Measuring the Impact of Regulation on Small Firms

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Abstract

Small firms are an important part of any economy, since they generate a large proportion of an economy's new jobs. Despite their apparent vitality, though, small firms are particularly vulnerable to the adverse effects of government regulation. Analyzing the impact of regulation on small firms is especially important for federal agencies in the U.S., since federal law requires agencies to conduct such studies. This study sets forth a simple economic theory of regulatory impact, and presents some tools that a regulatory body can use to evaluate the potential impact of a new regulation on small firms.

Keywords: economic impact analysis; regulatory economics; small business economics; Regulatory Flexibility Act; Small Business Regulatory Enforcement Fairness Act.

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I. Introduction

Understanding and describing the impact of regulation on different parts of an economy is an important aspect of good practice in policy analysis (Arrow, Cropper, Eads et al., 1996). In the U.S., impact analysis is more than merely good practice, since an Executive Order of the President requires federal agencies to consider "distributive impacts" in the design and choice of regulations (EO12866, 1993).

The "small business sector" is one part (among many) of an economy that warrants special attention in the analysis of regulatory impact. (See Julien, 1993 for a review of the role of small firms in economic theory.) Small firms play a crucial role in an economy, with various studies showing that small firms employ the majority of workers in the private sector (SBA, 1998b), generate many new jobs (Davis, Haltiwanger and Schuh, 1996; SBA, 1998b; Kirchhoff and Greene, 1996), and play a significant role in innovation (Acs and Audretsch, 1990). Despite the apparent vitality of the small business sector, small firms have a low survival rate (Storey, 1994). Being inherently unstable in general, small firms are more vulnerable to changing economic conditions. Moreover, the changes wrought by a regulation often have the greatest impact on small firms: the cost of complying with a regulation typically involves a fixed cost; when that occurs, a regulation will impose the highest average cost on the smallest firms (Hopkins, 1995; Hopkins, 1996).

Adverse impacts on small firms can affect an entire industry and the economy as a whole. A regulation that changes the cost structure of an industry can alter the structure, performance and dynamics of that industry, by increasing the minimum efficient scale of production (Pittman, 1981; Kohn, 1988), by decreasing the survival rate and the number
of firms in an industry (Pashigian, 1984), and by increasing barriers to entry (Dean, Brown and Stango, 2000). Although regulation is often directed at – and fully justified by – social goals unrelated to small firms, regulation can also have indirect and unintended consequences that can "redistribute power and competitive advantage among firms ... [with] important impacts on social welfare" (Dean, Brown and Stango, 2000, p. 73). (SBA, 1998a, provides a survey of the effects of federal regulations on small firms in the U.S. Donez, 1997, gives a brief review of several economic studies of the impact of regulation on small firms.)

Analyzing the impact of regulation on small firms is not only good practice in policy analysis – it may also be a political or administrative necessity. In the U.S., the Regulatory Flexibility Act of 1980 (RFA) requires federal agencies to consider the impact of new regulations on small firms, and to conduct periodic reviews of existing regulations. Indeed, small firms in the U.S. have their own dedicated agency, the U.S. Small Business Administration (SBA), charged with promoting and protecting the interests of small firms in both the federal government and the economy as a whole. For example, federal agencies in the U.S. often rely on SBA to define the term "small firm" for regulatory or administrative purposes; SBA's definition varies by industry, but typically it defines a "small firm" as one with fewer than 500 employees (SBA, 1998b).

Despite the importance of analyzing the impact of regulation on small firms, agencies face formidable barriers to conducting such analyses. Government agencies typically have limited resources for conducting economic analyses of any kind, let alone analyses directed at particular segment (e.g., small businesses) that may be outside their domains of expertise or authority. Conducting economic analyses is especially a problem
for local or regional governments, since analytical expertise is typically concentrated at
the national level. Government agencies can turn to consultants for analytical support,
but outside experts are expensive, and in some cases appropriate analytical tools may
simply not exist.

For a given level of compliance cost, the adverse impact on small firms increases
as the proportion of fixed compliance cost increases. But even though small firms may
be particularly affected by a regulation, if the total compliance cost is small enough, the
adverse impact on small firms may not be large enough to warrant special attention. In
the U.S., the RFA recognizes and makes allowances for this possibility. The act states
that if an agency can certify to SBA that a proposed regulation does not have a
"significant economic impact on a substantial number of small entities", then that
regulation is not subject to the further analytical and administrative requirements of the
RFA (United States Code, Title 5, Section 605). The act, though, does not provide any
guidance on what constitutes a "significant economic impact on a substantial number of
small entities", and a wide variety of "screening" procedures have evolved across the U.S.
government (Wittenberg and Arnold, 1999). (See Verkuil, 1982, for a review of the
RFA, and Sargentich, 1997, for a review of later amendments.)

In sum, policy analysts need tools for use in determining whether or not a
regulation warrants detailed study of its effect on small firms, and also for use in detailed
studies when time, funding, personnel or advanced analytical tools are lacking. Such a
toolkit should satisfy at least eight criteria.

1. Each tool should require only information that is readily accessible.
2. Some tools should be easy to use, for use when only a preliminary or screening analysis is needed.

3. Often a screening analysis leads to more detailed or advanced study. To foster efficient analysis, the tools should fit together in a hierarchical manner, so that the more sophisticated tools can call upon information collected at earlier stages.

4. Like all economic methods, the tools require compromises between reality and simplicity, but even the simplest tools should be based on sound economic principles.

5. The tools should cover short-run, long-run, and intermediate impacts.

6. The tools should clearly indicate the extent to which a firm can pass on higher costs to consumers, and the resulting effects of such "cost pass-through".

7. The tools should enable analysts to predict (and therefore avoid) hostile receptions for new regulations.

8. Both the nature and the results of the tools should be easy to explain to policy makers and to the small business sector.

Since agencies in the U.S. are required by law to analyze the impact of regulations on small firms, the U.S. government is the obvious place to look for best practice in this kind of impact analysis. A recent review of screening methods studied 35 regulatory impact analyses conducted by 13 different offices of the U.S. federal government (Wittenberg and Arnold, 1999). The review concluded that "many agencies do not conduct rigorous screening analyses to quantify the impact of a rule on small entities" (Wittenberg and Arnold, 1999, sec. III). Nine of the offices measured impact by comparing compliance costs to revenues; two offices considered only compliance cost,
and did not compare it to any measure of a firm's performance (e.g., revenue); and two offices conducted only a "qualitative" analysis of impact.

The review also concluded that the method in use at the U.S. Environmental Protection Agency (EPA) was "consistent with, or more advanced than, those used by other federal agencies" (Wittenberg and Arnold, 1999, sec. VIII). EPA's method compares compliance cost to revenue, and presumes the impact to be insignificant if the ratio of cost to revenue is sufficiently small (EPA, 1999b); the agency uses this and other information as a guide in the determination of whether the regulation is subject to the further analytical and administrative requirements of the RFA. This method satisfies many of the criteria mentioned previously: it is simple, feasible, easy to communicate, and is apparently effective (at least as measured by the absence of complaints about it). The method, however, is based only on a weak measure of ability to pay, does not include "cost pass-through", and is at best only a short-run indicator of impact.

Perhaps the obvious extension of the cost-revenue ratio is a compliance cost to "profit" ratio. Sound economic principles would support a measure based on profit, for both short-run and long-run impacts. The compliance cost-to-revenue ratio is motivated as a short-run measure of ability to pay, but profit is arguably better than revenue as a standard of ability to pay. A compliance cost-to-profit ratio also indicates a regulation's long-run impact – the number of firm's that exit the industry – since profitability is a factor in the rate of survival of both small and large firms. A simple cost-profit ratio, though, does not include cost pass-through.

