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Preventing Biological Invasions: Doing Something vs. Doing Nothing

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Abstract: Both biologists and economists are concerned about invasive species. There are several well-documented instances in which biological invaders have done extensive damage. This has led some economists to conclude that biological invaders should be treated as a form of "pollution", and that the activity generating the "pollution" — international trade — should be taxed to the point at which its marginal benefits balance its marginal costs. This prescription presumes a convex objective, however, and this assumption may not hold. If the expected damage done by a biological invader is the damage caused in the event an invasion occurs times the probability of invasion, and the probability of invasion is necessarily bounded by one, there may be situations in which fighting potential invaders is a lost cause and should be abandoned. A simple model illustrates that such results can arise even when potential damages are significant. In general the optimal policy is discontinuous in the magnitude of potential damages. For low-to-medium levels of damage it is best essentially to do nothing to prevent invasion, but at a certain threshold level of potential damages there is a discontinuous policy response from doing nothing to adopting aggressive measures. The dividing line between laissez faire and aggressive responses is associated with the relationship between gains from trade and potential damages. For potential damages in excess of the maximum potential gains from trade aggressive responses are generally warranted, while damages below that threshold often do not merit intervention. These results point to the need to better understand the true costs of biological invasion. It is especially important to distinguish between tangible and intangible values and decide what weight society assigns to the latter.

Keywords: Invasive species; nonconvexity; cost-benefit analysis; laissez faire

Subject areas: Economic damages/benefits; international trade

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1. Introduction

There has been considerable recent interest in invasive species. While organisms have been moving to new environments throughout history, modern technologies of transport have sparked new concerns regarding biological invasions. Deliberate introductions of organisms that reproduce explosively in their new surroundings can still be a problem in some places. Accidental stowaways are a larger issue, however. Plants, animals, and microbes can be unintentionally transported intermixed with commercial products, in packaging materials, as parasites on larger organisms, in the ballast water of ships, or even clinging to the landing gear of airplanes. Organisms that would never have been moved from one region to another, or would have perished during the long and arduous trip centuries ago, may now be whisked from one corner of the globe to another in a matter of days, or even hours.

The great majority of creatures that make such trips either do not survive the journey or fail to gain a foothold following their arrival; the majority of those that do survive blend in innocuously in their new environments (Mack, et al. 2000). A small fraction, however, when liberated from native predators, competitors, diseases, or nutritional constraints expand explosively. When they do, they may exterminate native species via predation or competition, alter ecological regimes, degrade infrastructure, reduce agricultural productivity, or cause illnesses in humans or domestic animals. Modern examples include the explosion of zebra mussels in the Great Lakes, which has caused great expense and damage by encrusting submerged structures, or the expansion of brown tree snakes in the South Pacific, where they have decimated populations of ground-nesting birds.
Several commentators have suggested that tougher measures be adopted to prevent further biological invasions. Many economists have also now written on the topic, and a commonly held view is that biological invaders are a form of "pollution" (for surveys of important contributions, see Olson 2005 and Lovell et al. 2005). They are the unintentional byproduct of an otherwise socially beneficial activity, international trade. As such, it has been suggested that restrictions should be placed on trade or, when possible, on the quantities of biological stowaways transported, so as to equate the marginal benefits of trade with its marginal costs in terms of expected biological damage (see, e.g., McAusland and Costello 2004, as well as the work surveyed in Olson and Lovell et al.)

In this paper I look more closely at the "equate marginal benefit to marginal cost" prescription for managing potential biological invaders. Such a prescription presumes well-behaved cost and benefit functions, but there are good reasons to believe that the costs of invasive species – the expected damages from their introduction – will not be convex. Once an invasive species is introduced and becomes established, its control and eradication may become very difficult and expensive, if not impossible. For this reason, biologists often argue for concentrating resources on the prevention of invasions, since the control of invaders that have already become established is often not feasible (see, e.g., Mack, et al. 2000). If this is the case, it makes little sense to fight lost causes. Once the first wave of invaders has become established, second and third waves have little additional impact.

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1 The Federal Court for the Northern District of California seems also to have adopted the view that invasive species should be treated as "pollutants". In 2006 it ruled that ballast water could no longer be exempted from treatment as a pollutant under the Clean Water Act.
The nonconvexity I explore here arises because the expected damage from the arrival of an invader is the anticipated damage in the event it becomes established times the probability of that event occurring. That probability increases in the volume of trade, but must increase at a decreasing rate: it is bounded by one. Consequently, the marginal expected damage declines in the volume of trade. At high trade volumes, and with a high enough probability that an invader arrives in any given shipment, it becomes a near certainty that an invasion will occur, and further shipments contribute very little on the margin to the probability of invasion.

Restricting trade is one way to reduce the likelihood of invasion, but there are other, generally better, ways to achieve that end. Cargos can be inspected, quarantined, or fumigated; the first two measures can be applied to human travelers as well. Many aquatic invasive species arrive in the ballast water of ocean-going vessels. Measures that have been suggested for treating ballast water include exchange at sea, heating in transit, ultraviolet radiation, filtration, ozonation, and deoxygenation (Lovell, et al., 2005). In the model I develop below I suppose that the probability that a biological invader will survive its journey to a new environment depends on the number of "treatments" undertaken to eliminate it. More treatments add to the cost of shipping, but reduce the likelihood of invasion.

*If* it makes sense to incur the cost to prevent invasions, the optimal policy should be relatively effective. This observation is almost tautological, however: if invasions are a
cause for serious concern, we should respond to the threat seriously. A more important point is that the optimal policy toward potential invasions may be discontinuous. If anticipated damage in the event that an invasion occurs is relatively low, the optimal strategy will involve a corner solution. No effort should be made to reduce the likelihood of invasion, and, since invasion is a near certainty, there is little reason to restrict the volume of trade. However, for damages above a threshold level, a discontinuous policy response is optimal. Relatively stringent measures should be adopted to reduce the likelihood of invasion, and these measures are sufficiently expensive that they result in a discrete reduction in the volume of trade.

Readers familiar with the economic literature on invasive species will recognize that these results derive from earlier work. I am particularly indebted to two earlier papers. In the first, Richard Horan and his coauthors (2002) develop a model with the same essential probabilistic feature I exploit. They also treat shipments as Bernoulli trials, note the nonconvexity in expected damages, and note in passing the possibility of discontinuous policy responses. Their emphasis is on dealing with deep uncertainty regarding the extent of damages, however. I abstract from this important issue in order to pursue in greater detail the outlines of optimal policy.

