The shape of things to come: Why is climate sensitivity so unpredictable, and who cares anyway?

Gerard Roe & Marcia Baker

Dept of Earth and Space Sciences, University of Washington Seattle, WA.
Disclaimer (added by EPA)

This presentation by Gerard Roe and Marcia Baker on April 15, 2008 has neither been reviewed nor approved by the U.S. Environmental Protection Agency. The views expressed by the presenters are entirely their own. The contents do not necessarily reflect the views or policies of the U.S. Environmental Protection Agency, nor does mention of trade names or commercial products constitute endorsement or recommendation for use.
Climate change: not a new problem

Joseph Fourier, 1827:
On the temperatures of the terrestrial sphere and interplanetary space.

John Tyndall, 1860s:
“The solar heat possesses. . . the power of crossing an atmosphere; but, when the heat is absorbed by the planet, it is so changed in quality that the rays emanating from the planet cannot get with the same freedom back into space. Thus the atmosphere admits of the entrance of the solar heat, but checks its exit; and the result is a tendency to accumulate heat at the surface of the planet.”

Samuel Langley, 1870s+

Also Pouillet (1838); De Marchi (1895); Milankovitch (1924)…
XXXI. On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground. By Prof. Svante Arrhenius.*

I. Introduction: Observations of Langley on Atmospheric Absorption.
Climate sensitivity: an envelope of uncertainty

200,000+ integrations, 31,400,000 yrs model time(!);

Eq\textsuperscript{m}. response of global, annual mean sfc. T to 2 x CO\textsubscript{2}.

6,000 model runs, perturbed physics

Slab ocean, Q-flux
12 model params. varied

Sanderson et al., 2007
Stainforth et al., 2005

• Two questions:
  1. What governs the shape of this distribution?
  2. How does uncertainty in physical processes translate into uncertainty in climate sensitivity?
Climate sensitivity: an envelope of uncertainty

- Wide variety of models, methods, and reconstructions.
Climate sensitivity: estimates over time

Climate sensitivity \equiv \text{Equilibrium change in global mean, annual mean temperature given } CO_2 \rightarrow 2 \times CO_2

- Why is uncertainty not diminishing with time?

1. Arrhenius, 1896
2. Moller, 1963
3. Weatherald and Manabe, 1967
4. Manabe, 1971
5. Rasool and Schneider, 1971
6. Manabe and Weatherald, 1971
7. Sellers, 1974
8. Weare and Snell, 1974
9. NRC Charney report, 1979
10. IPCC1, 1990
11. Hoffert and Covey, 1992
12. IPCC2, 1996
14. IPCC3, 2001
15. Forest et al., 2002
16. Harvey & Kaufmann, 2002
17. Gregory et al., 2002
18. Murphy et al., 2004
19. Piani et al., 2005
20. Stainforth et al., 2005
21. Forest et al., 2006
22. Hegerl et al. 2006
23. IPCC4, 2007
24. Royer et al., 2007
Need feedback analysis!

- introduced concept of negative feedback.

- got the idea on Lackawanna ferry on his way to work.

- took nine years to get granted a patent.

“Our patent application was treated in the same manner one would a perpetual motion machine” Black, H.S. IEEE Spectrum, 1977

• Formalized framework for the evaluation of interactions in dynamical systems.
Feedback analysis:
Formal framework for evaluating the strength and relative importance of interactions in a dynamical system.
(Maxwell, 1863; Black, 1927; Cess, 1975; Charney et al., 1979; Hansen et al., 1984; Schlesinger & Mitchell, 1985)

Confusion abounds….