This study presents a new set of tools for analyzing the impact of regulation on small firms, with the tools satisfying many of the criteria mentioned above. The tools are
straightforward, consisting of formulae that an analyst could compute with a calculator. The tools are based on sound economic principles, namely those derived from perhaps the simplest economic model, that of a perfectly competitive market. So throughout this study I assume that firms are price-takers, and given the price of output, they choose the level of production to maximize profit. The discussion in sections II, III and IV uses the perfectly competitive model to analyze the impact of regulation from three different perspectives: the short-run, long-run, and intermediate impacts. I focus exclusively on the impact of regulation on a firm's profit, and so the methods presented here are limited to the impact of regulation on a small firm's owner. It is clearly important to have simple methods for assessing the impact of regulation on workers and consumers, but I focus on the impact on owners, since that is arguably the intent of the RFA.

Section II presents a tool for measuring the impact of regulation over the short-run: the ratio of compliance cost to a firm's gross profit. This impact indicator requires four pieces of information:

1. the ratio of marginal compliance cost to a firm's revenue ($\rho_M$);
2. the ratio of fixed compliance cost to a firm's revenue ($\rho_F$);
3. the short-run price elasticity of demand ($\varepsilon_{SR}$); and
4. the firm's gross profit margin ($\mu \equiv 1 - \text{variable cost/revenue}$).

The first two pieces of information are similar to the information required in impact analyses that are based only on cost-to-revenue ratios, but the analyst would also be required to distinguish between marginal and fixed compliance costs. As an example of its application, I use the ratio of compliance cost to gross profit (and the other indicators) to assess the impact of a regulation recently proposed by EPA. Appendix I contains an
outline of how an analyst could use the tools presented in a screening analysis. (See Arnold, 1995, for a general discussion of approaches for assessing the impact of regulation on small firms. Hall, 1997, provides a general review of the economics of environmental regulation.)

Section III presents a tool for measuring the impact of regulation over the long-run: the percentage of firms that may exit the industry as a result of the regulation. This indicator uses the same information as the short-run indicator (substituting the short-run for the long-run elasticity of demand).

Section IV presents a tool for measuring the intermediate impact of regulation: the length of time required for the industry to "recover" from the effects of the regulation. This indicator uses the same information as the long-run indicator, but in addition requires an estimate of the industry's growth-rate.

In sum, each indicator uses a form of the compliance cost to revenue ratio, which is the most common impact indicator in use in the U.S. federal government. The analysis here shows that the ratio of compliance cost to revenue is more than merely an indicator of ability to pay, and is arguably the fundamental indicator of the impact of regulation on small firms. It is easy to convert the cost-to-revenue ratio into a cost-to-profit ratio, an impact indicator which also incorporates the potential for firms to pass on some of the compliance costs. Although the impact indicators presented here require more information than the cost-to-revenue ratio, they offer at least one useful insight: for a given level of compliance cost, a regulation that imposes higher fixed costs has a greater adverse impact on small firms, in both the short-run and the long-run. This feature of the impact indicators may help policy-makers design or select regulations that have less
adverse impact on small firms (and reduce the social cost of regulation), with perhaps little or no sacrifice of the social benefits of regulation.

The study concludes in section V with a discussion of the interpretation of the tools. Appendix II contains a detailed explanation of the economic model and assumptions behind the indicators, and a detailed derivation of each indicator.

II. The short-run impact of regulation: the cost-to-profit ratio

The typical impact of a new regulation is to increase a firm's production cost. For example, EPA recently proposed a regulation of boat builders, aimed at reducing emissions of hazardous air pollutants. Boat builders use various resins and solvents that emit hazardous substances. EPA proposed that boat builders be required to use materials with less potential for hazardous emissions. These alternative materials are good substitutes for the materials currently in use, but they are more expensive, so the regulation would increase the cost of building a boat. (For more information on this proposed regulation, see EPA, 1999a, or the announcement in the U.S. Federal Register on 14/July/2000.)

A regulation may increase a firm's marginal cost, its fixed cost, or both. For example, the regulation of boat builders would increase the marginal cost of building a boat by the product of the added cost of the new materials and the quantity of materials used in each boat – an estimated $11 for a small sailboat (less than 30 feet). The proposed regulation would also require manufacturers to use more expensive equipment in the process of building a boat, also with the intent of reducing emissions of hazardous air pollutants. This new equipment is a form of capital, and so is a (short-run) fixed cost.
to a firm. The proposed regulation would thus increase a firm's fixed cost by the cost of the new equipment, less the salvage value of the equipment it replaces. Also, regulations often involve a record-keeping and reporting requirement. This also would impose a fixed cost if the cost of learning how to fill out the forms was the same for all firms, regardless of size (say, if there was one type of form for all firms). EPA estimated that the total fixed cost of the regulation of boat builders was approximately $5126 per year. (The analysis in EPA, 1999a, though, is not based on fixed and marginal costs, and does not clearly distinguish between the two. For the purposes of this study, I’ve made some assumptions in interpreting the information in that report. The data and results I present here are merely illustrative, and are not a definitive analysis of this proposed regulation.)

An increase in production cost can adversely affect workers, consumers and the owners of a firm. In the short-run, an increase in marginal cost would cause a profit-maximizing, price-taking firm to reduce the rate of production, and perhaps layoff workers as a result. The market supply curve would shift up, increasing the market price in the short-run, adversely affecting consumers. And depending on the elasticity of supply and demand, the increase in price and decrease in production costs (that is, through lower production) may not be sufficient to cover the new variable cost, so the increase in production cost can result in lower profits for the firm's owners.

An increase in fixed cost, in contrast, does not affect consumers or workers in the short-run, and only affects only a firm's owners, through reduced profit. A decrease in profit can drive a firm out of the industry in the long-run (as I discuss in section II), with adverse impacts on workers and consumers, as well as on the firm's owners. (Of course a firm could close immediately following the increase in fixed cost, but closing is
nonetheless a long-run decision.) Figure 1 shows graphically the effects of higher marginal and fixed costs in the short-run and long-run.

As mentioned in the introduction, this study focuses exclusively on the impact of regulation on the owners of a small firm. I measure the impact on owners as the change in profit, as argued in the previous paragraphs. There are various ways to define a firm's profit, but standard economic theory supports the use of "gross profit". Accountants define gross profit as revenue less "cost of goods sold" (Friedman, 1987, p. 154); cost of goods sold includes raw materials, labor, and other factors that are "less clear cut, such as overhead" (Friedman, 1987, p. 131). For the purposes of this study, I interpret the cost of goods sold as total variable cost, so I define gross profit as revenue less variable cost.

Gross profit is a convenient standard to measure the impact of regulation, since gross profit (in my definition) is linked to a firm's fixed cost. To see the link between the two, note that short-run variable cost is \[ C(q) - c \], where \( q \) is a firm's rate of production of a firm, \( C(q) \) is the total production cost, and \( c \) is the short-run fixed cost. Gross profit is then

\[
\pi \equiv pq - [C(q) - c] = [p - C(q)/q]q + c ,
\]

where \( p \) is the market price of a firm's output. In a perfectly competitive industry, long-run equilibrium requires that price equals average total production cost \( (p = C(q)/q) \), so \( \pi = c \). That is, in long-run equilibrium, a firm's gross profit must cover its short-run fixed cost.

The link between gross profit and fixed cost is important since a firm's fixed cost plays a key role in analyzing the long-run impact of regulation, as I discuss in section III:

- regulation typically involves a fixed cost of compliance;
- an increase in fixed cost increases the minimum efficient scale of production;
- increasing the minimum efficient scale can force some firms out of the industry in the long-run.

Hence, in the following I measure the short-run impact of a regulation as the net cost of the regulation relative to a firm's equilibrium level of gross profit; I refer to this impact measure as the "cost-to-profit ratio".

