APPENDICES C THROUGH F

APPENDIX C
ERROR ANALYSIS

Estimating flow in a pipe or open channel is generally accomplished by measuring two or more variables and relating them with an equation to calculate the flow. The continuity equation relates flow to area and velocity:

$$Q = A \times v \tag{C.1}$$

where,

A: Area
v: Velocity

For a rectangular channel, the cross-sectional area can be calculated as the water depth multiplied by the width of the channel.

$$A = H \times w \tag{C.2}$$

where,

H: Depth
W: Width

Velocity can be directly measured with a mechanical current meter or Doppler technology. Estimating flow in the rectangular channel requires three measured variables; each will have an error associated with it:

$$Q = H \times w \times v \tag{C.3}$$

For depth and width measurements, the accuracy will usually be expressed as absolute error governed by the tolerance of the measuring device (i.e. measured depth $\pm$ X cm). For velocity, the error in measurement will most likely be a relative error expressed as a percent of the measured value (i.e. measured velocity $\pm$ X %). The total error in the calculated flow measurement will include all of the errors associated with the individual measurements as illustrated in the following example:

Equipment tolerances provided by manufacturers generally are based on laboratory data under ideal conditions (e.g. steady state, laminar flow), which may not be representative of installed conditions. A recent USGS study compared several flow monitoring devices designed specifically for stormwater application, and found the error in the observed measurements ranged from 12 to 28 percent.

The actual error is most likely somewhat less than the maximum error and mathematical formulas have been described by Taylor (1997), which describe how error propagates when variables (with associated errors) are combined.

If variables x_i (for I=1 to n) are measurements with small but known uncertainties δx_i and are used to calculate some quantity q, then δx_i cause uncertainty in q as follows.

If q is a function of one variable, $q(x_1)$, then

$$\delta q = \left| \frac{dq}{dx_1} \right| \delta x_1 \qquad \text{(C.4)}$$

If q is the sum and/or difference of x_is then

$$\delta q = \left[\sum_{i=1}^{n} (\delta x_i)^2 \right]^{1/2} \qquad \text{(for independent random errors)} \quad \text{(C.5)}$$

Estimates of δq from Equation C.2 are always less than or equal to:

$$\delta q = \sum \delta x_i$$

where x_i are measured with small uncertainties δx_i.

If q is the product and quotient of x_is then

$$\delta q = \left[\sum_{i=1}^{n} \left(\frac{\delta x_i}{x_i} \right)^2 \right]^{1/2} \qquad \text{(for independent random errors)} \quad \text{(C.6)}$$

Estimates of δq from Equation C.6 are always less than or equal to:

$$\delta q = \sum \frac{\delta x_i}{|x_i|} \qquad \text{(C.7)}$$

This approach can be directly applied to the analysis of error propagation. Examples for applying this method to flow measurement follow.

Relative Error in Flow Versus Relative Error in Head

Errors in flow measurements are most often caused by field conditions that are inconsistent with the conditions under which rating curves for flow devices were calibrated. However, even under ideal conditions, errors in flow measurement can be significant. This section discusses calculations for estimating the theoretical error associated with flow measurement equipment under ideal circumstances. It can be seen that errors, particularly in low flow measurements, can be quite large.

Equations relating the head (H) measured in a primary device to discharge (Q) (i.e., Rating Equations) fall into four general forms:

1) $Q = aH^{d}$
2) $Q = a(H + c)^{d}$
3) $Q = a(bH + c)^{d}$
4) $Q = a + b_1 H + b_2 H^{2} + b_3 H^{3} + \cdots + b_n H^{n}$

The first rating equation is a straight forward application of error propagation for a power function. This equation is

$$\delta Q = Q\left(d \frac{\delta H}{H} \right) \tag{C.8}$$

Flow and head can only be positive values and the power for Rating Equation 1 is always positive (i.e., flow increases proportionally to head, not decreases), thus the absolute value sign is omitted in the above equation. The relative error in flow equals the relative error in head multiplied by the exponent d.

Rating Equations 2, 3, and 4 require an equation relating the error in flow to the derivative of the flow equation and the error in the measured head, which is:

$$\delta Q = \left| \frac{dQ}{dH} \right| \delta H \tag{C.9}$$

Before applying this equation, the derivatives of Rating Equations 2, 3, and 4 are taken with respect to H.