• gets definitions of feedbacks wrong…

Feedback analysis: basics

forcing, $\Delta R$  \hspace{1cm} \rightarrow \hspace{1cm} \text{reference climate system} \hspace{1cm} \rightarrow \hspace{1cm} \text{response, $\Delta T$}

Climate sensitivity \textit{parameter} defined by: $\Delta T_0 = \lambda_0 \Delta R$

Reference climate system:

- Blackbody (i.e., no atmosphere).
- Terrestrial flux $= \sigma T^4$ (Stefan-Boltzmann)
- $\lambda_0 = (4\sigma T^3)^{-1} = 0.26 \text{ K (Wm}^{-2}\text{)}^{-1}$

$\Rightarrow \Delta T_0 = 1.2 \ ^\circ \text{C for a doubling of CO}_2$
Feedback analysis: basics

- defn: input is a function of the output

(n.b. Feedbacks are only meaningful when defined against a reference state.)

\[ \Delta T = \lambda_0 (\Delta R + c_1 \Delta T) \]
Feedback analysis: basics

- defn: input is a function of the output

(n.b. Feedbacks are only meaningful when defined against a reference state.)

\[ \Delta T = \lambda_0 (\Delta R + c_1 \Delta T) \]

Additional radn forcing due to system response to \( \Delta R \)
Feedback analysis: basics

- defn: input is a function of the output

\[ \Delta T = \lambda_0 (\Delta R + c_1 \Delta T) \]

So now

\[ \Delta T = \lambda_0 \Delta R + c_1 \lambda_0 \]

Additional rad\textsuperscript{n} forcing due to system response to \( \Delta R \)

Rearrange for \( \Delta T \)

\[ \Rightarrow \Delta T = \frac{\lambda_0 \Delta R}{1 - c_1 \lambda_0} \]
Feedback analysis: technobabble

Feedback factor: \( f = c_1 \lambda_0 \)  

(Gain is proportion by which system has gained)

Gain = \( \frac{\text{response with feedback}}{\text{response without feedback}} = \frac{\Delta T}{\Delta T_0} \)

From before \( \Delta T = \frac{\lambda_0 \Delta R}{1 - c_1 \lambda_0} = \frac{\Delta T_0}{1 - f} \)

And since \( \Delta T = G \Delta T_0 \):

\[
G = \frac{1}{1 - f}
\]
Feedbacks: gain curve

Range of possibilities:

- $-\infty < f < 0$: $G < 1 \implies$ response damped $\implies$ NEGATIVE fdbk.
- $0 < f < 1$: $G > 1 \implies$ response amplified $\implies$ POSITIVE fdbk.
- $f > 1$: $G$ undef. $\implies$ Planet explodes…
Feedback analysis: more than one feedback

\[ \Delta T = \lambda_0 (\Delta R + c_1 \Delta T + c_2 \Delta T) \] (two nudges)

Gives:

\[ \Delta T = \frac{\lambda_0}{1 - c_1 \lambda_0 - c_2 \lambda_0} \Delta R \]
Feedback analysis: more than one feedback

And so in general for N feedbacks:

\[ G = \frac{\Delta T}{\Delta T_0} = \frac{1}{1 - \sum_{i=1}^{N} f_i} \]
Climate feedbacks: calculating from models

Want to consider effect of variations in:
a) water vapor; b) clouds; c) sea-ice; d) snow cover; etc..

For $i^{th}$ climate variable: 

$$c_i \Delta T = \delta R \left( \begin{array} {l} j, j \neq i \end{array} \right) \frac{\partial R}{\partial \alpha_i} \left( \begin{array} {l} j, j \neq i \end{array} \right) \frac{d\alpha_i}{dT} \Delta T$$

So feedback factors: 

$$f_i \approx \lambda_0 \left( \frac{\Delta R}{\Delta \alpha_i} \right)_{j, j \neq i} \cdot \frac{\Delta \alpha_i}{\Delta T}$$

$\alpha_i$ - can be a lumped property (like clouds, sea ice, etc.),
    - or individual model parameter (like entrainment coefficient)
    - can also calculate spatial variations in $f_i$ if desired.
Climate feedbacks: estimating from models

From suites of GCMS:

Individual feedbacks uncorrelated among models, so can be simply combined:

Soden & Held (2006):
\[ \bar{f} = 0.62; \sigma_f = 0.13 \]

Colman (2003):
\[ \bar{f} = 0.70; \sigma_f = 0.14 \]

• How does this uncertainty in physics translate to uncertainty in climate sensitivity?
Climate feedbacks: estimating from models

From suites of GCMS:

Individual feedbacks uncorrelated among models, so can be simply combined:

Soden & Held (2006):
\[ \bar{f} = 0.62; \sigma_f = 0.13 \]

Colman (2003):
\[ \bar{f} = 0.70; \sigma_f = 0.14 \]

• How does this uncertainty in physics translate to uncertainty in climate sensitivity?
Uncertainty: it all depends on where you are.