The net cost of the regulation depends on how much the market price increases, and on how much production cost falls when output falls as a result of the price increase. Estimating the increase in market price and the fall in production cost requires detailed information on the market demand curve (for example, the demand for boats) and on firms' production functions (the production function for boat construction). This kind of information, though, is typically difficult to obtain, if it is available at all. To avoid these information constraints, in the following I derive upper and lower bounds for the impact of regulations on the cost-to-profit ratio, over the entire range of equilibrium market prices, and over a limited but still quite general range of production functions.

One can establish an upper bound on the cost-to-profit ratio by adding the marginal cost of the regulation to the firm's marginal production cost prior to the regulation – that is, by holding the firm's marginal production cost constant, except for the added cost of the regulation. (Note that I am not assuming that production occurs at constant marginal cost in general – that would be inconsistent with the perfectly competitive model. I am merely holding marginal production cost fixed at the initial level, as a means to calculate an upper bound on the impact. This is also a simple way to illustrate the logic involved in the more general case, but without the lengthy algebraic
manipulations required in the general case.) Suppose the regulation causes each firm's marginal production cost to increase from $b$ to $b + b'$, and suppose that the resulting short-run increase in the equilibrium market price is a fraction $f \leq 1$ of the increase in marginal cost. If $p_0^*$ is the initial equilibrium market price (for example, the price of a boat), then the new (short-run) equilibrium market price is

$$ p_0^* + f b' = p_0^* (1 + f b' / p_0^*) = p_0^* (1 + f \rho_M) . $$

(I use a "*" to indicate a long-run equilibrium level, and a "0" subscript to indicate an initial level.)

Note that $\rho_M$ is the ratio of variable compliance cost to revenue – one component of the standard compliance cost to revenue ratio. That is, if $q_0$ is the original level of production per firm (number of boats built), then

$$ \rho_M = \frac{b'}{p_0^*} = \frac{q_0 b'}{q_0 p_0^*} . $$

For example, the average cost of a sailboat is $22,379, so the variable compliance cost to revenue ratio for EPA's proposed regulation is $11/22379 = .05$ percent.

If the market price increases by $f \rho_M$ percent, then the market demand would fall by $\varepsilon_{SR} f \rho_M$ percent, where $\varepsilon_{SR}$ is the short-run elasticity of demand. For example, EPA reported that the elasticity of demand for small sailboats (less than 30 feet) is -1.9 (EPA, 1999a). If the price of a new sailboat increases by .05 percent, then the demand for sailboats would fall by at most $1.9*.05 = .095$ percent. I assume, as in the standard model of perfect competition, that the fall in market demand is distributed equally among firms, so that each firm's output falls by $\varepsilon_{SR} f \rho_M$ percent. The short-run equilibrium production per firm is then $(1 + \varepsilon_{SR} f \rho_M)q_0$. 

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So if price increases to \((p_0 + fb')\) and production per firm falls to \((1 + \varepsilon_{sf}f \rho_u)q_0\), then holding marginal production cost constant, the firm's new equilibrium gross profit would be

\[
\pi_1 = (p_0 + fb')(1 + \varepsilon_{sf}f \rho_u)q_0 - (b + b')(1 + \varepsilon_{sf}f \rho_u)q_0
\]

\[
= (1 + \varepsilon_{sf}f \rho_u)((p_0 + fb')q_0 - (b + b')q_0) = (1 + \varepsilon_{sf}f \rho_u)((p_0 - b)q_0 + (f - 1)b'q_0)
\]

\[
= (1 + \varepsilon_{sf}f \rho_u)(\pi_0^* + (f - 1)b'q_0),
\]

where

\[
\pi_0^* = (p_0^* - b)q_0
\]

is the firm's equilibrium gross profit before the regulation.

A regulation's full effect on a firm also includes the fixed cost of the regulation. Let \(c'\) denote the added fixed cost of the regulation (that is, the net cost of the new boat-building equipment). Before the regulation, the firm's gross profit would have been used to pay the firm's other costs (interest payments, return to equity and taxes). The new equipment would be required for the firm to stay in business, so the firm would have to pay for the equipment out of its new gross profit. So the net cost of the regulation is the decline in the funds available to pay the firm's other costs (including return to equity):

\[
c' + (\pi_0^* - \pi_1) = c' + \pi_0^* - (1 + \varepsilon_{sf}f \rho_u)(\pi_0^* + (f - 1)b'q_0)
\]

\[
= c' - \varepsilon_{sf}f \rho_u \pi_0^* - (1 + \varepsilon_{sf}f \rho_u)(f - 1)b'q_0.
\]

Hence, the ratio of compliance cost to equilibrium gross profit, holding marginal production cost constant, is at most

\[
g_c = \frac{c' + (\pi_0^* - \pi_1)}{\pi_0^*} = \frac{c'}{\pi_0^*} - \varepsilon_{sf}f \rho_u - (1 + \varepsilon_{sf}f \rho_u)(f - 1)\frac{b'q_0}{\pi_0^*}.
\]
One can express gross profit as a fraction of revenue:

\[ \pi_0^* = \mu_0^* (p_0^* q_0^*), \]

where \( \mu_0^* \) is the firm's equilibrium gross margin (that is, the ratio of gross profit to revenue). Substituting expression (8) into expression (7) and simplifying gives

\[ g_c = \left[ \rho_f - \rho_u (\mu_0^* \varepsilon_{\text{grad}} f + (1 + \varepsilon_{\text{grad}} f \rho_u) (f - 1)) \right] / \mu_0^*, \]

where

\[ \rho_f = \frac{c'}{p_0^* q_0^*} \]

is the compliance cost to revenue ratio for the fixed-cost component of the regulation. As an example of \( \rho_f \), the smallest sailboat builder in EPA's analysis built around 300 boats per year. At a price of $22,379 per boat, the firm's revenue would be $6.7 million. The regulation would impose a fixed cost per firm of $5,126, so \( \rho_f = .07 \) percent.

Viewing \( g_c \) as a function of \( f \), one can find the extreme values for \( g_c(f) \) over the interval \( 0 \leq f \leq 1 \). The graph of \( g_c(f) \) is a parabola, with global minimum at

\[ f_c^* = \frac{1 + \varepsilon_{\text{grad}} (\mu_0^* - \rho_u)}{-2 \varepsilon_{\text{grad}} \rho_u}. \]

Hence, the maximum cost-to-profit ratio occurs at either \( f = 1 \) if \( f_c^* \leq 0.5 \), or at \( f = 0 \) if \( f_c^* > 0.5 \). When \( f = 0 \) (no cost pass-through to consumers),

\[ g_c(f = 0) = \rho_f / \mu_0^* - \rho_u / \mu_0^*, \]

and when \( f = 1 \) (full cost-pass through),

\[ g_c(f = 1) = \rho_f / \mu_0^* - \rho_u \varepsilon_{\text{grad}}. \]

The value \( f = 1 \) can occur in equilibrium only if demand is insensitive to price.
Otherwise, \( f = 1 \) shows the relative effect on gross profit if firms attempt to pass all of their increased marginal costs, with the market still in short-run disequilibrium.

Since the maximum impact will involve either no cost pass-through or full cost pass-through, one can see from expressions (12) and (13) that an upper bound for the cost-to-profit ratio is

(14) \[ \bar{g} = \frac{\rho_r}{\mu_0^*} + \rho_M \max\{-\epsilon, 1/\mu_0^*\}. \]

Advocates of regulation often dismiss the impact of regulation on firms, casually assuming that firms can easily pass on to consumers the added costs of regulation. As evident from expression (14), though, passing on the costs can result in lower profits than absorbing the costs, if the price elasticity of demand is greater than the inverse gross profit ratio.

