts to clear the market. In addition to perishability, seafood harvest is often limited by gear and effort restrictions and depends in a large part on capital investments that are made long before the boat leaves the dock. For these reasons, inverse demand systems have been the preferred approach to analyze the demand for seafood, regulation of commercial fisheries, and welfare impacts of changes in seafood harvests (e.g. Holt and Bishop 2002; Park et al. 2004).

The capability of these models to capture preferences between source regions and substitution possibilities is limited, however, by the size of the demand system which would require an equation for every species and every region from which those species are harvested. As such, analyses of this sort tend to focus on a single harvest region, ignoring substitution possibilities from other sources (e.g. Beach and Holt 2001; Holt and Bishop 2002) or summing regional harvests into an aggregate supply thereby assuming away preferences between regions (e.g. Eales et al. 1997; Wong and McLaren 2005). When faced with a large number of

commodities analysts estimating direct demands have instead invoked weak separability and used multi-stage budgeting to model consumer choice in a series of steps (Strotz 1959; Gorman 1959; Brown and Heien 1972; Fan et al. 1995; Edgerton 1997). In the first step consumers allocate expenditures among commodity groups and then further divide those expenditures between the elementary commodities in subsequent steps.

This paper is the first to develop a two-stage inverse demand system and derive the total quantity flexibilities that describe how the market clearing price for one good is affected by a supply shock of another good in a different commodity group.[1] We show how the utility parameters estimated in the two stages can be used to estimate consumer welfare impacts of a change in supply by expanding upon a distance function approach developed in a single-stage framework by Kim (1997). To demonstrate the contribution of a two-stage model we compare the welfare estimates with those estimated by single-stage alternatives.

To examine how regional preferences and the availability of substitutes affect consumer welfare estimates we consider recent regulations intended to reduce excess nutrient and sediment loads to the Chesapeake Bay. Excess nitrogen and phosphorous loadings to the Bay are fueling the growth of algae blooms which lead to areas of low dissolved oxygen and in extreme cases create dead zones in which oxygen levels are so low that fish and shellfish that are unable to escape them die. Algae blooms also harm the aquatic grasses that juvenile fish and crabs rely on

---

[1] In this specific application, the first stage inverse demand system does not estimate expenditure shares of total consumption for broad commodity groups but rather focuses on one such commodity group and finds the market clearing prices of the elementary commodities in the second stage. The estimation approach and expression derived for flexibilities, however, would apply to a multi-stage budgeting approach in which the first stage estimates a demand system for total expenditures.

for habitat. Sediment pollution blocks sunlight when suspended in the water and can smother

grasses and oyster beds when it comes to rest on the bottom. In January 2013, a panel of experts

was convened by the US Environmental Protection Agency (EPA) and asked how the

populations of 14 different species in the Chesapeake Bay would respond to the nutrient and

sediment reductions required by the new regulations (the details of which are reported by

Newbold, Massey, and Moore, 2017). We use the expert predictions for commercially harvested

species, including the iconic striped bass, blue crab, and eastern oyster, to estimate the resulting

consumer welfare impacts. Our results show that limiting the scope to one harvest region or

aggregating harvests across regions will produce biased estimates of welfare impacts from

harvest changes.

Consumer Welfare Estimation in Quantity Space

The distance function is dual to the expenditure function and can be considered a normalized

money metric utility function (Kim 1997) that measures how the quantity vector must be scaled

in order to reach a reference utility level. Given the quantity vector X and a direct utility

function $U = U(X)$, the distance function $D(U, X)$ is defined as

$$D(U,X) = \max_{t} \left\{ t > 0 : U(X/t) > 0 \right\}, \tag{1}$$

which gives the maximum amount, t, by which the quantity vector must be divided in order to

just reach the indifference surface.

    Exact measures of consumer welfare can be calculated from the distance function by

finding the difference between $D(U,X)$ evaluated at different quantity vectors, say X^0 and X^1,

given a reference utility level U^0 and U^1. Specifically, the normalized compensating and equivalent surplus from exogenous quantity changes are

$$cs = D(U^0, X^1) - D(U^0, X^0) \tag{2}$$

$$es = D(U^1, X^1) - D(U^1, X^0) \tag{3}$$

Since (2) and (3) are normalized by total expenditures, the non-normalized, or absolute, consumer welfare measures are found by multiplying by total expenditures Y. Utility is not observable so the distance function cannot be estimated directly. To estimate the parameters of the distance function it is necessary to choose a functional form and derive the resulting system of inverse demands. These demands can be estimated empirically and are then used to recover the parameters needed for welfare analysis.

A number of functional forms for the distance function and associated inverse demand system have been used in the literature (see Brown et al. 1995 for a review). A popular choice is the Inverse Almost Ideal (IAI) specification (Eales and Unnevher 1994; Moschini and Vissa 1992). We adopt the version used by Moschini and Vissa 1992.[2]

$$\ln\left[D(U, X)\right] = a(X) - Ub(X) \tag{4}$$

where, $a(X) = a_0 + \sum_i a_i \ln(X_i) + (½)\sum_i \sum_j g_{ij}\ln(X_i)\ln(X_j)$, $b(X) = b_0 \prod_i X_i^{b_i}$, $i=(1,...,N)$ and $j=(1,...,N)$ index goods in the demand system, and X_i are elements of the quantity vector X.

---

[2] This is different from the specification of Eales and Unnevehr (1994), $\ln[D(U,X)] = (1-U)\cdot\ln[a(X)] + U\cdot\ln[b(X)]$. Footnote 1 in the appendix explains how the distance function could be written to resolve the difference between the two specifications.

Because $D(U,X)$ is homogeneous of degree one in X, the following restrictions apply: $\sum_i a_i = 1$

, $\sum_i g_{ij} = \sum_j g_{ij} = 0$, and $\sum_i b_i = 0$. The symmetry property further requires $g_{ij} = g_{ji}$.

According to Shephard's theorem, the first derivatives of the distance function with respect to the quantities yield normalized compensated (Hicksian) inverse demands. For a specific market, i, the compensated inverse demand, $H_i(U,X)$, is

$$\frac{\partial D(U,X)}{\partial X_i} = H_i(U,X) = \frac{P_i(U,X)}{Y}, \tag{4}$$

where $P_i(U,X)/Y$ is the price normalized by income. Applying Shephard's theorem to the IAI form of the distance function, recognizing that $\dfrac{\partial \ln[D(U,X)]}{\partial \ln[X_j]} = \dfrac{\partial D(U,X)}{\partial X_j} \cdot \dfrac{X_j}{D(U,X)}$, and that

utility maximization requires that $D = 1$, yields the compensated (or Hicksian, denoted by the superscript h) inverse demand system in expenditure share form

$$W_i^h = \alpha_i + \sum_j \gamma_{ij} ln\left(X_j\right) - U\beta_j b(X). \tag{5}$$

Using $D = 1$, inverting the distance function in (4), yielding $U(X) = a(X)/b(X)$, and plugging it into the inverse demands, yields

$$W_i = \frac{X_i P_i}{Y} = \alpha_i + \sum_j \gamma_{ij} ln\left(X_j\right) - \beta_i \ln Q \tag{7}$$

Where lnQ is a "quantity index," $\ln Q \equiv a(X)$. Expression (7) represents a non-linear system with cross-equation restrictions on the parameters to satisfy homogeneity and symmetry.

A single-stage model such as (7) could be used to capture the substitution patterns between multiple species harvested from multiple regions by estimating an inverse demand

equation for each species from each region. Such a model quickly becomes intractable, however, as the number of species and regions increases. Many of the species in this analysis are harvested in four or five different regions which would create a demand system with 80 equations or more and thousands of parameters in order to account for cross-quantity effects.

A more tractable alternative is to use a two-stage budgeting approach such as the one applied by Edgerton (1997) to a direct demand model. In a two-stage direct demand model, consumers allocate total expenditure among broad categories of goods and then further divide those expenditures among the elementary goods in the groups. Consumers take prices as given and quantity demanded adjusts to exogenous price shocks. In an inverse demand model, *quantities* of the elementary goods are determined exogenously and prices adjust to clear the market. In the first stage allocation of the inverse demand model, the exogenous supplies of the elementary goods determine the aggregate quantities, X_i, and the aggregate quantity index, $\ln Q$. This change in $\ln Q$ affects all normalized prices and budget shares among the commodity groups. Market clearing prices for elementary goods are determined in the second stage conditional on the aggregate quantity allocations of the first stage.