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]
Uncertainty: it all depends on where you are.

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]
Uncertainty: it all depends on where you are.

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]
Uncertainty: it all depends on where you are.

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]
Uncertainty: it all depends on where you are.

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]
Uncertainty: it all depends on where you are.

\[ \delta T = G \cdot \Delta T_0 \]

\[ \delta T \sim G^2 \cdot \Delta T_0 \cdot \delta \bar{f} \]

- Uncertainty in climate sensitivity strongly dependent on the gain.
Climate sensitivity: the math

Let pdf of uncertainty in feedbacks $h_f(f)$:

Also have:

So can write:

Assume Gaussian $h(f)$:

Gives
Climate sensitivity: the picture

\[ \Delta T = \frac{\Delta T_0}{1 - \bar{f}} \]

for:
\[ \bar{f} = 0.65 \]
\[ \sigma_f = 0.14 \]
Climate sensitivity: the picture

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]

\[ f = 0.65 \]

\[ \sigma_f = 0.14 \]
Climate sensitivity: the picture

![Graph showing climate sensitivity](image)

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]

for:

- \( \bar{f} = 0.65 \)
- \( \sigma_f = 0.14 \)

- Skewed tail of high climate sensitivity is inevitable!
Climate sensitivity:
GCM from linear sum of feedback factors

\[ f = 0.62; \sigma_f = 0.13 \]
\[ \bar{f} = 0.70; \sigma_f = 0.14 \]
Climate sensitivity: comparison with climateprediction.net

![Graph showing probability density for climate sensitivity (°C) and different models]

- Soden and Held (2006)
- Colman (2003)
- Climateprediction.net
Climate sensitivity: comparison with climateprediction.net

- GCMs produce climate sensitivity consistent with the compounding effect of essentially-linear feedbacks.
Climate sensitivity: comparison with studies

- $h_{\Delta T}(\Delta T)$ works pretty well.
Climate sensitivity: can we do better?

- How does uncertainty in climate sensitivity depend on $\sigma_f$?
Climate sensitivity: can we do better?

<table>
<thead>
<tr>
<th>$\bar{f}, \sigma_f$</th>
<th>$\Delta T$</th>
<th>2 to 4.5 °C</th>
<th>4.5 to 8 °C</th>
<th>&gt; 8 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65, 0.3</td>
<td></td>
<td>29%</td>
<td>14%</td>
<td>13%</td>
</tr>
<tr>
<td>0.65, 0.2</td>
<td></td>
<td>43%</td>
<td>18%</td>
<td>12%</td>
</tr>
<tr>
<td>0.65, 0.1</td>
<td></td>
<td>55%</td>
<td>20%</td>
<td>8%</td>
</tr>
<tr>
<td>0.65, 0.05</td>
<td></td>
<td>95%</td>
<td>5%</td>
<td>0%</td>
</tr>
</tbody>
</table>

- Not much change as a function of $\sigma_f$
## Climate sensitivity: can we do better?

<table>
<thead>
<tr>
<th>$\bar{f}, \sigma_f$</th>
<th>$\Delta T$</th>
<th>2 to 4.5 °C</th>
<th>4.5 to 8 °C</th>
<th>&gt; 8 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.65, 0.3</td>
<td>29%</td>
<td>14%</td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td>0.65, 0.2</td>
<td>43%</td>
<td>18%</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td>0.65, 0.1</td>
<td>55%</td>
<td>20%</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>0.65, 0.05</td>
<td>95%</td>
<td>5%</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

- Not much change as a function of $\sigma_f$

- Science is here

- Need to get here!
Climate sensitivity: can we do better?

• Combination of mean feedback and uncertainty at which a given climate sensitivity can be rejected.