To calculate \( \bar{g} \) for the proposed regulation of boat-builders, the only additional information one needs (beyond that already discussed) is \( \mu_0^* \). EPA reported that the average gross margin for the boat-building industry was at least 20 percent from 1990 to 1996. Since

(15) \[ 1/\mu_0^* = 5 \geq 1.9 = -\epsilon, \]

the worst impact occurs with no cost pass-through, and so

(16) \[ \bar{g} = (\rho_r + \rho_M)/\mu_0^* = (0.0005 + 0.0007)/2 = 0.006. \]

That is, the proposed regulation would not consume more than 0.6 percent of a firm's equilibrium gross profit.

To calculate a lower bound on the cost-to-profit ratio, one needs an idea of how production cost could change as production falls. A convenient but still quite general form for the marginal cost function is
the parameters $a$, $b$ and $d$ are positive and would vary across industries, but would be constant within an industry. The parameter $a$ governs the slope of the marginal cost curve; note that when $d = 1$, the marginal cost is linear in $q$ and the slope of marginal cost is $a$. The parameter $b$ is the component of marginal cost that does not vary with output; marginal cost would be constant if $a = 0$. The parameter $d$ governs the shape of the marginal of the marginal cost curve, and can affect the structure of the industry; this warrants some further explanation.

Figure 2 shows the average cost curve for three levels of $d$, expressed relative to the minimum average cost, and holding the minimum efficient scale constant (see Appendix II for a mathematical derivation of the curves in Figure 2). Note that as $d$ decreases towards zero, the average cost curve becomes flatter, and the minimum efficient scale is less precisely determined. As $d$ decreases, output per firm can depart more from the optimum, and firms may still remain competitive. Hence, smaller values of $d$ could result in a wider distribution of production per firm.

Following the same logic as in the derivation of the upper bound (but with a lot more algebra), one can show that, with the marginal cost function in expression (17), higher marginal and fixed production costs would consume

\[
g(f, d) = g_c(f) + (1 + \varepsilon_{sf} f \rho_u) \left\{ \frac{1}{d} \left[ (1 + \varepsilon_{sf} f \rho_u)^d - 1 \right], d > 0 \right\} + \log(1 + \varepsilon_{sf} f \rho_u), d = 0 \]

percent of the equilibrium gross profit (Appendix II). Note that $g_c(f)$ is the same function (expression (9)) used to calculate the upper bound on the cost-to-profit ratio. The term $(1 + \varepsilon_{sf} f \rho_u)$ is less than one, so the second term in expression (18) is always negative; this
second term thus accounts for the change in marginal cost as production departs more from the initial level. Hence,

\begin{equation}
(19) \quad g(f, d) \leq g_c(f) \leq \overline{g},
\end{equation}

so $g_c(f)$ is an upper bound on the cost-to-profit ratio, at least for the class of production functions considered here.

For every value of $f$, the minimum of $g(f, d)$ over $d$ occurs at $d = 0$. Hence, over the class of production functions considered here, a lower bound on the cost-to-profit ratio is

\begin{equation}
(20) \quad g(f, d = 0) = g_c(f) + (1 + \varepsilon_{\text{tr}} f \rho_M) \log(1 + \varepsilon_{\text{tr}} f \rho_M) \leq g(f, d).
\end{equation}

Unlike the minimum of $g_c(f)$, the minimum of $g(f, d = 0)$ over $f$ does not have an algebraic solution. Since $g(f, d = 0)$ is a function of one variable only, though, one could easily find the minimum numerically, say by plotting $g(f, d = 0)$ over the range $0 < f < 1$.

For example, in the proposed regulation of boat builders, the plot of the $g(f, d = 0)$ is effectively a straight line sloping down from $f = 0$, with minimum value 0.0035 at $f = 1$. In this case, the maximum percent change in output, $\varepsilon_{\text{tr}} \rho_M$, is small enough so that there would be virtually no change in production cost (over the class of production functions considered here). The cost-to-profit ratio is thus driven by the term $g_c(f)$, and one can find a lower bound the cost-to-profit ratio by finding the minimum of $g_c(f)$. Expression (11) gives the global minimum of $g_c(f)$. If $\rho_M$ is small (as in the proposed regulation of boat builders), the global minimum is likely to occur outside the range $0 \leq f \leq 1$, and the minimum within this range will occur at the endpoints – that is, at either full cost-pass through ($f = 1$) or at no cost pass-through ($f = 0$). In this case, a lower bound on the cost-
to-profit ratio is then

\begin{equation}
g = \frac{\rho_f}{\mu_0^*} + \rho_{\mu\epsilon} \min\{-\epsilon, 1/\mu_0^*\}.
\end{equation}

Further refinement of cost-to-profit ratio requires information on the market or firm supply curve, in the form of the parameter \(d\). Ideally the analyst would obtain a specific value for \(d\) through published research on the industry, but realistically such information is unlikely to be available. Absent information on the value of \(d\), one may be able to approximate the value of \(d\) through simple economic reasoning. As discussed previously, smaller values of \(d\) could result in a wider distribution of production per firm. Hence, for an industry where output per firm varies widely (as in the boat building industry), the analyst could set \(d = 0\); for an industry where output per firm (or plant) is less variable, the analyst could set \(d = 2\) (or higher).

III. The long-run impact of regulation: potential closures of small firms

The long-run provides a profit-maximizing firm with its only opportunity to recover an increase in fixed cost. As discussed in section II, in the short-run, a profit-maximizing firm can recover at least some of the increase in marginal cost, by decreasing output and letting the market price rise. But since profit-maximization requires only that price equals marginal cost, an increase in fixed cost would not affect the profit-maximizing level of output, and so neither the level of production nor the market price would be affected. Firms can recover an increase in fixed cost only by allowing the market to evolve over the long-run to a point where price covers all costs, including those imposed by a new regulation.
Figure 1 gives a graphical representation of the long-run equilibrium in a market that has experienced an increase in costs. A key assumption of the perfectly competitive model is that the market is in long-run equilibrium when every firm's revenue covers all of its costs, including a sufficient return to equity; or in other words, when market price equals average cost. Profit-maximization requires further that price equal marginal cost, so long-run equilibrium requires that a firm produce where average and marginal cost are equal – that is, where average cost is minimized. The level of production that has minimum average cost is called the "minimum efficient scale", which I denote by \( q^* \). Since the long-run price is determined by minimum average cost, the demand curve then determines the total market output. Dividing the equilibrium market output by the minimum efficient scale gives the equilibrium number of firms.

This simple model gives clear predictions of the long-run implications of higher production costs. As shown in Figure 1, when marginal cost increases, both the average cost and the marginal cost curves shift up; when fixed cost increases, the average cost shifts up, and the minimum efficient scale increases. Higher average costs means higher equilibrium prices, which in turn means lower market output. Lower market output combined with higher production per firm means that the equilibrium number of firms must fall. (See Martin, 1993, for a review of the economic theory of entry and exit beyond the simple model I use here. Siegfried and Evans, 1994, review the empirical literature on entry and exit.)

Hence, one measure of the long-run impact of a regulation is the relative change in the equilibrium number of firms; I will refer this as the "exit ratio". The analysis of an
increase in marginal cost alone is simpler, so I consider the effect of marginal cost on the exit ratio first.