The second paper is a widely cited contribution by Carol McAusland and Christopher Costello (2003). In most respects I follow their approach closely. I suppose that a social decision maker wishes to maximize the excess of the gains from trade, as defined by consumer and producer surpluses, over the expected damage from invasion. I also follow
their lead in supposing that each shipment may be regarded as a Bernoulli trial with a certain probability of causing an invasion. Finally, I follow McAusland and Costello in supposing that costly measures can be taken to reduce the likelihood of invasion, and hence, that the policy variables under the decision-maker’s control are the volume of shipments and the stringency of treatments to reduce the likelihood of invasion.²

The only substantive difference between my model and that of McAusland and Costello is that I follow Horan, et al. (2002) in supposing that once an invader has arrived the damage is done and cannot be exacerbated by subsequent arrivals. McAusland and Costello suppose that repeated introductions of invaders will cause further damage. In short, I treat expected damage from biological invasion as the probability that an invasion occurs times the damage from an invasion, while McAusland and Costello treat expected damage from biological invasion as the expected number of invaders that become established times the damage from an invasion.

This small change in assumptions can have important implications, however. As in McAusland and Costello (2003) I find that there can be corner solutions in which the marginal cost of reducing the likelihood of invasion by treating shipments is larger than the marginal benefit in terms of reduced expected damages. McAusland and Costello then suggest that the optimal policy would rely on tariffs or other trade restrictions to reduce

² I refer to “treatments” as the steps taken to eliminate potential invaders, while McAusland and Costello (2003) refer to inspections and detection of the presence of invaders. As detected invaders will be eliminated (and undetected ones will not), however, the approaches are essentially the same.
invasions by discouraging trade. In contrast, I find that little should be done to fight what is, at that point, a lost cause. Rather than relying on tariffs, the optimal policy essentially becomes *laissez faire*. My result does not arise because the damages are *de minimis*. To the contrary, numerical examples are easily constructed in which damages from invasion of nearly the same magnitude as the gains from trade do not justify aggressive countermeasures. There is, however, a “bright line” to guide policy. Aggressive measures to prevent invasions are generally optimal when the damages from invasion are greater than the gains from trade, while a *laissez faire* approach may be appropriate even when expected damages are a significant fraction of the gains from trade.

These findings beg the question of whether the modeling assumption of treating shipments containing potential invaders as Bernoulli trials is valid. Once an invader has become established, is it reasonable to say that subsequent reintroductions have no effect? Like most extreme conditions, this is unlikely to be strictly and universally true. The larger the established population of an invasive organism is, the more likely it is to persist. On the other hand, however, surely it is true that there must be diminishing returns in the contribution of subsequent reintroductions beyond some level of infestation. Moreover, there seems to be at least implicit acceptance in the biological literature of the proposition that small initial populations of invaders are sufficient to cause very big problems (e.g.,

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3 My results really augment or extend those of McAusland and Costello (2003) rather than refuting them. When there is a small enough probability of invasion that the likelihood of duplicated invasions is negligible my results confirm those of McAusland and Costello.
Mack et al. 2000), as well as explicit adoption of this view in the economic literature on invasives (Horan et al. 2002).^4^ 

It may be useful to anticipate two other objections. The first, that the model I develop in this paper considers only a single invasive species, can be addressed with the unexceptionable observation that the sum of concave functions is concave. If the introduction of each of several potential invaders can be treated as a Bernoulli trial, it does not seem too unreasonable, at least for the purposes of generating a reasonably tractable illustrative model, to lump them all together as a “representative invader”.^5^ 

The second objection is that at least some invasive species can be eradicated, albeit, typically, at some significant expense. This should also not be a problem for my model if the response to the detection of an invasion is generally insensitive to its scope. If, as often seems to be the case, the detection of any invader results in the adoption of comprehensive measures to control it, those costs of control will be triggered regardless of the number of times invaders arrive before control efforts begin.

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^4^ One note that might be added here is that absolute, if not necessarily relative, rates of population growth will be greater the greater the initial population of invaders, so the damage invasive species may cause may come sooner if there are more arrivals. It is not clear that such considerations would greatly change my results at reasonable discount and growth rates, however.

^5^ Another interesting perspective on the problem of exotic species is presented in recent work by Brock, et al. (2007). In their paper the authors argue that the damages from invaders are convex, and thus that trade restrictions should increase in the volume of trade. The rationale is that people have convex preferences over the numbers of extant species, and invaders reduce local biological diversity. In other words, Brock et al. presume a different damage function than I or McAusland and Costello have posited. There is likely no way to resolve this point a priori, but this underscores one of my most important conclusions: that it is important to think carefully about damages.
The remainder of this paper is laid out in several sections. In the next section I present and analyze a model of trade and the prevention of biological invasions. Conditions for optimal policy selection are developed in the third section. The potential for more than one (local) optimum is considered in the fourth section. The model is, admittedly, somewhat inelegant, as the functional forms I have chosen do not admit closed form solutions. Thus I combine analytical derivation of some results with numerical demonstrations of others. Some calculations are deferred to an appendix. Policy implications are discussed in the fifth section. A final section briefly concludes.

2. The Model

In this section I develop a simple model of trade and biological invasion following templates developed by Horan et al. 2002 and McCausland and Costello 2004. The model's elements are:

- A probability, $\pi$, with which any single shipment contains a biological invader;
- Damage, in the amount $D$, that results should an invader arrive;
- Gains from trade, as measured by conventional consumer and producer surplus;
- The number of shipments, $S$, accepted in any period, which may be varied as a policy choice variable; and
The intensity of treatments, which can, for a cost, be increased so as to reduce the likelihood of invasion. The intensity of treatments is also a policy variable under the control of social decision-makers.

I will suppose that $\pi$ is the probability with which an invasion can result from any given shipment. If $S$ shipments are received, then, the probability that an invasion occurs from among $S$ shipments is

$$1 - (1 - \pi)^S.$$ \hfill (1)

It will be helpful to define some terms before proceeding. It would be cumbersome to write “the probability that an invasion results from any particular shipment,” each time I refer to the quantity I have denoted by $\pi$, and the “probability that an invasion results from a collection of $S$ shipments,” which I have listed in expression (1). I will, then, refer to $\pi$ as “the probability of invasion”, and spell out "from a collection of $S$ shipments" on the less frequent occasions when I need to describe the probability over multiple shipments.