• Need to get cross hairs below a given line to reject that $\Delta T$ with 95% confidence.

• Need to get cross hairs below a given line to reject that $\Delta T$ with 95% confidence.
Summary:

- Climate change is unpredictable because climate change is inescapable.
  - Uncertainty is inherent in a system where the feedbacks are substantially positive.
  - Fat tail of the possibility of extreme climate sensitivity is inevitable, and has severe policy & planning consequences (e.g., Weitzman, 2008)

- The unpredictability of climate is predictable.
  - Compounding effect of essentially linear feedbacks dominates system sensitivity.

- If you know the feedback factors, and their uncertainties don’t need $10^4$ GCMs (or $10^7$ model years!).
  - Results suggest a simple relationship between forcing, feedbacks, and response
Paleoclimate speculations?

- What if feedback strengths change as a function of mean state?

\[ \Delta T = \frac{\Delta T_0}{1 - f} \]

- Dramatic changes in physics are not necessary for dramatic changes in climate sensitivity!
Can it be this simple?

• What is right about these ideas?
  
  - Very likely accounts for skewed tail of climate sensitivity pdfs.
  
  - From a modeling perspective, reducing uncertainties model parameters have limited effect on reducing uncertainty in climate sensitivity.

• What is wrong about these ideas?
  
  - \( h(f) \) cannot strictly be Gaussian.
  
  - feedback framework is a linear analysis in a very nonlinear world.
  
  - conclusions come from a modeling perspective. Observations of what actually happens have not been used!
Climate sensitivity: other approaches.

Using observations (Allen et al., 2006)

Estimate $\lambda$ from global energy budget:

$$\lambda(\Delta R - \Delta Q) \sim \Delta T$$

$\Delta R = \text{climate forcing}$; $\Delta Q = \text{energy imbalance}$; $\Delta T = \text{temperature change}$
Climate sensitivity: other approaches.

Using observations (Allen et al., 2006)

Estimate $\lambda$ from global energy budget:

$$\lambda \sim \frac{\Delta T}{\Delta R - \Delta Q}$$

$\Delta R = \text{climate forcing};$
$\Delta Q = \text{energy imbalance};$
$\Delta T = \text{temperature change}$

Example: for present day.

$\Delta T = 0.65 \pm 0.025 \degree C;$
$\Delta Q = 0.85 \pm 0.08 \text{ W m}^{-2};$
$\Delta R = 1.8 \pm 0.42 \text{ W m}^{-2}$

Lessons:
- Uncertainties in forcing dominate and still produce skewed tails.
- Forcing uncertainty comes from solar variability, volcanoes, aerosols, etc.
- True for any past climate reconstruction & also for modern…
Climate sensitivity: other approaches.

Combining different estimates (e.g. Annan & Hargreaves, 2006; Crucifix, 2006; Sherwood & Forest, 2007)

- In principle, climate sensitivity can be derived from multiple time intervals (little ice age, last ice age, modern, etc)

Bayesian estimates depend very sensitively on prior assumptions and the independence of different information.
Time dependent climate change:
However, climate sensitivity is an equilibrium measure of climate, and climate change is a time dependent problem…

- It takes a very long time for the full pdf of climate response to be realized
**Time dependent climate change:**
However, climate sensitivity is an equilibrium measure of climate, and climate change is a time dependent problem...

**IPCC CO₂ emissions scenarios,**
Time dependent climate change:
However, climate sensitivity is an equilibrium measure of climate, and climate change is a time dependent problem…

IPCC CO$_2$ concentration scenarios:

How does the envelope of response evolve in time?
Time dependent climate change: The role of the ocean

- The ocean heat uptake acts as a (transient) negative feedback.
Time dependent climate change:
The role of the ocean
Time dependent climate change: The role of the ocean

Addition of (transient) ocean feedback

- Ocean -ve feedback strongly reduces the width of the envelope
Time dependent climate change: The role of the ocean

- Equations for a climate model.....