Suppose, as in the previous section, that a new regulation caused a firm's marginal cost to increase by a constant amount $b'$. In this case, the marginal and average cost curves shift up, but the minimum efficient scale of production does not change. In the long-run, firms would produce at the same level as before the regulation, so the long-run equilibrium price is $p_0^* + b'$. If market price increases by $b'$, then the industry's equilibrium output $Q^*$ would fall by the factor $(1 + \varepsilon_{LR} \rho_M)$, where $\varepsilon_{LR}$ is the long-run price elasticity of demand. So the equilibrium number of firms in the industry changes from

$$N_0^* = \frac{Q_0^*}{q_0^*}$$

(22) to

$$N_1^* = \frac{Q_1^*}{q_0^*} = (1 + \varepsilon_{LR} \rho_M)\frac{Q_0^*}{q_0^*} = (1 + \varepsilon_{LR} \rho_M)N_0^*.$$  

(23) Hence, the equilibrium number of firms changes by

$$x = \frac{N_1^*}{N_0^*} - 1 = \varepsilon_{LR} \rho_M$$

(24) percent. So in this case, the percentage change in the equilibrium number of firms is the same as the percentage change in output per firm. (This case would not necessarily apply to the boat-building industry, since the proposed regulation of boat builders involves a fixed cost.)

Although the reasoning is the same, measuring the exit ratio is slightly more complicated when fixed cost increases as well. In that case, since the minimum efficient scale of production $q^*$ increases, marginal cost would increase by more than $b'$ (assuming that marginal cost is not constant). Suppose that every firm has the same marginal cost
function, as defined in expression (17). Then one can show that the minimum efficient scale changes by the factor

\[ \delta = \left( 1 + \frac{1}{\mu_0} \rho_s \right)^{1/(d+1)} \]

and that the equilibrium number of firms decreases by

\[ x(d) = \frac{1}{\delta} \left( 1 + \epsilon L R_\mu + \epsilon L R \mu_0 \ast \left[ (1 + \frac{1}{\tau})(\delta^d - 1), d > 0 \right] \log \delta, d = 0 \right) - 1 \]

percent (Appendix II). Note that \( x(d) \approx \epsilon L R \rho M \) when \( \delta \approx 1 \) (that is, when there is only a small change in the minimum efficient scale), so the more general value of \( x(d) \) reduces to the simple one in expression (24).

The exit ratio \( x(d) \) requires less information than the cost-to-profit ratio, \( g(f,d) \). Unlike \( g(f,d) \), \( x(d) \) does not depend on the rate of cost pass-through, since firms pass on all costs (both marginal and fixed) in the long-run. (In general, both firms and consumers suffer a welfare loss, though, as measured by producer and consumer surplus.) Like \( g(f,d) \), \( x(d) \) does depend on the parameter \( d \), but a set of upper and lower bounds for \( x(d) \) may be sufficient in a simple regulatory impact analysis.

The bounds on \( x(d) \) are simpler than the bounds on \( g(f) \), since \( x(d) \) increases as \( d \) increases – hence, the minimum value of \( x(d) \) occurs at \( d = 0 \). For small values of the scale change (\( \delta \approx 1 \)), one can approximate \( \log(\delta) \) as \( (\delta - 1) \), so the lower bound on \( x \) is approximately

\[ x \equiv \frac{1 + \epsilon L R (\rho_u + \rho_f)}{1 + \rho_f / \mu_0 \ast} - 1 \leq x(d). \]

Note that the numerator in expression (27) is simply the change in output resulting from a price increase equal to the average compliance cost, assuming no change in scale. The
denominator in expression (27) is the maximum effect on the minimum efficient scale, resulting from the increase in fixed cost (i.e., maximized with respect to the given marginal cost function). As $d$ increases above $d = 0$, $\delta^d$ approaches $(1 + \rho_\gamma/\mu_0^*)$, and $\delta$ approaches 1, so the upper bound on $x$ is simply

\begin{equation}
(28) \quad x(d) \leq \bar{x} \equiv \varepsilon_{\text{LR}}(\rho_u + \rho_r).
\end{equation}

For example, consider once again the proposed regulation of boat builders. If the price elasticity of demand is $-1.9$ (EPA, 1999a does not report whether the elasticity is short- or long-run), and the total compliance cost to revenue ratio is $\rho_u + \rho_r = 0.0012$, the regulation could cause the equilibrium number of firms to fall by at least 0.23 percent. The maximum change in the minimum efficient scale is $\frac{1}{x} \rho_r = 0.0035$ percent, so the regulation could cause the equilibrium number of firms to fall by at most 0.58 percent.

IV. The intermediate impact of regulation: length of the adjustment period

In a dynamic industry, the change in the equilibrium number of firms could overestimate the true, long-run impact of the regulation. If an industry is growing, then a regulation may reduce the equilibrium number of firms without reducing the actual number of firms. Instead, the regulation may merely retard the entry of new firms, by reducing the profits of existing firms and reducing the incentives for entry. If an industry is declining, then firms are gradually leaving the industry as demand shrinks exogenously. A new regulation would merely hasten their exit. Some firms will leave sooner than they would otherwise, but this is only a short-run (in fact, immediate) impact, and the long-run exit rate does not change.
To provide an additional perspective on the impact of a regulation, one could measure the "intermediate" impact as the length of time required for the existing firms in an industry to recover from the effects of the new regulation (this does not account for the impact on firms that would have entered in the industry, but were deterred by the new regulation). In a growing industry, the industry could be said to recover from the impact of the regulation when the equilibrium number of firms returns to the level prior to the regulation. In a declining industry, the industry could be said to "recover" from the impact of the regulation (even though it is in long-run decline) in the time required for the equilibrium number of firms, absent the regulation, to equal the equilibrium number of firms immediately following the regulation.

Suppose that the equilibrium industry output is changing at the annual rate $\gamma$ (via exogenously increasing demand). One can show that the length of the transition to the new equilibrium is

$$t = \left\lfloor \frac{1}{\gamma} \log(1 + x) \right\rfloor$$

years (Appendix II), where $x$ is the exit ratio (expression (26)). For a growing industry ($\gamma > 0$), $t$ measures the length of time that firms experience a decline in profits; after time $t$, firms would earn profits in excess of the equilibrium level (if the industry is growing, profits must be above the equilibrium level in order to attract new firms to the industry). For a declining industry ($\gamma < 0$), $t$ measures how much sooner firms are induced to close, given that they would close anyway in the exogenous decline of the industry. For an industry where demand is not growing ($\gamma = 0$), the intermediate impact is "infinite", since some firms must exit the industry, and so the industry never "recovers" from the impact.
of the regulation. Figure 3 gives a graphical representation of the value of \( t \) for both growing and declining industries.

The intermediate impact indicator \( t \) uses the same information as the short- and long-run impact indicators, with the addition of only the annual growth rate \( \gamma \). Like the short- and long-run impact indicators, \( t \) also requires the marginal cost parameter \( d \), but one can derive upper and lower bounds on \( t \). When \( x \) is negative, \( \log(1+x) \) is negative, so \( |\log(1+x)| \) is a decreasing function of \( x \). Hence, the bounds on \( t \) are

\[
\frac{1}{\gamma} \log \left( 1 + \varepsilon_x \left( \rho_u + \rho_f \right) \right) \leq \frac{1}{\gamma} \log \left( \frac{1 + \varepsilon_x \left( \rho_u + \rho_f \right)}{1 + \rho_f / \mu_*} \right),
\]

For example, EPA reported that the average annual growth rate of the sailboat building industry was 11 percent between 1991 to 1996 (\( \gamma = 0.11 \)). At that rate of growth, the sailboat-building industry could recover from the proposed regulation in one to three weeks (0.02 and 0.05 times 365 days).

V. Discussion

The tools presented in this study provide simple and straightforward means to evaluate the impact of regulations (or other imposed costs) on small firms. The tools are based on standard economic theory, and as such they provide an easily accessible and applicable framework for combining and interpreting a variety of economic data, such as demand elasticities, cost to revenue ratios, profit margins, and industry growth rates.