While none of the following results is dependent on the approximation I am about to introduce, the approximation does simplify subsequent calculations. Note, then, that

$$1 - (1 - \pi)^S = 1 - \left[ \left( \frac{1/\pi - 1}{1/\pi} \right)^{1/\pi} \right]^{S} = 1 - e^{-\pi S}. \hfill (2)$$
If, as seems reasonable, the probability of invasion is small, the approximation will be fairly close. Again, however, the following results do not hinge on this assumption.\footnote{Approximating $(1 - \pi)^S$ by $e^{-\pi S}$ means I can work with derivatives with respect to $S$ and $\pi$ of the form $-\pi e^{-\pi S}$ and $-S e^{-\pi S}$, respectively, rather than the exact, but unwieldy, forms $\ln(1 - \pi)(1 - \pi)^S$ and $-S(1 - \pi)^{S-1}$.}

Suppose that if the invader arrives and becomes established it will cause damages whose net present value is $D$. It will also be helpful to distinguish terms for “the damage anticipated in the event that an invader becomes established,” which I have denoted by $D$, and "the expectation of the damage that will be caused if $S$ shipments are received", which would be expressed in my notation as $[1 - (1 - \pi)^S]D$. These notions are distinguished by using the word “damage” by itself to denote the former, and “expected damage” to denote the latter.

Following McAusland and Costello (2004), I will suppose that the benefits from trade may be measured in the conventional fashion as the area under consumer demand less the cost of providing the good. Such costs of provision will include both the c. i. f.\footnote{“Cost, insurance, freight”: the cost of production plus whatever other expenses are incurred to bring the goods to their intended market.} costs of the goods themselves and the costs of whatever treatment is given to shipments to reduce the likelihood of biological invasion.

I will suppose that demand is of the linear form

$$p = \bar{p} - bS,$$  \hspace{1cm} (3)
where $\bar{p}$ and $b$ are constants and $S$ the volume of shipments accepted per time period.

Let $cS$ be the cost of producing and delivering $S$ shipments for some constant $c$, and let

$$K(\pi, \pi, S)$$

be the cost of treating the $S$ shipments so that the probability of invasion is reduced from an initial value $\pi$ to $\pi$. Then the benefits of engaging in trade, *gross of potential damages*, is

$$\int_{0}^{S} (\bar{p} - bs) ds - cS - K(\pi, \pi, S).$$

(4)

The linear demand specification is somewhat restrictive, but inasmuch as it is a first-order approximation to any demand function, it seems a reasonable construct for deriving illustrative results.

Note also that expression (4) does not necessarily mean that we must suppose that production and shipping costs are constant; I could have supposed that marginal costs were, say, $\bar{c} + \chi S$. The slope-of-marginal-cost term $\chi S$ could be subsumed in the integrand in (4), and all the same results would follow. I mention this now, as there are at times concerns with divisions of the gains from trade, and I want to emphasize that I do not necessarily suppose that all gains would accrue to consumers in the importing country.
It will be helpful to impose some structure on the treatment cost function $K(\pi, \pi, S)$. A natural way to do this is as follows. Suppose that authorities can choose among "treatments" that reduce the probability of damage. In ocean shipping, for example, common treatments in practice or under discussion include ballast water exchange, heating in transit, ultraviolet radiation, filtration, ozonation, and deoxygenation (Lovell, et al., 2005). Let $\phi_i$ be the probability with which a potential invader survives the $i^{th}$ treatment, and suppose that the $\phi_i$'s are statistically independent. Let $\pi$ be the probability with which an invader is present and would survive absent any treatment. Then the probability that an invader would survive a sequence of $T$ treatments is

$$\pi(T) = \pi \prod_{i=1}^{T} \phi_i.$$  \hfill (5)$$

Let $\kappa_i$ be the cost of the $i^{th}$ treatment, and suppose that treatment costs are linear in the number of treatments. Then the cost of reducing the probability of invasion from $\pi$ to $\pi(T)$ in any single shipment is $\sum_{i=1}^{T} \kappa_i$, and the cost of all treatments performed on all shipments received in a single period is

$$K[\pi, \pi(T), S] = \sum_{i=1}^{T} \kappa_i S.$$  \hfill (6)$$

Note the assumptions implicit in (6). First, a full course of $T$ treatments is given to every shipment received in any period. That is, it is not feasible to check and see if one of the preliminary treatments has "worked" by killing potential invaders, thus obviating the need
for further treatments. Second, policy makers commit to pursuing the same sequence of treatments on all shipments received in a given time period. They cannot learn immediately that an invader has slipped in and curtail subsequent treatments. Note also that these considerations will define the length of “period” of the model: the time to which policy makers commit to a treatment regime or, equivalently, the time required to learn that past treatments did not succeed in repelling all invaders.

The cost of giving all shipments a "marginal treatment" would then be

\[
K[\pi, \pi(T + 1), S] - K[\pi, \pi(T), S] = \kappa_{T+1}S. \tag{7}
\]

The reduction in the probability of invasion associated with the "marginal treatment" would be

\[
\pi(T + 1) - \pi(T) = \pi \left( \prod_{i=1}^{T+1} \phi_i - \prod_{i=1}^{T} \phi_i \right) = (\phi_{T+1} - 1)\pi(T). \tag{8}
\]

Thus we might approximate the derivative of the cost of treatment with respect to its efficacy by dividing (7) by (8):

\[
\frac{\Delta K(\pi, \pi, S)}{\Delta \pi} = \frac{\kappa_{T+1}}{(\phi_{T+1} - 1)\pi} S. \tag{9}
\]
Expressions (5) – (9) are a somewhat more general form of the specification I will use in what follows. If all the \( \kappa \)'s and \( \phi \)'s are the same, then

\[
\pi(T) = \pi \phi^T
\]  

(10)

and

\[
K(\pi, \pi, S) = \kappa TS = -\kappa \ln \phi \ln(\pi/\pi)S = k \ln(\pi/\pi)S,
\]  

(11)

where \( k = -\kappa/\ln \phi \).\(^8\)