For Rating Equation 2:

$$\frac{dQ}{dH} = ad(H + c)^{d-1} \tag{C.10}$$

For Rating Equation 3:

$$\frac{dQ}{dH} = abd(bH + c)^{d-1} \tag{C.11}$$

For Rating Equation 4:

$$\frac{dQ}{dH} = b_1 + 2b_2 H^{1} + 3b_3 H^{2} + \cdots + nb_n H^{n-1} \tag{C.12}$$

Prior to applying the equation to the derivatives of Rating Equations 2, 3, and 4 the equation is modified by dividing each side of the Equation by the flow (Q). This yields an equation for the relative error in the flow on the left hand side.

$$\frac{\delta Q}{Q} = \left|\frac{dQ}{dH}\right| \frac{\delta H}{Q} \tag{C.13}$$

Substituting flow Rating Equation 2 for Q and the derivative of Rating Equation 2 for dQ/dH into the right hand side of the above equation, yields:

$$\frac{\delta Q}{Q} = ad(H+c)^{d-1} \frac{\delta H}{a(H+c)^{d}} \tag{C.14}$$

which reduces to:

$$\frac{\delta Q}{Q} = \frac{d}{\left(1+\frac{c}{H}\right)} \frac{\delta H}{H} \tag{C.15}$$

Equation C.11 relates the relative error in the flow to the relative error in the head.

A similar analysis for Rating Equation 3 yields:

$$\frac{\delta Q}{Q} = \frac{d}{\left(1+\frac{c}{bH}\right)} \frac{\delta H}{H} \tag{C.16}$$

Determining an equation for the relative error for Rating Equation 4 is more cumbersome, but is calculated the same way:

$$\frac{\delta Q}{Q} = b_1 + 2b_2H^1 + 3b_3H^2 + \cdots + nb_nH^{n-1} \frac{\delta H}{a + b_1H + b_2H^2 + b_3H^3 + \cdots + b_nH^n} \tag{C.17}$$

Rearranging yields:

$$\frac{\delta Q}{Q} = \frac{b_1 + 2b_2H^2 + 3b_3H^3 + \cdots + nb_nH^n}{a + b_1H + b_2H^2 + b_3H^3 + \cdots + b_nH^n} \frac{\delta H}{H} \tag{C.18}$$

Equation C.4, C.11, C.12, and C.14 relate the relative error in flow to the relative error in head for four common equations describing flow through a primary device. While the equations can be unwieldy, it is a relatively simple exercise to enter them into a spreadsheet program to estimate the error in flow based on estimated error in head and other variables. Most primary devices have a relatively simple flow equation that is sufficiently accurate throughout most of the

flow range for the device, which allows for the use of an error equation related to one of the Rating Equations.

The equations relating the relative error in the estimate of flow to the relative error in the measurement of head can also be expressed in terms of absolute errors by multiplying each side of the equations by Q. For example the flow Equation 3 becomes:

$$Q \times \frac{\delta Q}{Q} = \frac{d}{\left(1 + \frac{c}{bH}\right)} \frac{\delta H}{H} \times a(bH + c)^d = abd(bH + c)^{d-1} \delta H \qquad \text{(C.19)}$$

An Example of Error Analysis for a BMP

The following example illustrates how estimates of error propagation can be applied to flow measurements. This example assumes a stormwater BMP has two separate sources of inflow and one outflow. The flow measurement devices and errors are listed in Table 1.

Table C.1: Example of inputs for estimation of errors in flow measurement devices

Station	**Variable**	**Equipment**	**Measured Value or formula**	**Accuracy**
Inlet 1	Width	Tape Measure	3 meters	± 0.025 meters
	Depth	Pressure Transducer	1.2 meters	± 0.007 meters
	Velocity	Doppler	0.071 meters/sec	± 4 %
Inlet 2	Depth	Bubbler	0.12 meters	± 0.001 meters
		0.457 m (1.5') Palmer-Bowlus Flume	Q (L/s) = $1076.4(H + 0.005715)^{1.8977}$	± 3 %
Outlet	Depth	Pressure Transducer	0.70 meters	± 0.007 meters
		45° V notch weir	Q (L/s) = $571.4H^{2.5}$	± 6 %

For Inlet 1, the flow calculation is:

$$Q_{inlet-1} = (3)\, m \times (1.2)\, m \times (0.071)\ \ m/s$$
$$Q_{inlet-1} = 0.2556\, m^3/s$$

The error associated with this measurement can be calculated using the equation for error of products and quotients (i.e., Equation C.6):