Mixed layer: \[ \rho C h_{ml} \frac{\partial T_{ml}'}{\partial t} + \frac{T_{ml}'}{\lambda} - \kappa \frac{\partial T'}{\partial z} \bigg|_{z=0} = \Delta R_f(t) \]

Deep ocean: \[ \frac{\partial T'}{\partial \alpha} = \chi \frac{\partial^2 T'}{\partial z^2} - w \frac{\partial T'}{\partial z} \]

- Forget the equations, the point is…
  - Climate feedbacks combine simply, so...
  - Can integrate a simple climate model to propagate the full range of uncertainty in feedbacks, ocean heat uptake, forcing etc…
Time dependent climate change:
The role of the ocean

Response of temperature to a doubling of CO₂
Time dependent climate change:
*Why does the tail grow so slowly?*

Let $h(T,t) =$ probability density at some $(T,t)$.

What governs how $h(T,t)$ varies with time? (can talk about later…)

---

**Figure:**
- **Equil. PDF**
- **Modified equil. PDF**
- **After 50 years**
- **After 100 years**
- **After 250 years**
- **After 500 years**

**Axes:**
- Y-axis: Probability density $\left( ^{\circ}C^{-1} \right)$
- X-axis: $\Delta T \left( ^{\circ}C \right)$

**Markers:**
- $h(2^{\circ}C, 50\text{ yrs})$
Time dependent climate change: 3 questions

1. What is the likelihood of reaching a given temperature at a given time?

- The larger the temperature contemplated, the more uncertain it is when that temperature will be reached (*policy implications*?).

![Graph showing probability density over time for different temperature targets](image)

Doubling of CO2 every 100 years
Time dependent climate change: 3 questions

2. Which uncertainties matter most?

Concentration scenarios controlled by:
- maximum concentration.
- time to maximum.
Time dependent climate change: 3 questions

2. Which uncertainties matter most?

- If you are above the line, there is a 1 in 20 chance of seeing that climate change.
Time dependent climate change: 3 questions

2. Which uncertainties matter most?

- All IPCC emissions scenarios yield a significant risk of dangerous climate change…
2. Which uncertainties matter most?

What happens if you halve the uncertainty in all climate model parameters?

- Getting smarter about climate, and reducing uncertainty, does not help that much.
- Uncertainty in emissions (and eventual concentrations) dominates.
Time dependent climate change: 3 questions

2. Why even care about climate sensitivity? (sense and sensitivity…)

- Constraining climate sensitivity not terribly relevant for predicting climate change…

(Allen and Frame, 2007)
Stabilization target of 450 ppm at 2100

High end sensitivities take a long, long time to be realized…
Time dependent climate change: 3 questions

2. Why even care about climate sensitivity? (sense and sensitivity…)

Constraining climate sensitivity not terribly relevant for predicting climate change…

(A llen and Frame, 2007)

Concentration target adjusted at 2050.

In the face of uncertain information, adaptation is the answer!
Conclusions II:

- We face a practical limit to the predictability of climate sensitivity - the fat tail is inevitable…

- Ocean heat uptake acts as a strong buffer.

- Growth of the fat tail is very slow.

- Uncertainty in emissions swamps uncertainty in climate feedbacks.

- Flexibility is key!
Time dependent climate change:
The role of the ocean

95% likelihood climate change

if you are above the line you have at least a 19 in 20 chance of that climate change
Time dependent climate change: The role of the ocean

95% likelihood of climate change

And if you halve the uncertainty…

if you are above the line you have at least a 19 in 20 chance of that climate change
What kind of uncertainties matter for projections?

Response to a step-function doubling of CO₂

95% range
What kind of uncertainties matter for projections?

Response to a step-function doubling of CO₂

95% range
What kind of uncertainties matter for projections?

Response to a step-function doubling of CO₂

95% range
Spatial patterns of feedbacks

• **cloud entrainment parameter** has biggest impact on climate sensitivity in *climateprediction.net* ensemble.

• entrainment ↓, upper level moisture ↑, clear sky greenhouse ↑

![Maps of spatial patterns of feedbacks](image)

Surface rad\textsuperscript{n} tendencies assoc. with entrainment

-\textsuperscript{ve} feedback (cooling)  
+\textsuperscript{ve} feedback (warming)
Spatial patterns of feedbacks

- **Ice fall speed** has 2nd biggest impact on climate sensitivity in *climateprediction.net* ensemble.