Adopting (11) in (4), then, gross gains from trade are

\[
\int_0^S (\bar{p} - bs)ds - cS - k \ln(\pi/\pi)S = [\bar{p} - cS - k \ln(\pi/\pi)]S - bS^2/2.
\]  

(12)

Expression (12) provides a per-period measure of the gross gains from trade. As noted above, the "period" is the interval over which treatment policy is fixed, or equivalently, the time it will take to determine that an invasion has occurred and that further measures to prevent it are futile. Let \( r \) be the discount rate applied over this period, and hence \( e^{-r} \) is the value of benefits or costs to be received after one period. I normalize the length of this

\(^8\) Note then that \( \partial k/\partial \pi = -\kappa/(\ln \phi) \pi = \kappa/(1-\phi) \), as \( \ln \phi = 1 - \phi \) for small \( \phi \). Compare to (9). Of course we could also derive (11) from the continuous analog to (9) when \( \kappa \) and \( \phi \) are constants by integration and imposition of the boundary condition \( \pi(0) = \pi \).
period to one, and note for use momentarily that the net present value of a perpetuity paying $1 per period and discounted at an exponential rate of \( r \) per period is \( \frac{1}{r} \).

If an invasion occurs, there will no longer be any point in paying to treat shipments to prevent the subsequent reintroduction of an invader that is already established. It will then be optimal to allow shipments until the price of the final shipment is equal to its c. i. f. marginal cost, which occurs when

\[
S = \frac{\bar{p} - c}{b},
\]

in which case the net present value of gains from trade would be

\[
\frac{1}{r} \int_0^{(\bar{p} - c)/b} (\bar{p} - bs)ds - cS = \frac{(\bar{p} - c)^2}{2br}.
\]

Recall also that if the invader arrives it imposes costs of net present value \( D \). Thus, denoting by \( W^0 \) the net present value of a policy that would restrict imports to a level \( S \) and reduce the probability of invasion to \( \pi \), we can write

\[
W^0 = \int_0^S (\bar{p} - bs)ds - cS - k \ln(\bar{p}/\pi)S + e^{-r} \left[ 1 - e^{-ks} \right] \left[ \frac{(\bar{p} - c)^2}{2br} - D \right] + e^{-(r + ks)}W^0,
\]

or solving for \( W^0 \),

\[
- 15 -
\]
\[ W^0 = \frac{\int (\bar{p} - bs) ds - cS - k \ln(\pi/\pi)S + e^{-r}(1 - e^{-\infty})\left[\frac{(\bar{p} - c)^2}{2br} - D\right]}{1 - e^{-(r+\infty)}}. \] (14)

3. Optimal Policy

I now want to describe an optimal policy. This involves the choice of two variables. The first is the volume of shipments to be accepted in any period in which an invasion has not yet occurred, \( S \). The second is the stringency with which shipments should be treated, as measured by the probability of invasion following treatment, \( \pi \).

Each variable might be chosen from a continuum of possible values. However, two constraints may bind. First, if invasion were likely, potential damage catastrophic, and treatment expensive, the optimal intervention would be to prohibit trade; that is, to set \( S = 0 \). We should, then, recognize the constraint

\[ S \geq 0. \] (15)

Second, if potential damages are not catastrophic, and the cost of treatment is high, the optimal intervention would be to \textit{not} treat shipments at all. So, the constraint

\[ \pi \leq \bar{\pi} \] (16)
may sometimes bind: the probability of invasion from any given shipment can be no higher than it is absent any treatment.

To find the conditions characterizing an optimal policy I differentiate (14) first with respect to $\pi$:

$$
\frac{\partial W^0}{\partial \pi} = \frac{kS/\pi - Se^{-(r+\pi)S}}{1 - e^{-(r+\pi)S}} \left[ W^0 + D - \frac{\bar{p} - c}{2br} \right] \geq 0, \tag{17}
$$

with strict equality obtaining when the optimal choice of $\pi$ is less than the upper bound $\pi^*$.

Assuming equality, cancelling common factors and rearranging,

$$
k/\pi^* = e^{-(r+\pi^*)S} \left[ W^0 + D - \frac{\bar{p} - c}{2br} \right], \tag{18}
$$

where the asterisk on $\pi^*$ denotes a value satisfying the first-order condition (17) as an equality.

The expression in square brackets in (18) cannot be negative, as this would imply that the probability of invasion would be negative. Note the condition on value implicit in the expression in square brackets: the expected net present value of following the optimal
treatment and shipment restriction policy is greater than the return that would result once
the invader is established and an unrestricted import strategy is pursued thereafter. Put
simply, the optimal policy must be preferable to being invaded, living with the
consequences, but placing no further restrictions on trade.

Next, differentiating the objective, (14), with respect to the volume of imports per period,
\( S \),

\[
\frac{\partial W^0}{\partial S} = \frac{\bar{p} - bS - c - k \ln(\pi/\pi) + \pi e^{-(r+S)}}{1 - e^{-(r+S)}} \left[ W^0 + D - \frac{(\bar{p} - c)^2}{2b} \right] \leq 0, \tag{19}
\]

with strict equality when a positive volume of shipments is accepted.

Assuming equality, canceling common factors, and rearranging,

\[
S = \frac{\bar{p} - c - k \ln(\pi/\pi) - \pi e^{-(r+S)}}{b} \left[ W^0 + D - \frac{(\bar{p} - c)^2}{2b} \right]. \tag{20}
\]

Note that if both first-order conditions (17) and (19) hold as equalities, then substituting
from (18) into (20),

\[
S^* = \frac{\bar{p} - c - k \ln(\pi/\pi^*) + 1}{b}, \tag{21}
\]
where the asterisk denotes that the solution is one in which the volume of shipments is positive and the likelihood of invasion is reduced *vis a vis* the untreated probability, \( \pi \).

As \( \pi < \bar{\pi} \) by assumption in the derivation of (21), (21) implies that

\[
S^* < \frac{\bar{p} - c - k}{b}. \tag{22}
\]

Alternatively, if constraint (16) binds, \( \pi = \bar{\pi} \), \( \ln(\bar{\pi}/\pi) = \ln(1) = 0 \). Suppose that it is optimal to import a positive quantity, and write (20) as

\[
\bar{S} = \frac{p - c - \pi e^{-(r + \bar{\pi})}}{b} \left[ W^0 + D - \frac{(\bar{p} - c)^2}{2br} \right]. \tag{23}
\]

Where \( \bar{S} \) denotes the optimal quantity of imports when the probability constraint binds. Under the assumption that expression (17) is an inequality when the constraint that the probability of invasion per shipment is no greater than \( \bar{\pi} \) binds,

\[
\bar{S} > \frac{\bar{p} - c + k}{b}. \tag{24}
\]

Combining expressions (22) and (24), we have
\[
S > \frac{\bar{p} - c + k}{b} > S^*.
\]  

(25)

When the probability constraint, (16), binds the optimal number of shipments to accept is always greater than it would be if the probability constraint does not bind.