Assuming that the errors are independent and randomly distributed, the relative error in q equals:

$$\frac{\delta q}{q} = \sqrt{\left(\frac{\delta w}{w}\right)^2 + \left(\frac{\delta H}{H}\right)^2 + \left(\frac{\delta v}{v}\right)^2} = 0.0413$$

$$\frac{\delta q}{q} = \sqrt{\left(\frac{.025}{3}\right)^2 + \left(\frac{0.007}{1.2}\right)^2 + (0.04)^2}$$

$$\delta q = 0.2556\, m^3/s \times 0.0413 = 0.011\ m^3/s$$

So that:

$$Q_{inlet-1} = 0.2556 \pm 0.011\ m^3/s$$

For the Palmer-Bowlus Flume installed in **Inlet 2**, the equation that describes flow (L/s) as function of water depth is:

$$Q_{inlet-2} = 1076.4 \times (H + 0.005715)^{1.8977}$$

Therefore:

$$Q_{inlet-2} = 1076.4 \times (0.12 + 0.005715)^{1.8977}$$
$$Q_{inlet-2} = 21.032 L/s = 0.0210\, m^3/s$$

The error associated with flow measurement above is proportional to the precision of the transducer used to measure the water depth (i.e., $\pm$ 0.007 meters) and the error intrinsic to the primary device (a relative error of 3%). Rating Equation 1 is used for this case; Equation C.8 can be used to determine the magnitude of relative error in the flow measurement as:

$$\frac{\delta Q}{Q} = \frac{d}{\left(1 + \frac{c}{H}\right)} \frac{\delta H}{H}$$

$$\frac{\delta Q}{Q} = \frac{1.8977}{\left(1 + \frac{0.005715}{0.12\, m}\right)} \frac{0.007\, m}{0.12\, m} = 0.11$$

$$\delta Q = 0.021 m^3/s \times 0.11 = 0.00231\, m^3/s$$

Relative error for the flume itself also has to be included. Since the error is a function of one variable, it can be calculated using Equation C.4:

$$\delta q = \left|\frac{dq}{dx}\right| \delta x = 0.03 \times 0.021\ m^3/s = 0.00063\ m^3/s$$

The total error is therefore the sum of errors associated with the measuring device (Equation C.5).

$$\delta q_{inlet-2(total)} = \sqrt{0.0023^2 + 0.00063^2} = 0.0024\ m^3/s$$

$$Q_{inlet-2} = 0.0210 \pm 0.0024\, m^3/s$$

For the **Outlet weir**, the flow can be calculated using the following equation:

$$Q = 571.4 \times H^{2.5}$$

$$Q = 571.4 \times 0.70^{2.5} = 234.25 L/s = 0.234\ m^3/s$$

This is also a power function (Rating Equation 1) and the error can be calculated similarly to the equation for the flume:

$$\delta Q = |2.5| \frac{0.007}{0.70} 0.234\, m^3/s = 0.059\, m^3/s$$

The error associated with the weir itself is a single variable as was the flume:

$$\delta q = 0.06 \times 0.234 m^3/s = 0.014\, m^3/s$$

The total error is the sum of the errors associated with the measuring device and is calculated as follows:

$$\delta q_{Outlet(total)} = \sqrt{0.059^2 + 0.014^2} = 0.061\, m^3/s$$

$$Q_{outlet} = 0.234 \pm 0.061\, m^3/s$$

Results of this error analysis are provided below in Table C.2.

Table C.2: Summary of examples demonstrating the propagation of errors in flow measurement

	Flow (m^3/sec)	Total Error (m^3/sec)	Total Relative Error (m^3/sec)
Inlet-1	0.255	± 0.011	4%
Inlet-2	0.021	± 0.0024	11%
Outlet	0.234	± 0.061	26%

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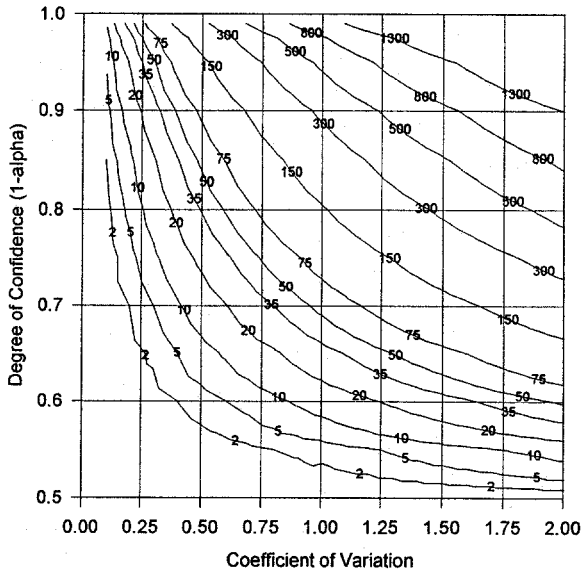
APPENDIX D