The most important contribution of the tools presented here is the emphasis on fixed and marginal (or variable) costs of compliance with regulation. In the U.S. federal government, the most commonly used measures of economic impact combine the fixed
and marginal compliance costs. Standard economic theory, though, indicates that fixed and variable costs have different impacts: if short-run supply is based on marginal cost, then fixed costs do not affect supply or market price, and firms pay all new fixed costs out of their profits; in the long-run, though, fixed costs can increase the scale of production, and so can lead to firm closures.

Since the procedures presented here are intended to be as simple as possible, they clearly have limitations. For example, the tools focus on the effect of regulation on the owners of small firms (that is, through the reduction in profits), and do not specifically consider the effects on employment (except indirectly, through firm closures). Also, the analysis does not consider the effects of regulation beyond the affected firms. There are, of course, other methods of assessing economic impact, such as input-output analysis (for example, see the Minnesota IMPLAN Group at www.mig-inc.com) or regional economic models (see Regional Economic Models, Inc., at www.remi.com). These methods are much more detailed and use more information, so one would expect them to be more reliable than the simple tools presented here. In deciding whether to use these tools instead, one should compare the marginal benefit of these tools (that is, the increase in reliability or in the confidence placed in the predictions) with the marginal cost of using them.

I regard the methods developed in this study – and indeed, all economic models – as analytical devices, rather than forecasting tools. In particular, the indicators measuring the change in the equilibrium number of firms do not necessarily predict the closure of that many firms, let alone the closure of any particular firms. Instead, the tools merely attempt to measure the change in one of the many different economic forces that
influence the decisions of small business owners and managers. The tools provide a measuring device analogous to, say, a thermometer measuring air temperature. Some people make decisions based in part on the day's temperature, but many other aspects of weather, as well as one's unique personal situation, also play a role those decisions. One would not expect the temperature – let alone a forecast of the temperature in the distant future – to be a reliable predictor of behavior, but it can provide useful information, nonetheless. One should use the same discretion in the interpretation and application of the measuring devices presented in this manuscript.

Further research should investigate not only the applicability of the indicators, by using them to analyze other regulations, but also their reliability, from both a modeling and an empirical perspective. From a modeling perspective, the tools could be evaluated by comparing their predictions with those derived from input-output or regional economic models. From an empirical perspective, the tools could be evaluated by applying them to existing regulations, and comparing their predictions to the corresponding real outcomes. A reliable investigation of that kind would have to explicitly consider the sources of uncertainty in each tool, to be able to determine if the errors were outside the range of errors that are expected or inherent in the tool.

VI. References


Executive Order 12866 (1993), Regulatory Planning and Review (Federal Register, 58 FR 51735; http://www.epa.gov/fedrgstr/eo/eo12866.htm).

Environmental Protection Agency (1999a), "Economic Analysis of the Proposed Boat Manufacturing NESHAP" (EPA-452/R).

Environmental Protection Agency (1999b), "Revised Interim Guidance for Rulewriters: Regulatory Flexibility Act as Amended by the Small Business Enforcement Fairness Act" (www.EPA.Gov/SBREFA).


Figure 1 shows the effect of increasing both the marginal cost and fixed cost. Figure 1.a) shows the effect on a firm: higher marginal and fixed costs causes the marginal cost (MC) to shift up, and the average cost (AC) curves to shift up and out. Figure 1.b) shows the corresponding effect on a market: the market supply curve shifts up, creating disequilibrium in the short-run; the supply curve continues to shift up in the long-run, as firms exit, until a new equilibrium is reached with price equal to the minimum average cost.
Figure 2 shows the average cost of production (relative to the minimum) that results from a percentage deviation from the least-cost level of production. Note that when $d = 0$, the average cost is relatively insensitive to the level of production; an industry with this type of cost function could support a wide range of firm (or plant) sizes.
Figure 3 shows the effect of a regulation on the equilibrium number of firms. Figure 3.a) shows that in a growing industry, the equilibrium number of firms eventually returns to its level before the regulation. Figure 3.b) shows that in a declining industry, the equilibrium number of firms would eventually fall to the level immediately following the regulation.
Appendix I: How to conduct a screening analysis using the tools in this manuscript

The following lists the steps that one would follow to use the ideas of this study in a "screening analysis" of a proposed regulation. The steps follow EPA's revenue test as closely as possible. In particular, I convert the ratio of compliance costs to profit described in section II into a compliance cost-to-revenue ratio, by multiplying the cost-to-profit ratio by the gross margin. This puts the cost-to-profit ratio on the same scale as the cost-to-revenue ratio, and so one can apply the same thresholds that EPA currently uses (one and three percent) to determine when an impact is "significant".

Short-run analysis: conduct the revenue test.

1. Estimate the compliance cost:
   - Capital Cost (i.e., fixed compliance cost).
   - Operating and Maintenance (O&M) cost (i.e., the marginal or variable compliance cost).

2. Obtain data on
   - Revenue per firm.

3. Calculate
   - Revenue Ratio = (Capital Cost + O&M Cost) / Revenue.

4. If Revenue Ratio < 0.01, then the regulation does not have a significant economic impact.

Otherwise, ...
Re-calculate the Revenue Ratio, incorporating the potential impact of cost pass-through.

5. Obtain data on ...
   - Gross Margin = (Revenue – Variable Cost) / Revenue.
     - "Variable Cost" is the firm's total variable cost before the regulation.
     - Potential data sources: US Census; The Almanac of Business and Industrial Ratios.
     - For an explanation and justification of using gross profit in a regulatory impact analysis, see the discussion surrounding expression (1).
   - Elasticity of demand.
     - Potential data sources: trade associations; Global Trade Analysis Project.

6. Determine the maximum potential impact on a firm:
   - Revenue Ratio_{\text{max}} = \frac{\text{[Capital Cost + O&M } \times \text{ max}(1, \text{–Elasticity} \times \text{Gross Margin})]}{\text{Revenue}}.
     - This formula results from multiplying the cost-to-profit ratio in expression (14) by the Gross Margin; see the discussion in section II for a full explanation of the cost-to-profit ratio.

7. Calculate the potential impact on demand:
   - Change in Output_{SR} = \frac{\text{O&M Cost}}{\text{Revenue}} \times \text{Elasticity}.
     - The term in brackets () above is the percent change in market price; multiplying by the elasticity gives the percent change in demand, and hence output.

8. Calculate the minimum potential impact on a firm:
   - Revenue Ratio_{\text{min}} = Revenue Ratio_{\text{max}} –
     \( (1 + \text{Change in Output}_{SR}) \times (\text{–Change in Output}_{SR}) \times \text{Gross Margin}.\)
- For an explanation and justification, see section II, particularly the discussion surrounding expression (20).

9. If Revenue Ratio$_{\text{min}} > 0.03$, then the regulation has a "significant" impact.

- Note that if Revenue Ratio$_{\text{min}} > 0.03$, then this is equivalent to

\[
\text{Revenue Ratio} > 0.03 + (1 + \text{Change in Output}_{\text{SR}}) \times (\text{Change in Output}_{\text{SR}}) \times \text{Gross Margin},
\]

so this test is quite different from the previous one (step 4).

Otherwise, ...

End of the short-run analysis. Evaluate long-run impact: minimum number of firm closures.

10. Calculate minimum percentage of firms that go out of business:

- Firm Closures$_{\text{min}} = -\text{Elasticity} \times \text{Revenue Ratio}.$

- The "Revenue Ratio" in this calculation is that from step 3.

- For an explanation and justification, see section III, particularly the discussion surrounding expression (24).

11. If Firm Closures$_{\text{min}} > 0.03$, then the regulation has a "significant" impact.

- The threshold of 0.03 (same as the Revenue Ratio) seems appropriate, since the percentage of firm closures is based on the Revenue Ratio.