4. Multiple Solutions and Comparative Analyses

Any candidate solution for the optimal policy choice will depend on the parameters of demand, cost, and treatment expense, \(\bar{p}, b, c, \) and \(k\), respectively. They will also depend on the values assumed for probability of invasion in the absence of treatment, \(\pi\), and the damage anticipated in the event of invasion, \(D\). I will focus here on the latter.

While it is difficult to establish analytically that there can be a value of anticipated damages, \(D\), for which an interior and a constrained optimum yield equal values of expected welfare, it is easy to find numerical examples illustrating the possibility (see Figure 1). Moreover, it can be shown that such a point occurs for at most a single value of \(D\). For lower damages than this critical value the optimal outcome is one in which shipments are not treated and invasives are treated as a "lost cause". For higher values, the optimal outcome involves aggressive treatments of shipments. Note that it may be optimal to essentially “do nothing” about invasions even if the damage they are almost sure to
cause is substantial. In Figure 1 expected damage up to about 60% of the gross gains from unrestricted trade would optimally be met with no response.\(^9\)

Some interesting characterizations of the two candidate optima arise from consideration of the second-order conditions for the optimization of the welfare objective, (14), with respect to \(\pi\) and \(S\). The Hessian matrix corresponding to the first-order conditions (17) and (19), when they hold as equalities, is

\[
\frac{\partial^2 W^0}{\partial \pi^2} = \frac{1}{1 - e^{-(r + S)\pi}} \begin{pmatrix} kS(\pi S - 1)/\pi^2 & kS \\ kS & k\pi - b \end{pmatrix}
\]  

(see Appendix for derivation).

A sufficient condition for (17) and (19) to characterize a local optimum, when they hold as equalities, are that the diagonal elements of (26) be negative and its determinant be positive. That is, that

\[
\pi S \leq 1; \quad (27)
\]

\[
k\pi \leq b; \quad (28)
\]

\(^9\) This is not exactly right. Because the “marginal shipment” contributes some to expected damage, it would be “optimal” to impose some very minor penalty on it, but this amount would be so small that one has to wonder if it would be worth the administrative costs of its imposition. Figure 1 is generated presuming shipments are restricted to integer amounts. In the absence of invasion concerns it would be optimal to import 50 shipments, and with invasion concerns as depicted it is still better to import 50 than 49 shipments.
and

\[ 1 - \pi S \geq \frac{k\pi}{b}. \]  

(29)

Condition (27) means that if there is an interior solution in which shipments are treated to reduce the likelihood of invasion, the policy must be effective in a well-defined sense. As \( e^{-\pi S} \) is the probability with which an invasion does not occur in any given period when the optimal policy is followed, expression (27) implies that the probability that an invasion results from any of the \( S \) shipments received in a given time period must be less than \( 1 - 1/e \approx 63\% \). Of course, we might well expect that the probability of an invasion under an optimal policy would be considerably less than 63\%. At the very least, though, expression (27) shows that “band-aids” or “empty gestures” that impose some restrictions without providing much assurance of effectiveness cannot be optimal.

I will assume that expression (28) is satisfied unless the corner solution of prohibiting trade, \( S = 0 \), is optimal. This is reasonable; expanding trade ad infinitum is not plausible. A sufficient condition for (28) to hold is that \( \bar{\pi} \leq b/k \), since the actual value of \( \pi \) can be no greater than the untreated probability of invasion, \( \bar{\pi} \). As we will see below, expression (29) rules out a situation in which cutting the probability of invasion would make importing larger quantities more attractive. Heuristically, the cost parameter, \( k \), must be large enough that shipments decline in the intensity of treatment.
The practical implication of assuming that (28) is satisfied is that, so long as the threat is not so grave as to motivate curtailing trade entirely, there will always be a unique interior solution with respect to the choice of the volume of imports to accept, whether or not steps are taken to treat shipments to reduce the chance of invasion. Thus, if we identify two interior extreme points of the optimization problem, one constrained by the requirement that $\pi \leq \overline{\pi}$ and the other not, they must be connected by a path that traverses a saddle point.

At such a saddle point the second-order conditions for optimization are, of course, not satisfied. In particular, at a saddle point characterized by policies $(\pi^p, S^p)$, inequality (27) is reversed:

$$\pi^p S^p > 1. \quad (30)$$

Since I presume that condition (28) is always satisfied, this implies that condition (29) is not satisfied at the saddle point:10

$$1 - \pi^p S^p - \frac{k\pi^p}{b} < 0. \quad (31)$$

Now by totally differentiating the first-order conditions (17) and (19) when they hold as equalities, it is possible to derive the following comparative static results (see Appendix for details):

\[ 10 \] I have made what seems to be the innocuous assumption that condition (29) is satisfied when (27) and (28) are.
\[
\frac{d\pi}{dD} = \frac{1}{k\pi b + \pi s - 1} \frac{\pi^2 e^{-(r + s\pi)}(1 - e^{-r})}{k(1 - e^{-(r + s\pi)})}.
\]

(32)

and

\[
\frac{dS}{dD} = \frac{k/\pi b}{k\pi b + \pi s - 1} \frac{\pi^2 e^{-(r + s\pi)}(1 - e^{-r})}{k(1 - e^{-(r + s\pi)})}.
\]

(33)

As the second fraction in expressions (32) and (33) must be positive, both \( \pi \) and \( S \) must be decreasing in the level of expected damages, \( D \), when evaluated at an interior optimum, and increasing in expected damages when evaluated at an interior saddlepoint.

Heuristically, and not surprisingly, the optimal strategy for dealing with the likelihood of invasion becomes more conservative if the expected damage from invasion increase.

Similarly, the potential interior and constrained optima grow "farther apart" in the sense that the saddle point that must separate them moves farther from the interior optimum when potential damages increase.