The two-stage budgeting approach requires that we assume weak separability between harvests of different species. That is to say, a change in harvest of a given species, i, from a particular region, r, can affect the market clearing price for a different species, j, but only through the effect that the supply shock has on the aggregate quantity and the normalized prices

and, thus, the expenditure shares determined in the first stage. [3] The welfare impact of a change in the quantity of species i in region r, $x_{(i)r}$ will depend on the new normalized market clearing price for that species and changes in the supply of other species to the extent that there are cross-quantity effects. The two-stage demand system allows us to account for the availability of substitutes - different types of fish and/or harvests from other regions - while recognizing that they may not be *perfect* substitutes. In the model description that follows time subscripts have been omitted but estimation relies on monthly harvest and dockside value data.
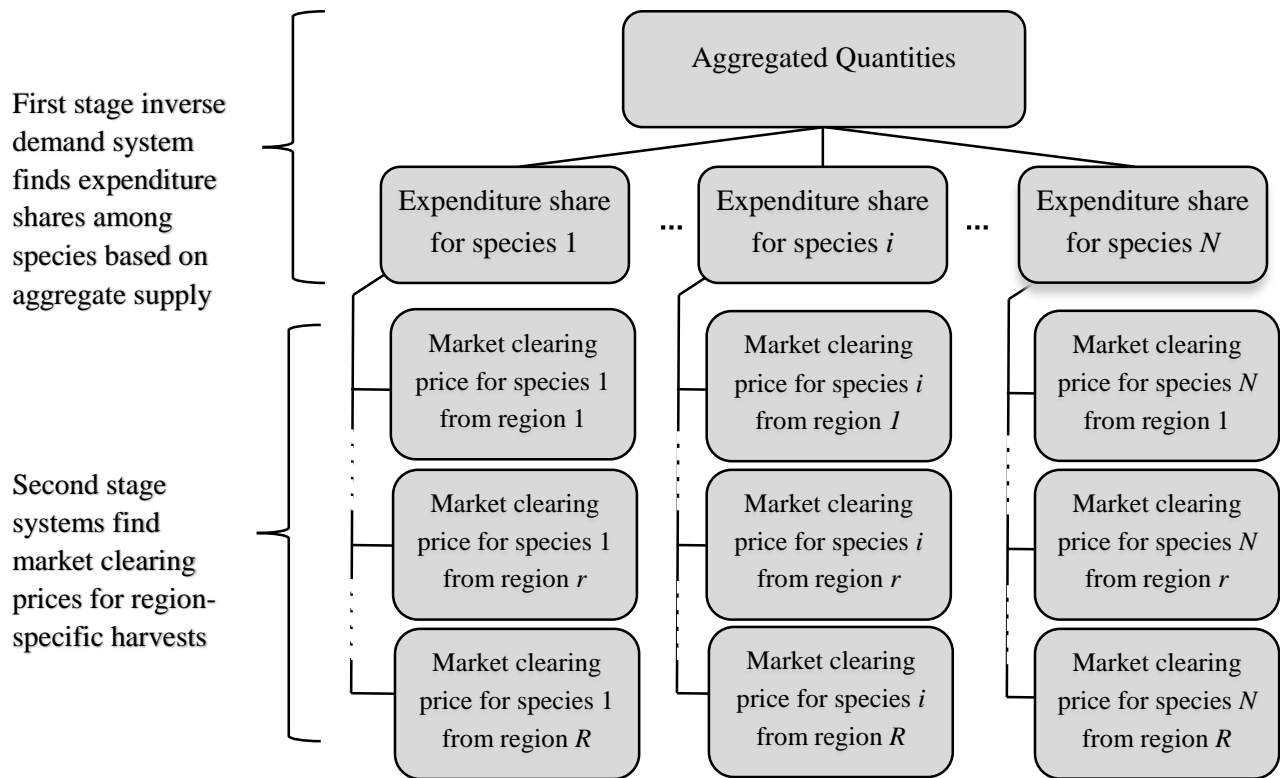
Figure 1: Two-stage inverse demand system for seafood expenditures

---

[3] In this model, a change in quantity affects the group expenditure through its impact on the normalized prices. This is because the quantity index, lnQ, in this model is only a function of quantities and not expenditures. As was pointed by our reviewer, a differential version of an inverse demand system (e.g., Brown et al. 1995) could represent real expenditures through d lnQ. In this case, consumers would directly allocate expenditures across commodity groups.

Substitution between different types of fish is captured by the first stage system which includes an inverse demand equation for each species. If a supply shift for one species has an effect on the market clearing price, and thus the expenditure share for another species, this information is required to project welfare changes based on the results of the second stage. A first stage IAI demand system with an equation for each species $i = (1,\ldots,N)$ will provide estimates of expenditure shares given harvest levels

$$W_i = \alpha_i + \sum_j \gamma_{ij} ln\left(X_j\right) - \beta_i \ln Q . \tag{8}$$

Evaluating expression (8) at the new quantity vector will provide the expected market clearing prices on all species of interest via the new expenditure shares W_i.

The second stage will estimate an inverse demand system for each species with an equation for each region that the species is harvested. This stage will capture the substitutability between harvests of the same species from different regions, $r = (1,\ldots, R)$. For example, if blue crabs harvested from any region other than the Chesapeake Bay are poor substitutes for Chesapeake Bay blue crab then welfare impacts from a change in Chesapeake Bay harvest will be larger than if very close substitutes were available. In fact, if consumers were completely indifferent regarding the source of the commodity, only the first stage inverse demand system would be necessary to estimate welfare impacts of a change in Chesapeake harvest because all that would matter to consumers is the change in total supply of that species. For species i, expenditure shares for region r will be estimated with the same IAI demand system described for the first stage

$$w_{(i)r} = \alpha_{(i)r} + \sum_s \gamma_{(i)rs} ln\left(x_{(i)s}\right) - \beta_{(i)r} \ln Q_{(i)} \tag{9}$$

9

The two-stage inverse demand system captures substitution patterns between fish species and harvest regions while keeping estimation tractable. The results of both stages enter the calculation of consumer welfare impacts from the expected supply changes. Equations (2) and (3) provide normalized welfare impacts, however, monetized impacts require forecasts of expenditures on each species. In this case, parameters estimated in the first stage allow us to estimate changes in the expenditures among species as a result of the changes in aggregate supply of each fish type. The baseline and post-policy expenditure shares for species i are

$$W_i^0 = \alpha_i + \sum_j \gamma_{ij} ln\left(X_j^0\right) - \beta_i \ln Q^0, \tag{10}$$

$$W_i^1 = \alpha_i + \sum_j \gamma_{ij} ln\left(X_j^1\right) - \beta_i \ln Q^1, \tag{11}$$

W_i^0 and W_i^1 are multiplied by total expenditures Y to provide estimates of baseline and post-policy expenditure allocations Y_i^0 and Y_i^1.

The coefficients estimated in the second stage provide the utility parameters to calculate welfare impacts from changes in regional harvests of a given species. Non-normalized, or absolute, compensating and equivalent surplus are calculated using

$$CS = \sum_i \left[Y_i^0 \left\{ D_i\left(U^0, \boldsymbol{X}_i^1\right) - D_i\left(U^0, \boldsymbol{X}_i^0\right)\right\} - \left(Y_i^1 - Y_i^0\right)\right], \tag{12}$$

$$ES = \sum_i \left[Y_i^1 \left\{ D_i\left(U^1, \boldsymbol{X}_i^1\right) - D_i\left(U^1, \boldsymbol{X}_i^0\right)\right\} - \left(Y_i^1 - Y_i^0\right)\right], \tag{13}$$

where

$\boldsymbol{X}_i^0 =$ baseline supply vector of species i
$\boldsymbol{X}_i^1 =$ post-policy supply vector of species i
$U_i^0 =$ baseline utility for species i
$U_i^1 =$ post-policy utility for species i.

10

So by appealing to weak separability we are able to forecast changes in market clearing prices of each fish type, estimate the welfare effects of regional supply changes, and sum the welfare impacts across fish types for total welfare changes.

Partial and Total Flexibilities

In an inverse demand system, quantity flexibilities are analogous to the price elasticities of a direct demand system and can be interpreted in similar ways. A quantity flexibility represents the change in the market clearing price of a particular good given a change in the supply of that good (in the case of own-quantity flexibilities) or another good (in the case of cross-quantity flexibilities). Similar to their direct demand counterparts, own-quantity flexibilities will be negative for normal goods.