- Note that if Firm Closures$_{\text{min}} > 0.03$, then this is equivalent to

\[
\text{Revenue Ratio} > 0.03 / -\text{Elasticity},
\]

so this test is quite different from the previous two.
Otherwise, ...

Calculate maximum number of firm closures.

12. Calculate the long-run change in output per firm:
   
   • Change in Output\(_{LR}\) = Capital Cost / (Revenue \times \text{Gross Margin}).

   - For an explanation, see section III, particularly the discussion surrounding expression (25); Appendix II provides further details.

13. Calculate maximum percentage of firms that go out of business:
   
   • Firm Closures\(_{\text{max}}\) = \(\frac{1 – \text{Firm Closures}_{\text{min}}}{1 + \text{Change in Output}_{LR}} – 1\)

   - For an explanation, see section III, particularly the discussion surrounding expression (27); Appendix II provides further details.

14. If Firm Closures\(_{\text{max}}\) < 0.01, then no significant impact.

   - Note that if Firm Closures\(_{\text{max}}\) < 0.01, then this is (approximately) equivalent to Revenue Ratio > Capital Cost / (Revenue \times \text{Gross Margin} \times –\text{Elasticity}), so this test is also different from the previous ones.

End of long-run analysis. Evaluate intermediate impact: the potential for firms to "grow out" of the added cost of the regulation.

15. Obtain data on the market's

   • Annual Growth of Production.

   - Source: US Census.

16. Calculate time required to "grow out" of the regulation:

   • Recovery Period\(_{\text{min}}\) = |Firm Closures\(_{\text{max}}\) / (Market's Growth Rate)|.

   • Recovery Period\(_{\text{max}}\) = |Firm Closures\(_{\text{min}}\) / (Market's Growth Rate)|.
- The recovery period is not on the same scale as the revenue ratios, so the 0.01 and 0.03 thresholds do not necessarily apply.

- For an explanation of the "recovery period", see section IV.
Appendix II: Technical notes in support of the study 'Measuring the impact of regulation on small firms'

Assumption I: All plants in the industry have the same technology, with average cost function

\[
(C(q) = a(q^d - 1)\right\{d > 0 \atop \log q, d = 0\} + b + c/q, (A1)
\]

where \(a, b, c\) and \(d\) are fixed, non-negative parameters, and \(q\) is a plant's rate of production measured in units of the minimum efficient scale, \(q^*\). That is, the plant is producing at minimum average cost when \(q = 1\). This (of course) requires that \(q^*\) is known, so that one can measure output.

Note: in the following, an asterisk (*) indicates a long-run equilibrium level; for example, \(q\) is the rate of output, and \(q^*\) is the long-run equilibrium rate of output (i.e., the minimum efficient scale).

Fact 1. The function \(\log(q)\) is the limit of the function \((q^d - 1)/d\), as \(d\) approaches zero, so one may use the parameter \(d\) as a continuous index of the range of average cost functions.

Proof: The function \((q^d - 1)/d\) is the Box-Cox function; many econometrics textbooks discuss the properties of this well-known function (for example, see Berndt, 1996).

Fact 2. The minimum efficient scale of a plant is

\[
q^* = (c/a)^{1/(d+1)}. (A2)
\]

Proof: The first-order condition for minimum average cost is
(A3) \( \bar{C}'(q,d) = a \begin{cases} \frac{q^{d-1}}{q-1}, & d > 0 \\ \frac{q^{-1}}{q-1}, & d = 0 \end{cases} - c / q^2 = a q^{d-1} - c / q^2 = 0. \)

Solving expression (A3) for \( q \) yields expression (A2).

Fact 3. The average cost function can be expressed in terms of the equilibrium level of gross profit:

(A4) \( \bar{C}(q) = \pi^* \begin{cases} \frac{1}{\pi}(q^d - 1), & d > 0 \\ \log q, & d = 0 \end{cases} + b + \pi^* / q. \)

The minimum average cost is then

(A5) \( \bar{C}(1) = b + \pi^*. \)

Proof: If the plant size \( q^* \) and the short-run fixed cost \( c \) are known, then using expression (A2), one can estimate \( a \) as

(A6) \( a = c / (q^*)^{d+1}. \)

Since \( q \) is measured in units of \( q^* \), \( q = q^* = 1 \), and

(A7) \( a = c = \pi^*. \)

Expression (A4) results from substituting \( \pi^* \) for \( a \) and \( c \) in expression (A1).

Fact 4. The marginal cost function is

(A8) \( C'(q) = \pi^* \begin{cases} \frac{1}{\pi}((d+1)q^d - 1), & d > 0 \\ 1 + \log q, & d = 0 \end{cases} + b. \)

Proof: Multiply average cost by \( q \) and take the derivative with respect to \( q \).
Fact 5. Producing at the rate $q$ increases the average production cost above the minimum by

$$
\pi^* \left\{ \left( \frac{1}{\alpha} - 1 \right) + \frac{1}{\alpha} (q^d - 1), d > 0 \right\} + \left( \frac{1}{\alpha} - 1 \right) + \log q, d = 0.
$$

(A9)

**Proof:** The difference between the average cost at $q$ and the minimum average cost is

$$
\bar{C}(q) - \bar{C}(1) = \pi^* \left\{ \left( \frac{1}{\alpha} (q^d - 1), d > 0 \right\} + b + \pi^*/q - (b + \pi^*). 
$$

(A10)

Expression (A9) results from simplifying expression (A10).

Assumption II: In addition to Assumption I, assume that ...

- The market is initially in long-run equilibrium, with market price $p_0$, and every plant producing at $q_0^* = 1$.
- Each plant's marginal cost increases by a constant amount $b'$, and each plant's fixed cost increases by $c'$.
- Each plant raises its output price by a fraction $f$ of the increase in marginal cost, or $(fb'/p_0)$ percent.
- Each plant's output falls by $\epsilon(fb'/p_0)$ percent, where $\epsilon$ is the price elasticity of demand.
- Market demand is unchanging.

Note: the following considers the effects of changes in the parameters of the industry. I use the subscripts 0 and 1 to indicate the initial and the new level of a parameter; for example, the equilibrium gross profit changes from $\pi_0$ to $\pi_1$. I use a prime (') to indicate a
change from the initial level, with the notation for the initial parameter unchanged; for example, the fixed component of marginal cost changes by $b'$, from $b$ to $b + b'$.

Fact 6. In the short-run, the scenario described in Assumption II changes the amount of funds available to cover other short-run fixed costs by

$$q_1 \left[ 1 - \left\{ \frac{1}{d} (q_1^d - 1), d > 0 \right\} \right] - \left( c' - b'(f - 1)q_1 \right) / \pi_0^* - 1$$

percent of the initial equilibrium gross profit, $\pi_0^*$, where

$$q_1 = 1 + \varepsilon(fb'/p_0)$$

**Proof**: Re-define the average cost function, incorporating the higher marginal and fixed cost:

$$\bar{C}_i(q) = \pi_0^* \left\{ \frac{1}{d} (q^d - 1), d > 0 \right\} + (b + b') + (\pi_0^* + c') / q.$$  

Of course, this cost function incorporates the previous one: set $b' = c' = 0$. A plant's variable cost as a function of output is

$$C_i(q) = q \bar{C}_i(q) - (\pi_0^* + c') = \pi_0^* q \left\{ \frac{1}{d} (q^d - 1), d > 0 \right\} + (b + b')q.$$  

Since initial output is $q_0^* = 1$, and since a plant's output falls by $\varepsilon(fb'/p_0)$, the new output per plant is

$$q_1 = 1 + \varepsilon(fb'/p_0).$$

Gross profit at the new output is
\[ \pi(q_1) = (p_0 + fb')q_1 - \pi_0 * q_1 \left\{ \frac{1}{d} (q_1^d - 1), d > 0 \right\} - (b + b')q_1 \]

\[ (A16) \]

Since the industry is initially in equilibrium, price equals minimum average cost, or

\[ (A17) \ p_0 = b + \pi_0 * \]

from expression (A5). Substituting \( p_0 - b = \pi_0 * \) into expression (A16) and factoring out \( \pi_0 * \) gives

\[ (A18) \ \pi(q_1) = q_1 \pi_0 * \left[ 1 - \left\{ \frac{1}{d} (q_1^d - 1), d > 0 \right\} \right] + (f - 1)b'q_1. \]

The amount available to cover other short-run fixed costs is \( \pi(q_1) - c' \). Expression (A11) results from dividing \( (\pi(q_1) - c') \) by \( \pi_0 * \) and subtracting one.