These points are underscored by the following calculation. Suppose that there is some level of anticipated damages from invasion \( D \) at which expected welfare is exactly the same at the interior optimum \( (\pi^*, S^*) \) as at the constrained optimum \( (\bar{\pi}, \bar{S}) \). Then totally differentiating expression (14) with respect to expected damages, \( D \), we have
\[
\frac{dW^0}{dD} = \frac{\partial W^0}{\partial \pi} \frac{d\pi}{dD} + \frac{\partial W^0}{\partial S} \frac{dS}{dD} - \frac{e^{-\tau}(1 - e^{-\pi S})}{1 - e^{-(\tau + \pi S)}} = -\frac{e^{-\tau}(1 - e^{-\pi S})}{1 - e^{-(\tau + \pi S)}} < 0. \quad (34)
\]

The first term on the right-hand side of the first equality is zero, either because welfare is optimized with respect to \( \pi \) in the unconstrained case or because \( \pi \) does not vary with potential damage in the constrained case. The second term is also zero by optimization with respect to the volume of shipments, \( S \). Not surprisingly, then, welfare declines in the damage expected in the event of an invasion whether the global optimum is constrained or unconstrained.

Note, however, that by assumption \( \pi^* < \bar{\pi} \), and from (25) \( S^* < \bar{S} \). Thus \( \pi^* S^* < \pi \bar{S} \), and

\[
\left. \frac{dW^0}{dD} \right|_{(\pi^*, S^*)} > \left. \frac{dW^0}{dD} \right|_{(\pi, \bar{S})}. \quad (35)
\]

Both derivatives are negative, and hence, the content of expression (35) is that the value of the objective when calculated at the constrained optimum is falling faster than it is when calculated at the interior optimum.

It is obvious that when potential damages are negligible the objective function achieves its maximum when \( \pi \) is constrained to be no greater than \( \bar{\pi} \). Heuristically, when \( D \) is close to zero the value of following the optimal policy, \( W^0 \), must be close to the net present value of...
pursuing a *laissez faire* policy, \((\bar{p} - c)^2/2b\). But then we can write the first-order condition with respect to \(\pi\), (17), as

\[
\frac{\partial W^0}{\partial \pi} = \frac{kS/\pi}{1 - e^{-(r+\delta S)}} > 0. \tag{36}
\]

As expression (36) must be positive for *any* value of \(\pi\), we conclude that \(\pi\) must be constrained by \(\bar{\pi}\) for small values of \(D\). So, for small values of \(D\) the global optimum of the welfare objective must be achieved where \(\pi\) is constrained. As \(D\) increases, however, an interior local optimum arises, and for a large enough value of the potential damages, the interior optimum is preferred to the constrained optimum.

5. **Discussion and Policy Implications**

It would be foolish to oversell the results of a simple model and make overly broad claims for their policy implications. Having said this, however, I think it reasonable to say that the model establishes a rebuttable presumption for a surprising assertion: the best policy response to many biological invasion threats is to do nothing.

As this is a radical prescription, it is appropriate to offer several caveats. As has been seen, a *laissez faire* approach would generally not be appropriate if potential damages exceed the maximum gains from trade. Some situations are "no brainers". Quarantine would be an
appropriate response to the outbreak of a dangerous and highly communicable disease, for example. Similarly, the availability of very inexpensive countermeasures argues for their use. For example, preflight inspections are an established routine in virtually all civil, commercial, and military aviation. There would be only minimal expense associated with adding an inspection for brown tree snakes coiled about the landing gear of aircraft embarking to the preflight checklist for flights between Pacific islands.\footnote{Another policy recommendation I might offer on the basis of my model is to look for the "knee of the curve" in considering preventative treatments. If reasonably effective treatments might be undertaken cheaply enough so as not to deter an appreciable volume of imports, there is little reason not to do them.}

Tougher questions arise when dangers are less clear and costs more burdensome. Economists have often appealed to the logic of option theory in confronting such matters. When faced with uncertainty concerning both the extent of damages and the likelihood of their occurrence, as well as the prospect of suffering irreversible damages, it is often prudent to exercise caution (see, e. g., Arrow and Fisher 1974).

While this advice is unexceptionable, there are some countervailing considerations in the case of potential biological invasions. There are many reasons to suppose that damages from biological invasions are not generally as bad as might be suggested by consideration of the more spectacular examples. Moreover, international trade is often distorted by other factors, which beg the question as to whether further impediments would be justified.

There can be little dispute that many invasive species have altered ecological and economic conditions in the environments in which they become established. In many instances, however, at least some positive aspects can be identified from their arrival. The notorious
zebra mussel, for example, is also a prodigious filterer, which results in clearer water in the
areas it invades. *Prosipis juliflora* – the mesquite tree native to Mexico and the
southwestern United States-- has spread over large areas of Africa and South Asia, where it
is regarded as a nuisance by many. However, because it repels grazing animals, thrives on
degraded land, coppices rapidly, and burns so well as to have been dubbed "wooden
anthracite", it has been credited with contributing to the aversion of an emerging fuelwood
crisis in India.

Another consideration in thinking about the damages caused by invasions is that there is an
increasingly blurred distinction between “invasive pathways” and “migratory corridors”.
With increased concern for climate change, many biologists fear that species will be unable
to move from the areas they now inhabit to habitats that will be more amenable to their
needs if the earth warms. How are we to resolve the tradeoff between encouraging the
survival of species that may no longer thrive in their original habitats and preventing the
extinction of others that might be threatened by the arrival of "invaders"?

The last question begs the more general issue of what is most objectionable about invasive
species. It is one thing to say that they cause tangible, measurable damage. It is another to
appeal to the maintenance of the biological *status quo* as an end in itself. The position is
philosophically precarious (Sagoff 2005). If the modern transport of exotic species that
may become established in new environments is objectionable because it is “unnatural”,
how broadly should we apply the principle? Was this also the case when the Europeans
brought new organisms to the New World? When the Polynesians spread across the
Pacific? When Asians ventured across the Bering land bridge? When hominids first spread from Africa to Asia and Europe? One might well respond, at least in the more historically recent cases, that the spread of disease and the extinction of indigenous species were tragic. My point, though, is that when we cannot now point to such specific harms, appealing to what’s “natural” would not appear to provide much useful guidance.