- Fall speed ↓, clouds/humidity ↑, greenhouse effect ↑

---

Surface rad^n Tendencies assoc. with fall speed
Effect of introducing uncertainty in the forcing on equilibrium climate response

Makes lower climate response more likely
Evolution of the three terms in the energy balance in response to a step function in forcing

- $\rho C \frac{dT}{dt}$ Mixed layer
- $\lambda T$ To space
- $\kappa \frac{dT}{dz}$ Into deep ocean

- Warming term rapidly diminishes to near zero…
Response to ramp forcing

CO2 doubling every 100 years
Response to ramp forcing

CO2 doubling every 100 years
What’s right about this?

• Very likely accounts for skewed tail of climate sensitivity pdfs.

• From a modeling perspective, reducing uncertainties model parameters have limited effect on reducing uncertainty in climate sensitivity.
What’s wrong about this?

• h(f) cannot strictly be Gaussian.
  
  *not a big deal, any reasonable h(f) will do.*

• feedback framework is a linear analysis in a very nonlinear world.

• conclusions come from a modeling perspective.
  
  *Observations of what actually happens have not been used!*
Where does our uncertainty in f come from?

1. Ignorance?!

2. Nonlinearities in climate feedbacks

From basic analysis:

\[ \Delta R = \frac{dR}{dT} \Delta T + O(\Delta T^2) \]

But can take quadratic terms…

\[ \Delta R = \frac{dR}{dT} \Delta T + \frac{1}{2} \frac{d^2R}{dT^2} \Delta T^2 + O(\Delta T^3) \]

giving…

\[ G = \frac{1}{1 - f - \frac{\Delta T}{2} \frac{df}{dT}} \]
Where does our uncertainty in f come from?

2. Nonlinearities in climate feedbacks.

- Stefan-Boltzmann, Clausius-Clapeyron nonlinearities give $\delta f \sim 0.02$ for $\Delta T \sim 4^\circ C$.

- Colman et al. (1997) nonlinearities in water vapor, clouds, and lapse rate feedbacks, giving $\delta f \sim 0.1$ for $\Delta T = 4^\circ C$. 
Where does our uncertainty in $f$ come from?

3. Climate sensitivity varies with mean state.
   - Senior and Mitchell (2000) climate sensitivity increases 40% under a global warming scenario.
   - Boer and Yu (2003) climate sensitivity decreases 20%.
   - Crucifix (2006) different models have very different changes in sensitivity between LGM and modern climates.

4. Chaotic climate system.
   - Lea et al. (2005); Knight et al. (2007) suggest small but identifiable effects.
Other approaches:

**Using observations** (Allen et al., 2006)

Estimate $\lambda$ from global energy budget:

$$
\lambda \sim \frac{\Delta T}{\Delta R - \Delta Q}
$$

$\Delta R$ = climate forcing; $\Delta Q$ = energy imbalance; $\Delta T$ = temperature change

**Example: for present day.**

- $\Delta T = 0.65 \pm 0.025 \, ^\circ\text{C}$; $\Delta Q = 0.85 \pm 0.08 \, \text{W m}^{-2}$; $\Delta R = 1.8 \pm 0.42 \, \text{W m}^{-2}$
- Uncertainties in forcing dominate and still produce skewed tails.

**Combining different estimates**

(e.g. Annan & Hargreaves, 2006; Crucifix, 2006; Sherwood & Forest, 2007)

Bayesian estimates: -

depends very sensitively on prior assumptions, and the independence of different information.
Time dependent climate change:
The role of the ocean

- The ocean heat uptake acts as a (transient) negative feedback.
Time dependent climate change:
The role of the ocean

Mixed layer: \[ \rho \text{Ch}_{ml} \frac{\partial T_{ml}'}{\partial t} + \frac{T_{ml}'}{\lambda} - \kappa \left. \frac{\partial T'}{\partial z} \right|_{z=0} = \Delta R_f(t) \]