For the IAI demand model, the first stage uncompensated partial own- and cross-quantity flexibility for species, i and j, $\partial \ln[P_i]/\partial \ln[X_j]$, is given by

$$E_{ij} = \frac{\partial \ln[P_i]}{\partial \ln[X_j]} = -\delta_{ij} + \left\{ \gamma_{ij} - \beta_i \left(W_j + \beta_j \ln Q \right) \right\} \left(\frac{1}{W_j} \right) \tag{14}$$

where δ_{ij} is a Kronecker delta such that $\delta_{ij}=1$ if $i = j$ and $\delta_{ij}=0$ otherwise. The second stage partial own- and cross-quantity flexibilities for two regions, r and s, supplying the same species, i, would be

$$\varepsilon_{(i)rs} = \frac{\partial \ln[p_{(i)r}]}{\partial \ln[x_{(i)s}]} = \left\{ \gamma_{(i)rs} - \beta_{(i)r} \left(w_{(i)s} + \beta_{(i)s} \ln Q_{(i)} \right) \right\} \left(\frac{1}{w_{(i)r}} \right) \tag{15}$$

The total quantity flexibility would be the percent change in the second stage price of a species, i, in some region, r, given a percent change in the quantity of *any* species, j, harvested in

any region, *r*. This is analogous to Edgerton's (1993) uncompensated total price elasticity for the

direct almost ideal demand model. For the IAI model, the total quantity flexibility,

$\partial \ln[p_{(i)r}]/\partial \ln[x_{(j)s}]$, is given by

$$\varepsilon_{(i)r,(j)s} = \delta_{ij}\left[-\delta_{rs} + \left\{\gamma_{(i)rs} - \beta_{(i)r}\left(w_{(i)s} + \beta_{(i)s}\ln Q_{(i)}\right)\right\}\left(\frac{1}{w_{(i)r}}\right)\right]$$
$$+ \left[\left\{\gamma_{ij} - \beta_i\left(W_j + \beta_j \ln Q\right)\right\}\left(\frac{1}{W_i}\right)\right]\left(\frac{x_{(j)s}}{X_j}\right)$$

(16)

The derivation of these flexibilities is given in the appendix. While the estimated flexibilities will

not be used in the welfare analysis directly, prior expectations on their signs and magnitudes

provide a theoretical check on the estimated inverse demand parameters.

Data

To better represent consumers' substitution patterns and simplify estimation, the Chesapeake

species are divided into three groups: high value fish, low value fish, and shellfish. Other

popular species that are not harvested from the Chesapeake but could substitute for Chesapeake

species are included in each group (table 1). High value fish are finfish with an average dockside

price greater than $1 per pound while low value fish have a dockside price of less than $1 per

pound and are generally harvested for uses other than human consumption. A two-stage inverse

demand system is estimated for each group. The presumption here is that changes in the supply

of Chesapeake species will affect demand for other species within that group but not across

groups. Total expenditures on each group are held constant.

12

NOAA National Marine Fisheries Service (NMFS) compiles landings and ex-vessel value data for all commercial fisheries. Both stages of the demand model are estimated with monthly harvest and dockside price data from the years 1990 to 2012. None of the species harvested commercially in the Chesapeake Bay is harvested from the Pacific so only Atlantic regions and Gulf of Mexico are included in the analysis. Not all species are harvested to a significant degree in all regions.

To project changes in supply, seven experts on fisheries and water quality in the Chesapeake Bay participated in a workshop in January, 2013. The panelists were asked to provide their best professional judgments of potential changes in Chesapeake Bay species' stock sizes under the required reductions in nutrient and sediment pollution relative to current water quality conditions, holding all other influences constant. The qualitative conclusions of each panelist were converted to percentages by EPA analysts and averaged to generate a single projection for each species. For this analysis we assume harvest changes will be proportional to the projected changes in stock sizes and report the results in table 1. It is worth noting that one Chesapeake stock, Menhaden, is expected to decrease as a result of the water quality improvements. Menhaden feed on the microalgae that benefit from excess nutrient pollution so reducing nitrogen and phosphorus loads to the Bay will likely decrease menhaden harvest. Also worth noting are the relatively large expected increases in blue crab and oyster harvests. In addition to management practices that will reduce nutrient and sediment pollution plans for cleaning up the Chesapeake Bay, plans include actively expanding oyster beds and restoring eel

grass beds that protect juvenile crabs from predation. Members of the expert panel providing

these projections were asked to consider those features of the implementation plans as well.

Table 1 Baseline Harvest and Projected Change after Water Quality Improvements
(tons per year)

	Baseline Harvest (1990-2012 Average[a])		Projected Change in Chesapeake Harvest[c]
	Chesapeake Bay	Combined Rest of Regions[b]	
High Value[d]			
Flounder	1,504	4,580	65
Striped Bass	2,358	2,248	101
Black Sea Bass	171	955	0
Eel	315	163	7
Cod		15,782	-
Grouper		3,043	-
Low Value[e]			
Spot	1,588	485	90
Perch	824	186	24
Croaker	4,771	466	138
Menhaden	199,200	538,470	-14,140
Herring		15,782	-
Mackerel		19,170	-
Shellfish			
Blue Crab	44,382	51,287	7,280
Oyster	710	10,569	117
Clam		26,640	-
Scallop		20,240	-
Shrimp		9,863	-
Lobster		39,250	-

[a] *Based on NOAA National Marine Fisheries Service (NMFS) landings and ex-vessel value data for all commercial fisheries.*
[b] *The other regions are New England, the Mid-Atlantic, the South Atlantic, and the Gulf of Mexico.*
[c] *Projected harvest changes are based on the best professional judgments of seven experts panelists in January 2013 on the potential changes in Chesapeake Bay species' stock sizes under the required reductions in nutrient and sediment pollution relative to current water quality conditions and assuming that harvest changes will be proportional to the projected changes in stock sizes.*
[d] *Finfish with an average dockside price greater than $1 per pound.*
[e] *Finfish with a dockside price of less than $1 per pound and generally harvested for uses other than human consumption.*

Results

The first and second stage systems of the inverse demand model are estimated via non-linear

seemingly unrelated regression. Estimation is performed using Stata's nonlinear seemingly

unrelated regression (*nlsur*) routine which relies on iterative feasible nonlinear generalized least

squares. The singularity of the cross-equation covariance matrix requires dropping one of the

equations from the system and relying on the adding up condition to identify the parameters.

The stability of the parameters using this approach was confirmed by reestimating each of the inverse demand systems, omitting a different equation, and reproducing the same results. Homogeneity and symmetry restrictions are imposed by estimating a subset of the model parameters and solving for the remaining restricted parameters.

With eighteen equations in the first stage and ten systems of three to five equations each in the second stage, hundreds of individual parameters are estimated in the three two-stage models. To present those results in a more manageable and meaningful way we report the partial first and second stage own-quantity flexibilities alongside total own-quantity flexibilities for comparison in tables 2 through 4. Numerical simulations of the complex non-linear flexibilities provide the standard deviations reported on the right-hand side of the tables[4]. For all 56 equations (18 in the first stages and 38 in the second) the estimated partial own-quantity flexibilities lie below zero, which is what one would expect for normal goods. Only species harvested from the Chesapeake require second stage inverse demand systems for welfare estimation so second stage flexibilities for other species are not estimated. Tables 2 through 4 also contain the expected change in market clearing prices given the expert panel's projection of harvest changes. Price changes are calculated by using the first and second stage model results to find the baseline and policy expenditure shares and then dividing by the exogenously determined supply vectors to find market clearing prices under each scenario. Something to note in table 2 is that since black sea bass harvests were not projected to change the only impact to

---

[4] To simulate distributions for each flexibility we take 1,000 draws from a multivariate normal distribution using the vector of point estimates of the inverse demand parameters γ and β and full covariance matrix for each inverse demand system. The draws are then plugged into expressions (14), (15), and (16) to simulate a distribution for each flexibility. The means and standard errors from these distributions provide the results shown in tables 2, 3, and 4. See Krinsky and Robb (1986) for more details on using numerical simulations to estimate statistical properties of elasticities and flexibilities.

market clearing price for that species is through the cross-quantity effects of the first stage. An

implication of weak separability is that the prices for all black sea bass harvests are affected

proportionally, hence identical percentage changes in price for all regions. Likewise, market

clearing prices for each of the non-Chesapeake species would be expected to change by the same

proportion regardless of harvest region and so reporting only one percentage change for each

species is sufficient.

Table 2 Own-quantity flexibilities and changes to market clearing prices for high value[a] finfish

	Own-Quantity Flexibilities			Standard Deviations			Percentage Change in Price[b]
	First Stage	Second Stage	Total	First Stage	Second Stage	Total	
Flounder	-0.612			0.011			
Chesapeake		-0.4720	-0.3775		0.0209	0.0212	-2.25%
New England		-0.6452	-0.5472		0.0134	0.0135	-0.84%
Mid-Atlantic		-0.6143	-0.5167		0.0144	0.0147	-0.67%
South Atlantic		-0.4949	-0.3970		0.0129	0.0133	0.12%
Striped Bass	-0.3842			0.0184			
Chesapeake		-0.8186	-0.3683		0.0371	0.0386	-1.62%
New England		-0.4479	-0.2735		0.1726	0.1727	1.01%
Mid-Atlantic		-0.7519	-0.6620		0.0468	0.0469	0.15%
South Atlantic		-0.6291	-0.5372		0.0759	0.0759	0.98%
Black Sea Bass	-0.3351			0.0205			
Chesapeake		-0.6840	-0.5262		0.0155	0.0164	-0.49%
New England		-0.2334	-0.1243		0.0184	0.0188	-0.49%
Mid-Atlantic		-0.5675	-0.3873		0.0191	0.0199	-0.49%
South Atlantic		-0.5976	-0.4269		0.0150	0.0161	-0.49%
Gulf		-0.4068	-0.3546		0.0597	0.0597	-0.49%
Eel	-0.7295			0.0552			
Chesapeake		-0.7141	-0.5306		0.0200	0.0425	-1.02%
Mid-Atlantic		-0.6381	-0.5698		0.0284	0.0321	-0.08%
South Atlantic		-0.5968	-0.5639		0.0309	0.0318	-0.34%
Cod	-0.5880			0.0162			-0.37%
Grouper	-0.6224			0.0143			-0.56%