Fact 7. Under Assumption II, the minimum efficient scale changes by a factor of

\[ (A19) \ \delta \equiv \left( 1 + \frac{c'}{\pi_0 *} \right)^{1/(d+1)}. \]

Proof: Suppose that short-run fixed cost increase from \( \pi_0 * \) to

\[ (A20) \ \pi_0 * + c' = \pi_0 *(1 + \frac{c'}{\pi_0 *}). \]

The original minimum efficient scale of production is

\[ (A21) \ q_0 * = \left( \frac{\pi_0 *}{a} \right)^{1/(d+1)}, \]

and so the new minimum efficient scale is

\[ (A22) \ q_1 * = \left( \frac{\pi_0 * (1 + \frac{c'}{\pi_0 *})}{a} \right)^{1/(d+1)} = \left( 1 + \frac{c'}{\pi_0 *} \right)^{1/(d+1)} \left( \frac{\pi_0 *}{a} \right)^{1/(d+1)} \equiv \delta q_0 *. \]
Fact 8. Under Assumption II, in the long-run the market price increases by

\[
\mu_0^* \left\{ (1 + \frac{1}{d})(\delta^d - 1), d > 0 \right\} \log \delta, d = 0 \right\} + \frac{b'}{p_0},
\]

percent, where \(\mu_0^*\) is the gross margin at the initial equilibrium.

**Proof**: Note that

\[
\pi_0^* + c' = \pi_0^* (1 + c'/\pi_0^*) = \pi_0^* \delta^{d+1}.
\]

The new average cost function is

\[
\bar{C}_1(q) = \pi_0^* \left\{ \frac{1}{d}(q^d - 1), d > 0 \right\} + (b + b') + (\pi_0^* + c')/q
\]

\[
= \pi_0^* \left\{ \frac{1}{d}(q^d - 1) + \delta^{d+1}q^{-1}, d > 0 \right\} + (b + b').
\]

(Note that output is still measured in units of the original minimum efficient scale.) The new minimum efficient scale is

\[
q_1^* = \delta q_0^* = \delta,
\]

so the new, minimum average cost is

\[
\bar{C}_1(\delta) = \pi_0^* \left\{ \frac{1}{d}((d + 1)\delta^d - 1), d > 0 \right\} + (b + b').
\]

The difference between the original and the new minimum average cost is

\[
\bar{C}_1(\delta) - \bar{C}(1) = \pi_0^* \left\{ \frac{1}{d}((d + 1)\delta^d - 1), d > 0 \right\} + (b + b') - (\pi_0^* + b)
\]

\[
= \pi_0^* \left\{ \frac{1}{d}((d + 1)\delta^d - 1), d > 0 \right\} + b' = \pi_0^* \left\{ (1 + \frac{1}{d})(\delta^d - 1), d > 0 \right\} + b'.
\]

Dividing expression (A28) by \(p_0\) gives
(A29) \( \left( \tilde{C}_i(\delta) - \tilde{C}(1) \right) / p_0 = \frac{\pi_0^*}{p_0} \begin{cases} (1 + \frac{1}{d})(\delta^d - 1), & d > 0 \\ \log \delta, & d = 0 \end{cases} + \frac{b'}{p_0}. \)

Let \( \mu_0^* \) denote the initial, equilibrium gross margin:

(A30) \( \mu_0^* = \frac{\pi_0^*}{p_0 g_0^*} = \frac{\pi_0^*}{p_0}. \)

Substituting expression (A30) into expression (A29) yields expression (A23).

Fact 9. Under Assumption II, the long-run equilibrium number of firms falls by

(A31) \( \frac{1}{\delta} \left[ 1 + \varepsilon \mu_0^* \begin{cases} (1 + \frac{1}{d})(\delta^d - 1), & d > 0 \\ \log \delta, & d = 0 \end{cases} + \varepsilon \frac{b'}{p_0} \right] - 1. \)

percent.

Proof: Let \( Q_0^* \) denote the initial equilibrium output of the industry. The equilibrium number of firms is initially

(A32) \( N_0^* = Q_0^*/q_0^*. \)

The percentage change in the long-run equilibrium price is given in expression (A23), so if \( \varepsilon_{LR} \) is the long-run elasticity of demand, the new equilibrium output of the industry is

(A33) \( Q_1^* = Q_0^* \begin{cases} 1 + \varepsilon \mu_0^* \begin{cases} (1 + \frac{1}{d})(\delta^d - 1), & d > 0 \\ \log \delta, & d = 0 \end{cases} + \varepsilon \frac{b'}{p_0} \end{cases}. \)

The new equilibrium number of firms is then

(A34) \( N_1^* = \frac{Q_1^*}{q_1^*} = \frac{Q_0^*}{\delta q_0^*} \begin{cases} 1 + \varepsilon \mu_0^* \begin{cases} (1 + \frac{1}{d})(\delta^d - 1), & d > 0 \\ \log \delta, & d = 0 \end{cases} + \varepsilon \frac{b'}{p_0} \end{cases}. \)

Dividing expression (A34) by \( N_0^* \) and subtracting 1 gives the percentage change in the equilibrium number of firms in expression (A31).
Assumption III: Same as Assumption II, except in addition ...

- The long-run supply curve has infinite price-elasticity (i.e., long-run supply is horizontal at price equal to the minimum average production cost).
- The demand curve is shifting out at the annual rate $\gamma$, and the price elasticity of demand is unchanging.

Fact 10. The industry can be said to "recover" from the impact of the higher production cost in

$$t = \frac{1}{\gamma} \log \left[ \frac{1}{\delta} \left( 1 + \varepsilon \mu_0 \ast \left\{ \begin{array}{l} (1 + \frac{1}{d})(\delta^d - 1), d > 0 \\ \log \delta, d = 0 \end{array} \right\} + \varepsilon \frac{b'}{p_0} \right) \right]$$

years.

*Proof*: Absent the change in costs, the long-run equilibrium number of firms in year $t$ would be

$$N_0 \ast (t) = \frac{e^{\gamma t} Q_0 \ast}{q_0 \ast}.$$  

With the change in costs, the long-run equilibrium number of firms in year $t$ would be

$$N_1 \ast (t) = \frac{Q_0 \ast e^{\gamma t}}{\delta q_0 \ast} \left[ 1 + \varepsilon \mu_0 \ast \left\{ \begin{array}{l} (1 + \frac{1}{d})(\delta^d - 1), d > 0 \\ \log \delta, d = 0 \end{array} \right\} + \varepsilon \frac{b'}{p_0} \right].$$

To measure the impact of higher cost in a growing industry ($\gamma > 0$), set $N_1 \ast (t) = N_0 \ast (0)$ and solve for $t$. To measure the impact of higher cost in a declining industry ($\gamma < 0$), set $N_0 \ast (t) = N_1 \ast (0)$ and solve for $t$. The solution to these equations is expression (A35).