Deep ocean: \[ \frac{\partial T'}{\partial \tau} = \chi \frac{\partial^2 T'}{\partial z^2} - w \frac{\partial T'}{\partial z} \]
Time dependent climate change: The role of the ocean

Nondimensionalized:

Mixed layer: \[ X \frac{\partial T}{\partial t} + (1 - f_a) T - f_o \frac{\partial T}{\partial z} \bigg|_{z=0} = 1 \]

Deep ocean: \[ \frac{\partial T'}{\partial t} = \frac{\partial^2 T'}{\partial z^2} - \frac{\partial T'}{\partial z} \]

Solution depends three nondimensional parameters \( f_a, X, & f_o \)

\[ X = \frac{\tau_{ml}}{\tau_o} = \frac{\text{mixed layer response time}}{\text{deep ocean mixing time}} \approx 10^{-2} \]
Time dependent climate change: The role of the ocean

Mixed layer: \[ X \frac{\partial T}{\partial t} + (1 - f_a)T - f_o \frac{\partial T}{\partial z} \bigg|_{z=0} = 1 \]

Deep ocean: \[ \frac{\partial T'}{\partial t} = \frac{\partial^2 T'}{\partial z^2} - \frac{\partial T'}{\partial z} \]

Solution depends three nondimensional parameters \( f_a, X, \) & \( f_o \)

\[ f_o = \rho C \lambda_o w = \text{ocean feedback factor} \approx -0.15 \]

\( w = \text{upwelling rate} \)
Time dependent climate change:
The role of the ocean
Time dependent climate change: The role of the ocean

Addition of (transient) ocean feedback

- Ocean -ve feedback strongly reduces the width of the envelope
Time dependent climate change: The role of the ocean

Analytical solution allows for extremely efficient Monte Carlo computation of the effect of parameter uncertainties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atmospheric feedback</td>
<td>0.65</td>
<td>0.15</td>
</tr>
<tr>
<td>Upwelling rate</td>
<td>4 m yr$^{-1}$</td>
<td>1.5 m yr$^{-1}$</td>
</tr>
<tr>
<td>Mixed layer depth</td>
<td>75 m</td>
<td>25 m</td>
</tr>
<tr>
<td>Ocean diffusivity</td>
<td>1.5 cm$^2$ s$^{-1}$</td>
<td>0.5 cm$^2$ s$^{-1}$</td>
</tr>
</tbody>
</table>
Time dependent climate change:
The role of the ocean

Response of temperature to a doubling of CO$_2$
Time dependent climate change:

Why does the tail grow so slowly?

Let $h(T,t) = \text{probability density at some } (T,t)$.

What governs how $h(T,t)$ varies with time?
Response to a ramp forcing

What is the likelihood of reaching a given temperature at a given time?

- The larger the temperature contemplated, the more uncertain it is when that temperature will be reached.

Doubling of CO2 every 100 years
Time dependent climate change:

Why does the tail grow so slowly?

Probabilities are conserved so can write a conservation equation:

\[
\frac{\partial h}{\partial t} + \nabla \cdot \left( h \frac{dT}{dt} \right) = 0
\]

Therefore:

\[
h(T,t) \times \frac{dT}{dt} = \text{flux of probabilities to higher } T
\]
Time dependent climate change: Why does the tail grow so slowly?

Probabilities are conserved so can write a conservation equation:

\[ \frac{\partial h}{\partial t} + \nabla \cdot \left( h \frac{dT}{dt} \right) = 0 \]

Can integrate from \( T_c \) to \( \infty \):

\[ \frac{dp_{\text{cum}}}{dt} \bigg|_{T>T_c,t} = h(T_c,t) \times \frac{dT}{dt} \bigg|_{T_c,t} \]

\( p_{\text{cum}} \bigg|_{T>T_c,t} = \text{cumulative probability of } T > T_c \)
Time dependent climate change: Why does the tail grow so slowly?

\[ h(T_c, t) \times \left. \frac{dT}{dt} \right|_{T_c, t} = \text{flux of probabilities to higher } T \]

- Flux in the tail diminishes quickly to low (but non zero) values.