[a] *Finfish with an average dockside price greater than $1 per pound.*
[b] *Predicted change in market clearing prices given the expert panel's projection of Chesapeake Bay species' stock sizes and assuming proportional harvest changes. Price changes are calculated by using the first and second stage model results to find the baseline and policy budget allocations and then dividing by the exogenously determined harvest vectors to find market clearing prices under each scenario.*

Table 3 Own-quantity flexibilities and changes to market clearing prices for low value[a] finfish

	Own-Quantity Flexibilities			Standard Errors			Percentage Change in Price[b]
	First Stage	Second Stage	Total	First Stage	Second Stage	Total	
Spot	-0.4124			0.0482			
Chesapeake		-0.7335	-0.3434		0.0083	0.0332	-1.16%
Mid-Atlantic		-0.6017	-0.5881		0.0468	0.0468	3.09%
South Atlantic		-0.6135	-0.3719		0.0191	0.0283	0.95%
Gulf		-0.0626	-0.0501		0.1188	0.1167	0.93%
Perch	-0.7806			0.0759			
Chesapeake		-0.7980	-0.6199		0.0092	0.0175	1.33%
Mid-Atlantic		-0.2601	-0.2473		0.0229	0.0229	1.86%
South Atlantic		-0.2063	-0.1784		0.0437	0.0437	2.01%
Croaker	-0.5963			0.0283			
Chesapeake		-0.9024	-0.6936		0.0092	0.0175	2.16%
Mid-Atlantic		-0.7064	-0.7064		0.0229	0.0229	4.27%
Gulf		-0.4376	-0.4017		0.0437	0.0437	3.99%
Menhaden	-0.7524			0.0081			
Chesapeake		-0.7520	-0.6821		0.0117	0.0119	3.92%
Mid-Atlantic		-0.1915	-0.1859		0.0930	0.0930	2.01%
South Atlantic		-0.7055	-0.6991		0.0245	0.0245	1.74%
Gulf		-0.7941	-0.6278		0.0073	0.0094	-0.13%
Herring	-0.7755			0.225			0.66%
Mackerel	-0.7187			0.202			0.06%

[a] Finfish with a dockside price of less than $1 per pound and generally harvested for uses other than human consumption.
[b] Predicted change in market clearing prices given the expert panel's projection of Chesapeake Bay species' stock sizes and assuming proportional harvest changes. Price changes are calculated by using the first and second stage model results to find the baseline and policy budget allocations and then dividing by the exogenously determined harvest vectors to find market clearing prices under each scenario.

Table 4 Own-quantity flexibilities and changes to market clearing prices for shellfish

	Own-Quantity Flexibilities			Standard Errors			Percentage Change in Price[a]
	First Stage	Second Stage	Total	First Stage	Second Stage	Total	
Blue crab	-0.4022			0.0402			
Chesapeake		-0.5186	-0.2962		0.0123	0.0191	-5.48%
Mid-Atlantic		-0.2611	-0.2345		0.0364	0.0366	-0.34%
South Atlantic		-0.4831	-0.3171		0.0177	0.0211	0.05%
Gulf		-0.5945	-0.4072		0.0162	0.0211	-2.11%
Oyster	-0.6780			0.0447			
Chesapeake		-0.2802	-0.2570		0.0236	0.0239	-8.07%
Mid-Atlantic		-0.5734	-0.5645		0.0261	0.0262	-1.98%
South Atlantic		-0.5515	-0.5409		0.0256	0.0256	-2.78%
Gulf		-0.9772	-0.6967		0.0040	0.0392	-3.77%
Clam	-0.1051			0.0833			-3.43%
Scallop	-0.3461			0.0281			-0.22%
Shrimp	-0.5741			0.0217			-0.28%
Lobster	-0.3863			0.0219			-0.11%

[a] *Predicted change in market clearing prices given the expert panel's projection of Chesapeake Bay species' stock sizes and assuming proportional harvest changes. Price changes are calculated by using the first and second stage model results to find the baseline and policy budget allocations and then dividing by the exogenously determined harvest vectors to find market clearing prices under each scenario.*

In order to evaluate consumer welfare, we first have to forecast the new expenditures on each species, Y_i^1, using the results of the first stage expenditure share equations and the post-policy quantity vectors based on the expert panel's projections. Table 5 shows the baseline expenditure share and the predicted change based on the new quantity vectors for each of the species groups.

Table 5 Impact of Harvest Changes[a] on Average Expenditures

Group	Species	Average Annual Expenditure Share 1990-2012[b]	Average Annual Expenditure 1990-2012[b] (thousands 2012$)	Change in Expenditure Share[c]	Change in Annual Species Expenditure (thousands 2012$)
High Value[d]	Flounder	0.2634	22,952	0.0003	31
	Striped Bass	0.1985	9,998	0.0021	211
	Black Sea Bass	0.0630	5,638	-0.0003	-28
	Eel	0.0562	4,496	0.0001	15
	Grouper	0.1544	12,356	-0.0005	-49
	Cod	0.2646	32,077	-0.0018	-180
Low Value[e]	Spot	0.0197	2,846	0.0001	88
	Perch	0.0295	1,161	-0.0004	127
	Croaker	0.0185	7,518	-0.0008	295
	Menhaden	0.6267	95,883	0.0017	-601
	Herring	0.1355	19,084	-0.0007	91
	Mackerel	0.1702	5,823	0.0001	1
Shellfish	Crab	0.1109	136,800	0.0049	7,359
	Oyster	0.0542	64,998	-0.0017	-2,523
	Clam	0.0105	29,588	-0.001	-1,558
	Scallop	0.2286	248,400	-0.0006	-833
	Shrimp	0.302	418,800	-0.0013	-1,849
	Lobster	0.2938	273,600	-0.0004	-597

[a] *Projected harvest changes are based on the best professional judgments of seven experts panelists in January 2013 on the potential changes in Chesapeake Bay species' stock sizes under the required reductions in nutrient and sediment pollution relative to current water quality conditions, holding all other influences constant, and assuming that harvest changes will be proportional to the projected changes in stock sizes.*
[b] *Based on NOAA National Marine Fisheries Service (NMFS) data.*
[c] *Changes in expenditure shares are found by evaluating the estimated expenditure equations at the post-policy harvest vectors and differencing the result with the baseline expenditure shares.*
[d] *Finfish with an average dockside price greater than $1 per pound.*
[e] *Finfish with a dockside price of less than $1 per pound and generally harvested for uses other than human consumption.*

These expenditure shares are sufficient to calculate the new market clearing prices since we hold total expenditures on the high value, low value, and shellfish categories constant. The change in expenditure shares shows the value of including substitute species that are not directly affected by the policy so far as cross-quantity effects cause market clearing prices on fish like cod, mackerel, and lobster to change despite no changes in their harvest. This implies welfare impacts in those markets that should also be accounted for.

Table 6 shows the compensating variation resulting from the expected changes in harvest for each of the species groups. Consumer welfare gains in the market for high value fish are relatively modest, about $800,000 per year. This is an expected result since the projected harvest increases for Chesapeake species are also modest – none greater than 5% of current levels. The impacts in the market for low value fish are in the opposite direction and somewhat larger, a loss of almost $2 million per year. The negative impacts are driven by the expected decline in menhaden harvest. The Chesapeake Bay catch of menhaden amounts to about 37% of the U.S. supply so one would expect the direct impact and the increase in the prices of substitutes to be substantial. The vast majority of consumer welfare impacts, however, are in the shellfish market. The expected increase in the supply of the iconic Chesapeake blue crab provides the largest increase in welfare, about $4.6 million per year, followed closely by that of Chesapeake Bay oysters, about $4.1 million per year. Consumers substituting toward these two species, which are relatively cheaper now because of the supply increases, decreases demand for substitutes harvested elsewhere and consequently lowers the market clearing price for other shellfish yielding additional consumer welfare gains. The total welfare gains in the shellfish market are $13.5 million per year. The estimated net welfare impact in all markets from the reduced sediment and nutrients is about $12.4 million per year.