There are also several reasons to believe that the world still engages in too little, as opposed to too much, international trade. It is not clear that imposing more regulation on an already distorted system will enhance its efficiency. It need hardly be noted that international trade is frequently distorted by protectionism. Some have said that biological invasion concerns are occasionally nothing more than fig leaves for what is, actually, rent-seeking behavior by parties more interested in securing industrial advantage (see, e. g., Margolis, et al. 2005, for a model of the political economy of biological invasion control, and Olson 2006 for a review of literature in this vein). It is interesting to note in this context that at least one empirical study finds that some interventions could not be justified 

*even if the invader against which they were directed were certain to arrive* absent the measures adopted (Orden, *et al.*, 2000, as cited by Olson 2006).

Another factor that merits consideration is that the distribution of the gains from trade is not equal. While I have modeled the supply of imported goods as perfectly elastic, I noted when I introduced this assumption that I made it for convenience. Nothing substantive would change if I introduced an upward-sloping supply curve. The implication of an imperfectly elastic supply is that inframarginal producers would accrue a surplus on the
goods they supply. Inasmuch as such producers may be located in the relatively poor developing world, it may not be unreasonable on a social welfare basis to afford them a greater weight in decision-making. As invasive species are environmental policy concerns, it particularly bears mentioning that the emerging majority view among academic economists seems to be that, contrary to the protests of some advocacy groups, trade enhances environmental performance (see, e.g., Bhagwati 2002; Frankel 2004). One of the main mechanisms for this improvement is through wealth effects: richer people care more about the environment, and trade makes people richer.

Perhaps the most compelling argument for questioning the damages now induced by biological invaders can be distilled from the wisdom of the great Yogi Berra. Among his celebrated malapropisms is “That place is so crowded nobody goes there anymore”. Some biologists motivate their increased concern with biological invasions by noting that modern transportation technologies have greatly increased the speed and volume at which invasive organisms can be transferred from one place to another. It is natural to ask, then, how many of the invaders that could survive the journey from one habitat to another have already done so. If they have arrived, the damage is done; if they have not, it may be because either the likelihood of their survival or the extent of the danger they pose has been exaggerated.12

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12 The point is easily made with a simple application of Bayes's Law. Suppose that one believes that the probability of invasion is \( \pi \) with probability \( q \) and zero with probability \( 1 - q \). If \( S \) shipments have been received to date and no invasion has occurred, then the posterior probability assigned to the belief that the probability of invasion is, in fact, \( \pi \), will be \( \frac{(1-\pi)^S}{(1-\pi)^S + (1-q)/q} \). As \( S \) becomes large the posterior probability vanishes for all priors \( q < 1 \).
For all of these reasons, it seems that we can have reasonable doubts that defending against the arrival of biological invaders should be a high priority in a world with myriad other priorities to address, both social and ecological. Further biological research might better inform policy choices, and could well prove the conjecture wrong. Inasmuch as the capacity for sophisticated biological research is also a scarce asset, however, we might ask if it could be better applied to other topics. At the least, this paper shows that further research would not be valuable if its contribution were only to pinpoint more exactly the magnitude of damages too small to motivate aggressive responses.

6. Conclusion

The model in this paper shows that the optimal policy can be essentially to do nothing to prevent biological invasions, even when the dangers such invasions pose may be relatively large. While one might wish we could say that the optimal policy response is to determine how large such damages really are and then respond accordingly, the biology of invasions does not appear to be sufficiently settled to make such determinations. I have suggested that several considerations might lead us to suppose that while the consequences of biological invasions are not well understood, we might still often take a laissez faire attitude toward them.

Such a conclusion begs a common question: how seriously should we take a model of a complex phenomenon built on a number of simplifying assumptions? Inasmuch as any
model must simplify very complex relationships to arrive at useful answers, reliance on some such simplifications is the only alternative to “paralysis by analysis”. I would defend the assumptions made in developing this model, inasmuch the demand functions, probabilities, and cost functions follow more-or-less canonical forms. They may not be accurate, but they are not unreasonable. If these results obtain under these assumptions, they are likely to obtain under others.

The more controversial and important aspects of the problem may involve not so much bioeconomics as political economy, however. The temptation to disguise protectionist sentiments as biological concerns has been noted by several authors. For this reason it seems appropriate to require a fairly high burden of proof before imposing (further) restrictions on international trade. In this respect, it may be a good thing that the model prescribes a discontinuity in policy responses to differing levels of potential damages. Inasmuch as it makes no sense to institute token measures that would reduce trade without appreciably affecting the likelihood of invasion, appeals to do just that might be more easily dismissed.

Perhaps most importantly, the model highlights the need to reach an understanding as to what the real costs of biological invasions are. While things like costs of control, damage to property, and loss of production are important, such costs are often not presented as the major motivation for social concern with invasive species. Rather, concerns such as threats to native species and modifications to invaded ecosystems are more prominent.13 The basic

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13 These considerations are, for example, listed before crop losses and disease as factors motivating concern with invasive species by Mack, et al. (2000; although, to be fair, it may have been that those authors felt the
prescription I offer in this paper is that if costs are high, aggressive measures should be
taken to prevent invasions, whereas if costs are not high – but also not necessarily
negligible – little should be done. Especially if the costs of concern are intangible, it is
important to reach a social consensus as to how important they are before taking expensive
measures to avoid them. The answer to such questions may lie in neither the biological nor
the economic realm, but rather, the political.
Appendix: Derivation of second-order conditions and comparative statics

Second-order conditions

I reproduce the first-order conditions for convenience:

\[
\frac{\partial W^0}{\partial \pi} = \frac{kS/\pi + S e^{-(r+S)} \left[ \frac{(\bar{p}-c)^2}{2br} - (W^0 + D) \right]}{1 - e^{-(r+S)}} \geq 0, \tag{17}
\]

\[
\frac{\partial W^0}{\partial S} = \frac{\bar{p} - bS - c - k \ln(\pi/\bar{p}) + \pi e^{-(r+S)} \left[ \frac{(\bar{p}-c)^2}{2b} - (W^0 + D) \right]}{1 - e^{-(r+S)}} \leq 0. \tag{19}
\]