Table 6 Annual Compensating Surplus from Harvest Increases[a]

Group	Species	Annual Compensating Surplus (thousands 2012$)	Standard Deviation
High Value[b]	Flounder	209	9.09
	Striped Bass	296	32.34
	Seabass	28	4.03
	Eel	44	8.89
	Grouper	49	6.24
	Cod	180	9.46
	High Value Total	**806**	**33.52**
Low Value[c]	Spot	32	1.84
	Perch	-36	22.10
	Croaker	-246	37.16
	Menhaden	-1,644	85.27
	Herring	-91	15.40
	Mackerel	-1	15.00
	Low Value Total	**-1,985**	**90.64**
Shellfish	Crab	4,646	610.72
	Oyster	4,078	268.63
	Clam	1,558	243.49
	Scallop	833	366.47
	Shrimp	1,849	480.83
	Lobster	597	274.72
	Shellfish Total	**13,561**	**324.85**
All Species Groups		**12,383**	**342.04**

[a] *Projected harvest changes are based on the best professional judgments of seven expert panelists in January 2013 on the potential changes in Chesapeake Bay species' stock sizes under the required reductions in nutrient and sediment pollution relative to current water quality conditions, holding all other influences constant, and assuming that harvest changes will be proportional to the projected changes in stock sizes.*
[b] *Finfish with an average dockside price greater than $1 per pound.*
[c] *Finfish with a dockside price of less than $1 per pound and generally harvested for uses other than human consumption.*

To demonstrate the value of the two-stage approach in this particular case the same data and harvest forecasts are used in two separate single-stage approaches to compare welfare estimates. Estimating a single-stage model forces the analysts to make one of two implicit assumptions that are not required in a multi-stage model. One option is to capture a large part of the market by aggregating the supply of each species into a single national supply (e.g. Nielsen et al. 2012; Cheng and Capps 1988). In this case the implicit assumption is that harvests from all regions are perfect substitutes and when faced with a change in supply consumers are indifferent

about where the change in harvest occurred. If, on the other hand, the analyst wants to capture

the value of a regional fishery and estimates an inverse demand system focusing on that single

region or, alternatively, multiple regions but for one type of fish (e.g. Lee and Thunberg 2013;

Hilmer et al. 2011; Park et al. 2004) then the availability of substitutes and associated cross-price

effects are not being captured. The direction of the bias introduced by each of these approaches

is ambiguous and will depend on a number of factors including the substitutability of aggregated

or omitted harvests. Table 7 presents the results of the single-stage welfare models and shows

the differences with the two-stage results in percentage terms.

Table 7 Annual Compensating Surplus Estimates from Harvest Increases[a] using Single-Stage Models

	Aggregated Supplies Model[b]		Chesapeake Only Model[c]	
Group	Compensating Surplus (thousands 2012$)	% Difference from two-stage model results	Compensating Surplus (thousands 2012$)	% Difference
High Value[d]	817	1.28%	633	-21.52%
Low Value[e]	-1,505	-24.18%	-1,416	-28.64%
Shellfish	13,464	-0.72%	9,965	-26.52%
Total	12,775	3.17%	9,182	-25.85%

[a] *Projected harvest changes based on the best professional judgments of seven experts panelists in January 2013 on the potential changes in Chesapeake Bay species' stock sizes under the required reductions in nutrient and sediment pollution relative to current water quality conditions, holding all other influences constant, and assuming that harvest changes will be proportional to the projected changes in stock sizes.*
[b] *The Aggregated Supplies Model is a single-stage IAI demand system using an aggregate national supply of each species to estimate the compensating surplus.*
[c] *The Chesapeake Only Model is a single-stage IAI demand system using only Chesapeake harvests to estimate the Compensating Surplus.*
[d] *Finfish with an average dockside price greater than $1 per pound.*
[e] *Finfish with a dockside price of less than $1 per pound and generally harvested for uses other than human consumption.*

In some cases the single-stage estimates are very similar to the two-stage estimates. Such

is the case with high value fish and shellfish when all regional supplies are aggregated into a

homogeneous national supply. The single-stage model deviates from the two-stage model by

about 1% for both species groups. There is a large difference, however, in the market for low

value fish. The single-stage model produces a 24% lower estimate of the welfare impact from

the projected harvest changes. This is an unexpected result since one might expect the source of

the harvest to matter least for these types of fish. It's hard to draw general conclusions from

these results however because the expected harvest changes examined here vary in direction and

magnitude across species so the cross-quantity effects are complex. When only Chesapeake

harvests are included in the analysis, excluding substitutes harvested outside the Chesapeake

(right-hand side of table 7), the differences from the two-stage model are similar across groups

and rather large. Again, we should resist drawing broad conclusions from these results but when

only the region in which the harvest increases occur is considered, the welfare effects are

between 21% and 29% smaller in magnitude than the two-stage model that includes harvests

from other regions. The smaller welfare impacts could be explained in part by the omission of

price effects for non-Chesapeake species. In the two-stage model and in the aggregated single-

stage model changes in Chesapeake harvest affect prices for species harvested elsewhere leading

to larger welfare gains in the high value and shellfish market and larger losses in the low value

market. These impacts are not captured when the analysis is restricted to Chesapeake harvests.

Conclusion

There are several features of the two-stage model that allow us to capture consumers'

preferences and substitution patterns among a large number of species harvested from four

regions on the Atlantic coast and the Gulf of Mexico. First, species that are commercially

harvested from the Chesapeake Bay were divided into three groups between which cross-

quantity effects are expected to be negligible. Then species that are not harvested from the

Chesapeake Bay but serve as substitutes for those that are were added to each of the groups. A

first stage inverse demand model was estimated for each of the groups that captured substitution

patterns between species but aggregated regional harvests into a single supply. Then a second

stage was estimated for each of the Chesapeake species with an inverse demand equation for

each harvest region. The results of the first stage were used to forecast market clearing prices

and expenditure shares among species after the increase in Chesapeake harvests. The second

stage expenditure shares, regional supply vectors, and the utility parameters estimated in the

second stage were used to estimate compensating variation for the harvest increase using a

distance function approach.

In addition to developing the welfare model we have derived expressions for the total

flexibilities of the two-stage IAI demand model that describe how market clearing prices respond

to supply changes in other commodity groups. These total flexibilities account for within-group

substitution and changes in relative prices between commodity groups. We have calculated the

partial flexibilities from the first and second stages to show how they differ from the total

flexibilities when both stages of budgeting are accounted for. Visual inspection of tables 2

through 4 show that the total own-quantity flexibilities tend to be smaller in magnitude than the

second stage flexibilities. This suggests that after a regional supply shift cross-species

substitution helps to redistribute demand and dampen the impact on prices.

We apply the two-stage welfare model to estimate compensating variation from increased

commercial harvests in the Chesapeake Bay resulting from pollution reduction programs recently

put in place there. The anticipated harvest increases for high value finfish are modest and the

estimated welfare impacts are accordingly small in magnitude. The large menhaden fishery in

the Chesapeake Bay and the judgement of experts that populations of menhaden would fall as a

result of reducing nutrient pollution lead to consumer welfare losses in the low value finfish market. The high value of the blue crab and oyster fisheries in the Bay combined with optimistic projections for their population and harvest lead to welfare gains that are an order of magnitude larger than the other two markets. When two alternative single-stage approaches are used to estimate the welfare impacts - one equivalent to the first stage, the other focusing on Chesapeake harvests only - we find some significant differences when compared with the two-stage model.

References

Barten AP, Bettendorf LJ (1989) Price formation of fish: An application of an inverse demand system. European Economic Review 33:1509-1525

Beach RH, Holt MT (2001) Incorporating quadratic scale curves in inverse demand systems. American Journal of Agricultural Economics 83:230-245

Brown, M, Heien, D (1972). The S-branch utility tree: a generalization of the linear expenditure system. Econometrica: Journal of the Econometric Society: 737-747

Brown MG, Lee J-Y, Seale Jr JL (1995) A family of inverse demand systems and choice of functional form. Empirical Economics 20:519-530

Cheng H-t, Capps O (1988) Demand analysis of fresh and frozen finfish and shellfish in the United States. American Journal of Agricultural Economics 70:533-542

Eales J, Durham C, Wessells CR (1997) Generalized models of Japanese demand for fish. American Journal of Agricultural Economics 79:1153-1163

Eales JS, Unnevehr LJ (1994) The inverse almost ideal demand system. European Economic
Review 38:101-115

Edgerton DL (1997) Weak separability and the estimation of elasticities in multistage demand
systems. American Journal of Agricultural Economics 79:62-79

Fan S, Wailes EJ, Cramer GL (1995) Household demand in rural China: a two-stage LES-AIDS
model. American Journal of Agricultural Economics 77:54-62

Gorman, WM (1959) Separable utility and aggregation. Econometrica: Journal of the
Econometric Society: 469-481

Hilmer CE, Holt MT, Bishop RC (2011) Bootstrapping your fish or fishing for bootstraps?
Precision of welfare loss estimates from a globally concave inverse demand model of
commercial fish landings in the US Great Lakes. American Journal of Agricultural
Economics 93:98-112

Holt MT, Bishop RC (2002) A semiflexible normalized quadratic inverse demand system: An
application to the price formation of fish. Empirical Economics 27:23-47

Kim HY (1997) Inverse demand systems and welfare measurement in quantity space. Southern
Economic Journal:663-679

Krinsky I, Robb AL (1986) On approximating the statistical properties of elasticities. The
Review of Economics and Statistics:715-719

Lee M-YA, Thunberg EM (2013) An inverse demand system for New England groundfish:
Welfare analysis of the transition to catch share management. American Journal of
Agricultural Economics 95:1178-1195

Moschini G, Vissa A (1992) A linear inverse demand system. Journal of Agricultural and
Resource Economics:294-302

Newbold S, Massey M, Moore C (2017) Commercial fishing and outdoor recreation benefits of
water quality improvements in the Chesapeake Bay. NCEE Working Paper.
https://www.epa.gov/environmental-economics/research-environmental-economics-ncee-
working-paper-series

Nielsen M, Smit J, Guillen J (2012) Price effects of changing quantities supplied at the integrated
European fish market. Marine Resource Economics 27:165-180

Park H, Thurman WN, Easley Jr JE (2004) Modeling inverse demands for fish: Empirical
welfare measurement in Gulf and South Atlantic fisheries. Marine Resource
Economics:333-351

Stiles ML, Kagan A, Lahr HJ, Pullekines E, Walsh A (2013) Seafood Sticker Shock Why you
may be paying too much for your fish. Oceana

Strotz, RH (1959) The Utility Tree--A Correction and Further Appraisal. Econometrica: Journal
of the Econometric Society: 482-488

Wong KG, McLaren KR (2005) Specification and estimation of regular inverse demand systems:
a distance function approach. American Journal of Agricultural Economics 87:823-834

Appendix: Two-Stage Inverse Almost Ideal (IAI) Demand System Derivation

The logarithmic distance function analogous to Deaton and Muellbauer's (1980) Almost Ideal

Demand System model as specified by Moschini and Vissa (1992)[5] is

$$ln[D(U,X)] = a(X) - U \cdot b(X) \tag{A.1}$$

where $\quad a(X) = \alpha_0 + \sum_i \alpha_i \ln(X_i) + (\tfrac{1}{2}) \sum_i \sum_j \gamma_{ij} \ln(X_i) \ln(X_j) \tag{A.2}$

$$b(X) = \beta_0 \, \Pi_i X_i^{\beta i} \tag{A.3}$$

X is the quantity vector, $X = \{X_1, \ldots, X_i, \ldots, X_N\}$, and U is utility. The distance function measures

how this quantity vector must be scaled in order to achieve the utility level, U. The following

restrictions ensure that $D(U,X)$ is homogenous of degree one

$$\sum_i \alpha_i = 1 \qquad \sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0 \qquad \sum_i \beta_i = 0 \qquad \gamma_{ij} = \gamma_{ji}$$

If we evaluate our quantity vector, X at its optimum, X^* for utility level U, then $D(U,X^*)=1$ and

$ln[D(U,X^*)]=0$. This implies that direct utility at the optimum is

$$U(X^*) = a(X)/b(X) \tag{A.4}$$

One property of the distance function is that the differentiation with respect to the quantity for a

given sector, i, gives the compensated (Hicksian) inverse demand function of prices in that

---

[5] This is different from the specification of Eales and Unnevehr (1994), $ln[D(U,X)] = (1-U) \cdot \ln[a(X)] + U \cdot \ln[b(X)]$, where $\ln[a(X)] = \alpha_0 + \sum_i \alpha_i \ln(X_i) + (\tfrac{1}{2}) \sum_i \sum_j \gamma_{ij} \ln(X_i) \ln(X_j)$, and $\ln[b(X)] = \beta_0 \, \Pi_i X_i^{-\beta i} + \ln[a(X)]$. This specification would require that $\beta_0 < 0$ in order to meet the requirement that a distance function is increasing in X and decreasing in U (Deaton 1979). The Moshini and Vissa (1992) specification, with a "– U," emphasizes the fact that this function is decreasing in U. One way to make the Moshini and Vissa function comport with the Eales and Unnevehr specification is to write the distance function as $ln[D(U,X)] = (1-U) \cdot a(X) - U \cdot b'(X)$, where $b'(X) = \beta_0 \Pi_i X_i^{\beta i} - a(X)$, which then reduces to our specification. In this case, if utility is scaled to $0 \le U \le 1$, then $a(X)$ is the value by which a quantity vector, X, must be divided to reach subsistence, U=0 and $b'(X)$ is the value by which a quantity vector must be divided to reach bliss, U=1.

sector, P_i, normalized by expenditure, Y, as a function of utility and quantity supplied (Deaton 1979),

$$\frac{\partial D(U,\boldsymbol{X})}{\partial X_i} = H_i(U,\boldsymbol{X}) = \frac{P_i(U,\boldsymbol{X})}{Y} \tag{A.5}$$

Substituting in direct utility, equation (A.4), gives the uncompensated (Marshallian) inverse demand of prices for that sector normalized by expenditure as a function of quantity,

$$M_i(\boldsymbol{X}) = \frac{P_i(\boldsymbol{X})}{Y} \tag{A.6}$$

The compensated (Hicksian, denoted by the superscript h) inverse budget share in terms of utility and quantities, W^h_i, for sector i evaluated at $\boldsymbol{X}^*$ so that $D(U,\boldsymbol{X}^*)=1$ is

$$\frac{\partial \ln[D(U,\boldsymbol{X})]}{\partial \ln[X_i]} = \frac{\partial D(U,\boldsymbol{X})}{\partial X_i} \cdot \frac{X_i}{D(U,\boldsymbol{X})} = \frac{P_i(U,\boldsymbol{X})}{Y} \cdot X_i = W^h_i \tag{A.7}$$

Two-stage budgeting

First stage

In our two-stage IAI model, the first stage (group level) is the fish species, indexed over $\boldsymbol{i}$=(1,…,N) and $\boldsymbol{j}$=(1,…,N).[6] The distance function is $ln[D(U,X)] = a(\boldsymbol{X}) - U \cdot b(\boldsymbol{X})$, where $\boldsymbol{X}$={X_1,…,X_i,…,X_N}. Capital letters are used for first stage variables. Using the functional forms given in equations (A.2) and (A.3) and taking the derivative of equation (A.1), the compensated budget share for species i is [7]

---

[6] In an effort to avoid excessive notation, i and j can serve as both indices over all species and to indicate an individual species.

[7] This utilizes the fact that $\partial f(x)/\partial \ln(x) = [\partial f(x)/\partial x] \cdot x$ and that $\gamma_{ij} = \gamma_{ji}$.

$$W_i^h = \frac{\partial \ln[D(U,X)]}{\partial \ln[X_i]} = \alpha_i + (\tfrac{1}{2})\Sigma_i\, \gamma_{ij} ln\left(X_i\right) + (\tfrac{1}{2})\Sigma_j\, \gamma_{ij} ln\left(X_j\right) - U\beta_0[\beta_i\Pi_j X_j^{\beta_j}\frac{1}{X_i}]X_i$$

$$= \alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - U\beta_0\beta_k\Pi_j X_j^{\beta_j} = \alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - U\beta_j b(X) \tag{A.8}$$

Substituting in direct utility, equation (A.4), gives the uncompensated budget share

$$W_i = \frac{X_i P_i}{Y} = \alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - \left[\frac{a(X)}{b(X)}\right]\beta_0\beta_i\Pi_j X_j^{\beta_j} = \alpha_k + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - \beta_i a(X)$$

$$= \alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - \beta_i \ln Q \tag{A.9}$$

where ln$Q$ is a "quantity index"

$$\ln Q \equiv a(X) = \alpha_0 + \Sigma_i\, \alpha_i ln\left(X_i\right) + (\tfrac{1}{2})\Sigma_i \Sigma_j\, \gamma_{ij} ln\left(X_i\right) ln\left(X_j\right) \tag{A.10}$$

For any species, i, the group level, uncompensated inverse demand is

$$M_i(X,Y) = P_i = \left(\frac{Y}{X_i}\right)W_i = \left(\frac{Y}{X_i}\right)\left\{\alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - \beta_i \ln Q\right\} \tag{A.11}$$

The group level expenditure on species i is

$$Y_i = X_i \cdot P_i = Y \cdot W_i = \left(Y\right)\left\{\alpha_i + \Sigma_j\, \gamma_{ij} ln\left(X_j\right) - \beta_i \ln Q\right\}, \tag{A.12}$$

and $$ln(Y_i) = ln(X_i) + ln(P_i) \tag{A.13}$$

Second stage

An analogous set of calculations can be done for the second stage (within-group) IAI, which are

regions in our model, indexed over **r**=(1,…, R) and **s**=(1,…,R).[8] The second stage distance

function for species, i, is $ln[D_i(U_i,x_i)] = a(x_i) - U_i \cdot b(x_i)$, where x_i is the quantity vector for

---

[8] Similar to what was done for the first stage, r and s can serve as both indices over all regions and to indicate a
specific region.