In what follows I assume that expressions (17) and (19) hold as equalities. To find the Hessian matrix of second derivatives, I differentiate expressions (17) and (19) again with respect to \( \pi \) and \( S \):

\[
\frac{\partial^2 W^0}{\partial \pi^2} = -kS/\pi^2 - Se^{-(r+S)} \frac{\partial W^0}{\partial \pi} - S^2 e^{-(r+S)} \left[ \frac{(\bar{p}-c)^2}{2br} - (W^0 + D) \right] - Se^{-(r+S)} \frac{\partial W^0}{\partial \pi} \cdot \tag{A1}
\]

\[
1 - e^{-(r+S)}
\]
\[
\frac{\partial^2 W_0}{\partial S^2} = \left[-b - \pi^2 e^{-(r+\pi S)} \left[\frac{(\bar{p} - c)^2}{2b} - (W_0 + D)\right] - \pi e^{-(r+\pi S)} \frac{\partial W_0}{\partial S}\right] - \pi e^{-(r+\pi S)} \frac{\partial W_0}{\partial S} \frac{1}{1 - e^{-(r+\pi S)}}
\]

(A2)

\[
\frac{\partial^2 W_0}{\partial S \partial \pi} = \frac{\partial^2 W_0}{\partial \pi \partial S} = \kappa / \pi - S e^{-(r+\pi S)} \frac{\partial W_0}{\partial S} + (1 - \pi S) e^{-(r+\pi S)} \left[\frac{(\bar{p} - c)^2}{2br} - (W_0 + D)\right] - \pi e^{-(r+\pi S)} \frac{\partial W_0}{\partial S} \frac{1}{1 - e^{-(r+\pi S)}}
\]

(A3)

Evaluating these expressions when \(\partial W_0 / \partial \pi = \partial W_0 / \partial S = 0\),

\[
\frac{\partial^2 W_0}{\partial \pi^2} = \frac{-kS / \pi^2 + \pi S^2 e^{-(r+\pi S)}}{1 - e^{-(r+\pi S)}} \left[\frac{(\bar{p} - c)^2}{2br} - (W_0 + D)\right]
\]

(A4)

\[
\frac{\partial^2 W_0}{\partial S^2} = \frac{\pi^2 e^{-(r+\pi S)} \left[W_0 + D - \frac{(\bar{p} - c)^2}{2b}\right]}{1 - e^{-(r+\pi S)}} - b
\]

(A5)

\[
\frac{\partial^2 W_0}{\partial S \partial \pi} = \frac{\partial^2 W_0}{\partial \pi \partial S} = \frac{\kappa / \pi + (1 - \pi S) e^{-(r+\pi S)} \left[\frac{(\bar{p} - c)^2}{2br} - (W_0 + D)\right]}{1 - e^{-(r+\pi S)}}
\]

(A6)

Substituting from (18),
\[
\frac{\partial^2 W^0}{\partial \pi^2} = \frac{kS}{\pi^2} \frac{\pi S - 1}{1 - e^{-(r+S)}} \quad (A7)
\]

\[
\frac{\partial^2 W^0}{\partial S^2} = \frac{k\pi - b}{1 - e^{-(r+S)}} \quad (A8)
\]

and

\[
\frac{\partial^2 W^0}{\partial S \partial \pi} = \frac{\partial^2 W^0}{\partial \pi S} = \frac{kS}{1 - e^{-(r+S)}} \quad (A9)
\]

The Hessian matrix is, then,

\[
\frac{\partial^2 W^0}{\partial \pi^2} = \frac{1}{1 - e^{-(r+S)}} \begin{pmatrix}
  kS\left(\pi S - 1\right)/\pi^2 & kS \\
  kS & k\pi - b
\end{pmatrix},
\]

As reported in expression (25) in the text.

**Comparative statics**

I reproduce the objective function for convenience, as expressions below involve the
derivatives of \( W^0 \) with respect to \( D \) and \( \pi \).
Differentiating the objective totally with respect to $D$, I have

$$\frac{dW^0}{dD} = \frac{\partial W^0}{\partial \pi} \frac{d\pi}{dD} + \frac{\partial W^0}{\partial S} \frac{dS}{dD} - \frac{e^{-r} \left(1 - e^{-\pi S}\right)}{1 - e^{-(r+\pi S)}} = -\frac{e^{-r} \left(1 - e^{-\pi S}\right)}{1 - e^{-(r+\pi S)}}.$$  \hspace{1cm} (A10)

Since either $\frac{\partial W^0}{\partial \pi}$ is zero by optimization or $d\pi/dD$ is zero by constraint, and similarly for $\frac{\partial W^0}{\partial S}$ and $dS/dD$.

Differentiating equation (18) totally with respect to $D$,

$$-k/\pi^2 \frac{d\pi}{dD} = e^{-(r+\pi S)} \left[ \frac{dW^0}{dD} + 1 \right] - \left( \pi \frac{dS}{dD} + S \frac{d\pi}{dD} \right) e^{-(r+\pi S)} \left[ W^0 - \left( \frac{(\overline{\rho} - \overline{c})^2}{2br} - D \right) \right]$$  \hspace{1cm} (A11)

Differentiating equation (21) totally with respect to $D$,

$$\frac{dS}{dD} = \frac{k}{\pi b} \frac{d\pi}{dD}.$$  \hspace{1cm} (A12)

Substituting from (A10), (A12), and (18) in (A11),
\[-k \sqrt[\pi^2] \frac{d\pi}{dD} = e^{-(r+\pi S)} \frac{1 - e^{-r}}{1 - e^{-(r+\pi S)}} - \left( \frac{k}{b} + S \right) \frac{k \, d\pi}{\pi \, dD}, \quad (A13)\]

or

\[
\frac{d\pi}{dD} = \frac{1}{k \pi b + \pi S - 1} \frac{\pi^2 e^{-(r+\pi S)} (1 - e^{-r})}{k (1 - e^{-(r+\pi S)})},
\]

Which is reported as equation (32) in the text. Equation (33) then follows by substituting (A12) in (32).
Figure 1:
An example with interior and constrained optima

Parameter values used to generate Figure 1
(see text for definitions)

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<td>$(\bar{p} - c)/b$ (optimal shipments absent concern over invasion)</td>
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<tr>
<td>$c$</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>$R$</td>
<td>0.10</td>
<td>$(\bar{p} - c)^2/2br$ (Net present value of gains from trade absent concern over invasion)</td>
</tr>
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<td>$\pi$</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
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</tr>
<tr>
<td>$D$</td>
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<td>Optimal policy choices</td>
<td>Unconstrained</td>
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REFERENCES