species i, $x_i = \{x_{(i)1}, \ldots, x_{(i)r}, \ldots, x_{(i)R}\}$. Small letters are used for second stage variables and the first stage subscript is written in parentheses when it appears with a second stage subscript. The within-group budget share, quantity index, and uncompensated inverse demand for any specific species, i, in region, r, are

$$w_{(i)r} = \frac{x_{(i)r} p_{(i)r}}{Y_i} = \alpha_{(i)r} + \sum_s \gamma_{(i)rs} ln\left(x_{(i)s}\right) - \beta_{(i)r} \ln Q_{(i)} \ , \tag{A.14}$$

$$\ln Q_{(i)} \equiv \alpha_{(i)0} + \sum_r \alpha_{(i)r} ln\left(x_{(i)r}\right) + (\tfrac{1}{2})\sum_r \sum_s \gamma_{(i)rs} ln\left(x_{(i)r}\right) ln\left(x_{(i)s}\right) \tag{A.15}$$

$$m_{(i)r}(x_i, Y_i) = p_{(i)r} = \left(\frac{Y_i}{x_{(i)r}}\right) w_{(i)r} = \left(\frac{Y_i}{x_{(i)r}}\right) \left\{\alpha_{(i)r} + \sum_s \gamma_{(i)rs} ln\left(x_{(i)s}\right) - \beta_{(i)r} \ln Q_{(i)}\right\} \tag{A.16}$$

In the two-stage budgeting context, equations (A.14)-(A.16) are conditional on the first stage budgeting allocation, given by Y_i. For this to be appropriate, the conditional, within-group inverse demand function for species i in region r must yield the same result as an unconditional demand function for that species and region. (Deaton and Muellbauer 1980; Edgerton 1997)

$$p_{(i)r} = m_{(i)r}(x_i, Y_i) = m_{(i)r}(x_i, Y_i(X, Y)) = M_{(i)r}(X, Y) \ , \tag{A.17}$$

To maintain consistency with the first stage quantity totals, the group level quantity, measured in pounds harvested for any given species, i, is the sum of all regional level harvests of that species, also measured in pounds,

$$X_i = \sum_r x_{(i)r} \tag{A.18}$$

Flexibilities

Quantity flexibilities

31

To evaluate the effect of a quantity shock of species i in region r, we define the quantity flexibility as the percentage change in price associated with a one percent change in quantity.[9] This is analogous to a price elasticity in a direct demand model. The within-group own-quantity flexibility is the percentage change in the price of species i in region r given a one percent change in the quantity of that species, i, in that region, r.[10]

$$
\varepsilon_{(i)rr} = \frac{\partial \ln[p_{(i)r}]}{\partial \ln[x_{(i)r}]} = \left[\frac{\partial p_{(i)r}}{\partial x_{(i)r}}\right]\left(\frac{x_{(i)r}}{p_{(i)r}}\right) = \left[\frac{\partial \left\{ w_{(i)r}\left(\frac{Y_i}{x_{(i)r}}\right)\right\}}{\partial[x_{(i)r}]}\right]\left(\frac{x_{(i)r}}{p_{(i)r}}\right)
$$

$$
= \left[\frac{-w_{(i)r}Y_i}{(x_{(i)r})^2} + \frac{\partial\left\{\alpha_{(i)r} + \sum_s \gamma_{(i)rs} ln\left(x_{(i)s}\right) - \beta_{(i)r}\ln Q_{(i)}\right\}\left(\frac{Y_i}{x_{(i)r}}\right)}{\partial[x_{(i)r}]}\right]\left(\frac{x_{(i)r}}{p_{(i)r}}\right) \tag{A.19}
$$

$$
= \left[\frac{-w_{(i)r}Y_i}{(x_{(i)r})^2} + \left\{\frac{\gamma_{(i)rr}}{x_{(i)r}} - \beta_{(i)r}\left(\frac{\alpha_{(i)r}}{x_{(i)r}} + \frac{\sum_s \gamma_{(i)rs} ln\left(x_{(i)s}\right)}{x_{(i)r}}\right)\right\}\left(\frac{Y_i}{x_{(i)r}}\right)\right]\left(\frac{x_{(i)r}}{p_{(i)r}}\right)
$$

$$
= -1 + \left\{\gamma_{(i)rr} - \beta_{(i)r}\left(w_{(i)r} + \beta_{(i)r}\ln Q_{(i)}\right)\right\}\left(\frac{1}{w_{(i)r}}\right)
$$

The within-group cross-quantity flexibility – the effect on the price of species i in region r given a change in the quantity of species i in some other region s – is similar but lacks the first term.

$$
\varepsilon_{(i)rs} = \frac{\partial \ln[p_{(i)r}]}{\partial \ln[x_{(i)s}]} = \left\{\gamma_{(i)rs} - \beta_{(i)r}\left(w_{(i)s} + \beta_{(i)s}\ln Q_{(i)}\right)\right\}\left(\frac{1}{w_{(i)r}}\right) \tag{A.20}
$$

The generic within-group quantity flexibility would then be

---

[9] The agricultural economics literature refers to this relationship as a "price flexibility" (for example, see Houck 1965). We use the term "quantity flexibility" here to emphasize the fact that this is an inverse model and that we are measuring the response due to a change in our independent variable, quantity

[10] This uses the results that $Y_i/(x_{(i)r} \cdot p_{(i)r}) = 1/ w_{(i)r}$ and $w_{(i)r} + \beta_{(i)r} \ln Q_{(i)} = \alpha_{(i)r} + \sum_s \gamma_{(i)rs} \ln(x_{(i)s})$.

$$\varepsilon_{(i)rs} = \frac{\partial \ln[p_{(i)r}]}{\partial \ln[x_{(i)s}]} = -\delta_{rs} + \left\{ \gamma_{(i)rs} - \beta_{(i)r} \left(w_{(i)s} + \beta_{(i)s} \ln Q_{(i)} \right) \right\} \left(\frac{1}{w_{(i)r}} \right) \tag{A.21}$$

Where δ_{rs} is a Kronecker delta equal to 1 if r=s and equal to zero otherwise.

A similar set of quantity flexibilities can be derived for the first stage inverse demands. The generic, group level quantity flexibility is the percentage change in the first stage price of species i given a one percent change in the first stage, total quantity of species j.

$$E_{ij} = \frac{\partial \ln[P_i]}{\partial \ln[X_j]} = -\delta_{ij} + \left\{ \gamma_{ij} - \beta_i \left(W_j + \beta_j \ln Q \right) \right\} \left(\frac{1}{W_j} \right) \tag{A.22}$$

It is important to recognize that the partial (within-group) quantity flexibility derived above, $\varepsilon_{(i)rs}$, is conditional on the first stage budgeting allocation in the same way as was discussed for equations (A.14)-(A.16). The total quantity flexibility, $\varepsilon_{(i)r,(j)s}$, would be the percentage change in the price of species i in region r given a one percent change in the price of any species, j, in any region, s. In other words, it is the change in the second stage price of a species in some region given a change in the quantity of any species harvested in any region. This is analogous to Edgerton's (1997) uncompensated total price elasticity for the direct Almost Ideal specification model. Deriving this total flexibility requires an estimation of the effect of a second stage quantity change on the first stage expenditures and is derived in detail below.

Expenditure flexibilities

In the direct Almost Ideal specification models, the expenditure elasticity – the effect on quantity given a change in expenditure – is $(\partial \ln[X_i]/\partial \ln[Y]) = 1 + (\beta_i/W_i)$ (see Eales and Unnevher 1994). The first term on the right-hand side is the direct effect of expenditure on quantity while the

second term is the effect of income as it affects the expenditure share. The second term appears because expenditure is an explicit argument in the share equation in the direct Almost Ideal specification model. In the IAI model, expenditure, Y, is not an argument in the share equation, W_i. Therefore, the second term is not present in the analogous expenditure flexibility for the IAI model.

The group level expenditure flexibility is

$$\Upsilon_i = \frac{\partial \ln[P_i]}{\partial \ln[Y]} = \left[\frac{\partial P_i}{\partial Y}\right]\left(\frac{Y}{P_k}\right) = \left[\frac{\partial\left\{W_i\left(\frac{Y}{X_i}\right)\right\}}{\partial Y}\right]\left(\frac{Y}{P_i}\right) = \left(\frac{W_i}{X_i}\right)\left(\frac{Y}{P_i}\right) = 1 \qquad (A.23)$$

Similarly, the within-group expenditure flexibility (conditional on Y_i) is

$$\psi_{(i)} = \frac{\partial \ln[p_{(i)r}]}{\partial \ln[Y_i]} = \left[\frac{\partial p_{(i)r}}{\partial Y_i}\right]\left(\frac{Y_i}{p_{(i)r}}\right) = \left[\frac{\partial\left\{w_{(i)r}\left(\frac{Y_i}{x_{(i)r}}\right)\right\}}{\partial[Y_i]}\right]\left(\frac{Y_i}{p_{(i)r}}\right) = \left(\frac{w_{(i)r}}{x_{(i)r}}\right)\left(\frac{Y_i}{p_{(i)r}}\right) = 1 \qquad (A.24)$$

Scale flexibilities

The expenditure flexibility derived above is the analog to the expenditure elasticity. However, in the inverse demand literature, the scale flexibility is sometimes conflated with this analog.[11] In fact, the scale flexibility is something different. In an inverse demand system, the scale flexibility measures the change in the price of a good given a change in <u>all</u> quantities. This is related to the distance function which measures how much the full quantity vector, **X**, must be scaled in order

---

[11] See Park and Thurman (1999) for a discussion of the link between scale flexibilities and income elasticities.

to achieve a given utility level, U. If λ is this scaling factor, then the scale flexibility is the percent change in the price of a good given a one percent change in this scaling factor.

The scale flexibility can be derived by first multiplying all of our quantities by λ. Our first stage share equation and Marshallian demand would be

$$W_i^* = \alpha_i + \sum_j \gamma_{ij} ln\left(\lambda X_j\right) - \beta_i \ln \lambda Q \tag{A.25}$$

$$\ln \lambda Q = \alpha_0 + \sum_i \alpha_i ln\left(\lambda X_i\right) + (\tfrac{1}{2})\sum_i \sum_j \gamma_{ij} ln\left(\lambda X_i\right) ln\left(\lambda X_j\right) \tag{A.26}$$

and
$$M_i^*(\lambda X, Y) = P_i^*(\lambda X) = \left(\frac{Y}{\lambda X_i}\right) W_i^* \tag{A.27}$$

$\lambda=1$ is the special case, assumed in our derivations above, in which the quantity vector is just sufficient to produce the utility level implied by equation (A.4).

The first stage scale elasticity is then[12,13]

$$\Phi_i = \frac{\partial \ln[P_i^*(\lambda X)]}{\partial \ln[\lambda]} = \frac{\partial \ln[M_i^*(\lambda X, Y)]}{\partial \ln[\lambda]} = \left[\frac{\partial M_i^*(\lambda X, Y)}{\partial \lambda}\right]\left(\frac{\lambda}{M_i^*}\right) = \left[\frac{\partial\left\{W_i^*\left(\frac{Y}{\lambda X_i}\right)\right\}}{\partial \lambda}\right]\left(\frac{\lambda}{M_i^*}\right)$$

$$= \left[\frac{-W_i^* Y}{\lambda^2 X_i} + \left\{\sum_j \frac{\gamma_{ij}}{\lambda} - \beta_i\left(\sum_i \frac{\alpha_i}{\lambda} + \sum_i \sum_j \frac{\gamma_{ij} ln(X_i)}{\lambda}\right)\right\}\left(\frac{Y}{\lambda X_i}\right)\right]\left(\frac{\lambda}{M_i^*}\right) \tag{A.28}$$

$$= \left[-W_i^* + \left\{\sum_j \gamma_{ij} - \beta_i\left(\sum_i \alpha_i + \sum_i \sum_j \gamma_{ij} ln(X_i)\right)\right\}\right]\left(\frac{Y}{M_i^* \lambda X_i}\right)$$

$$= -1 - \left(\frac{\beta_i}{W_i^*}\right)$$

---

[12] This uses the fact that $\sum_i \alpha_i = 1$, $\sum_j \gamma_{ij}=0$, and $Y/(M_i^* \lambda X_i) = 1/W_i$.
[13] Eales and Unnever (1994) derive a scale flexibility equal to $-1+\beta_i/W_i^*$, but recall that their specification requires that $\beta_i<0$ (see appendix footnote 1), so our specification is equivalent to Eales and Unnever if $\beta_i>0$.

The second stage scale flexibility gives the percentage change in the price of species i in region r given a one percent change in the scale parameter in region r.

$$\phi_{(i)r} = \frac{\partial \ln[m^*_{(i)r}]}{\partial \ln[\lambda]} = -1 - \left(\frac{\beta_{(i)r}}{w^*_{(i)r}} \right) \tag{A.29}$$

A scale flexibility might be appropriate to measure the effect of a shock in a region that affects all species in that region by the same percentage.

Aggregate quantity flexibilities

The final elasticity that we will need to derive the total quantity flexibility is the percent change in first stage aggregate *quantity* of some species, i, given a one percent change in quantity of that species in a single region, r.[14]

$$\xi_{(i),(i)r} = \frac{\partial \ln[X_i]}{\partial \ln[x_{(i)r}]} = \frac{\partial X_i}{\partial x_{(i)r}} \left(\frac{x_{(i)r}}{X_i} \right) = \frac{\partial \left(\sum_s x_{(i)s} \right)}{\partial x_{(i)r}} \left(\frac{x_{(i)r}}{X_i} \right) = \left(\frac{x_{(i)r}}{X_i} \right) \tag{A.30}$$

It should be obvious that a change in the quantity of one species, j, in some region, r, does not affect the aggregate quantity of a different species, i. That is $\xi_{(i),(j)r} = \partial \ln(X_i)/\partial \ln(x_{(j)r}) = 0$.

Total, flexibility

The total flexibility, $\varepsilon_{(i)r,(j)s}$, is then the percent change in the price of species i in region r given a one percent change in the quantity of *any* species j in *any* region s. We refer to this as the total flexibility because it includes the effect of the change in expenditures for each species. As we mentioned before for equation (A.17), for two-stage budgeting to be appropriate, the conditional,

---

[14] This is the inverse demand analog to $\partial P_s/\partial p_{sj}$ in Edgerton's (1997) direct demand model.

within-group inverse demand function for species i in region r must yield the same result as an

unconditional demand function for that species and region.

$$p_{(i)r} = m_{(i)r}(x_i, Y_i) = m_{(i)r}(x_i, Y_i(X,Y)) = M_{(i)r}(X,Y).$$

So, [15]

$$
\begin{aligned}
\varepsilon_{(i)r,(j)s} &= \frac{\partial \ln[M_{(i)r}(X,Y)]}{\partial \ln[x_{(j)s}]} = \frac{\partial \ln[m_{(i)r}(x_i, Y_i(X,Y))]}{\partial \ln[x_{(j)s}]} = \frac{\partial \ln[p_{(i)r}(x_i, Y_i(X,Y)]}{\partial \ln[x_{(j)s}]} \\
&= \frac{\partial \ln[p_{(i)r}]}{\partial \ln[x_{(j)s}]} + \frac{\partial \ln[p_{(i)r}]}{\partial \ln[Y_i]} \frac{\partial \ln[Y_i]}{\partial \ln[x_{(j)s}]} \\
&= \delta_{ij}\left[\varepsilon_{(i)rs}\right] + \frac{\partial \ln[p_{(i)r}]}{\partial \ln[Y_i]}\left[\frac{\partial \ln[X_i]}{\partial \ln[x_{(j)s}]} + \frac{\partial \ln[P_i]}{\partial \ln[x_{(j)s}]}\right] \\
&= \delta_{ij}\left[\varepsilon_{(i)rs}\right] + \psi_i\left[\delta_{ij}\xi_{(j),(j)s} + \frac{\partial \ln[P_i]}{\partial \ln[X_j]}\frac{\partial \ln[X_j]}{\partial \ln[x_{(j)s}]}\right] \\
&= \delta_{ij}\left[\varepsilon_{(i)rs}\right] + \psi_i\left[\delta_{ij}\xi_{(j),(j)s} + E_{ij}\xi_{(j),(j)s}\right] \\
&= \delta_{ij}\left[\varepsilon_{(i)rs}\right] + \psi_i\left[\delta_{ij} + E_{ij}\right]\xi_{(j),(j)s}
\end{aligned}
\tag{A.31}
$$

Substituting in the IAI functional forms for these flexibilities, the resulting total, unconditional,

within-group flexibility is

$$
\begin{aligned}
\varepsilon_{(i)r,(j)s} &= \delta_{ij}\left[-\delta_{rs} + \left\{\gamma_{(i)rs} - \beta_{(i)r}\left(w_{(i)s} + \beta_{(i)s}\ln Q_{(i)}\right)\right\}\left(\frac{1}{w_{(i)r}}\right)\right] \\
&+ \left[\left\{\gamma_{ij} - \beta_i\left(W_j + \beta_j\ln Q\right)\right\}\left(\frac{1}{W_i}\right)\right]\left(\frac{x_{(j)s}}{X_j}\right)
\end{aligned}
\tag{A.32}
$$

---

[15] Note that $\partial \ln(X_i)/\partial \ln(x_{(j)s}) \neq 0$ if i=j (i.e., if we are evaluating the effect of a regional quantity change on the aggregate quantity of the same species) so it is set equal to $\delta_{ij}\xi_{(j),(j)r}$.

Appendix References

Deaton A (1979) The distance function in consumer behaviour with applications to index
numbers and optimal taxation The Review of Economic Studies 46:391-405

Deaton A, Muellbauer J (1980) An almost ideal demand system The American economic review
70:312-326

Eales JS, Unnevehr LJ (1994) The inverse almost ideal demand system European Economic
Review 38:101-115

Edgerton DL (1997) Weak separability and the estimation of elasticities in multistage demand
systems American Journal of Agricultural Economics 79:62-79

Houck JP (1965) The relationship of direct price flexibilities to direct price elasticities Journal of
Farm Economics 47:789-792

Moschini G, Vissa A (1992) A linear inverse demand system Journal of Agricultural and
Resource Economics:294-302

Park H, Thurman WN (1999) On interpreting inverse demand systems: a primal comparison of
scale flexibilities and income elasticities American Journal of Agricultural Economics
81:950-958