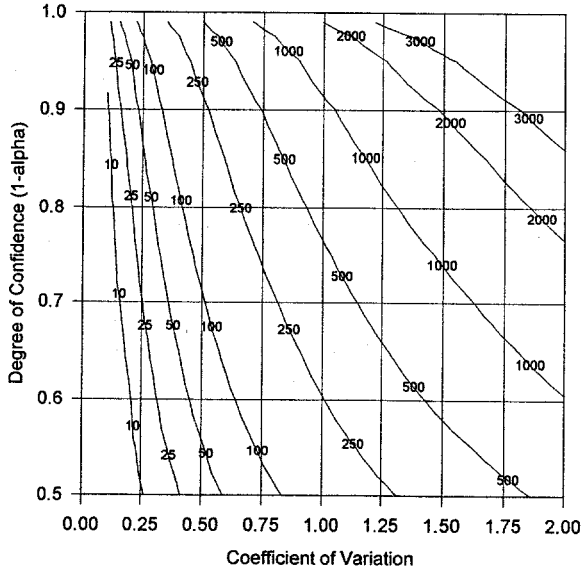
NUMBER OF SAMPLES REQUIRED FOR VARIOUS POWERS, CONFIDENCE INTERVALS, AND PERCENT DIFFERENCES

From R. Pitt and K. Parmer. *Quality Assurance Project Plan (QAPP) for EPA Sponsored Study on Control of Stormwater Toxicants*. Department of Civil and Environmental Engineering, University of Alabama at Birmingham. 1995. Reprinted in Burton, G.A. Jr., and R. Pitt. *Stormwater Effects Handbook: A Tool Box for Watershed Managers, Scientists, and Engineers*. ISBN 0-87371-924-7. CRC Press, Inc., Boca Raton, FL. 2002. 911 pages.

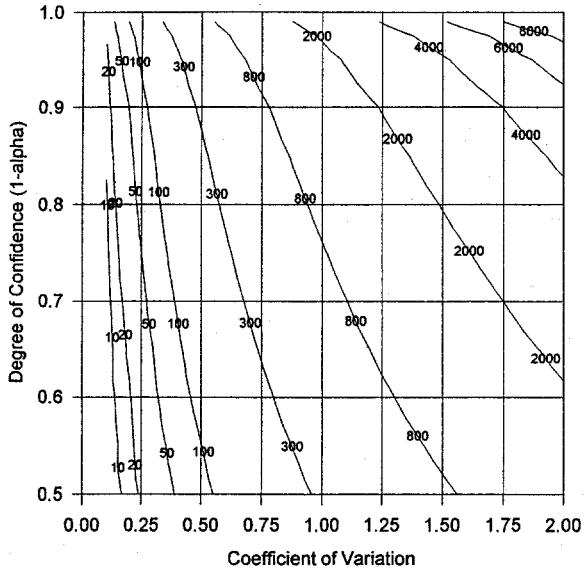
Number of Sample Pairs Needed
(Power = 0.5 Difference = 10%)



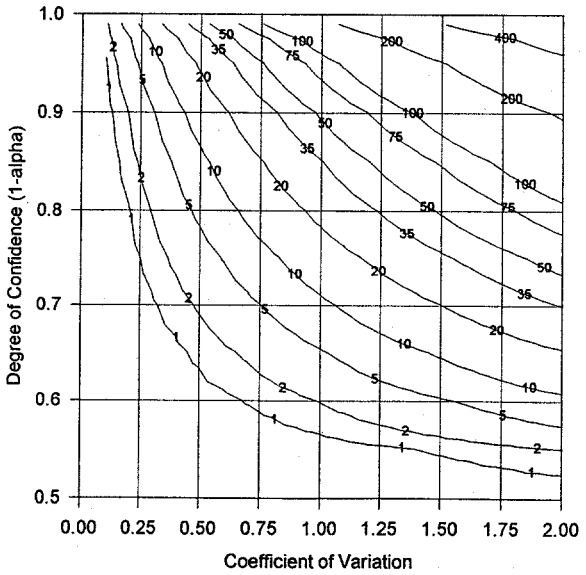
Number of Sample Pairs Needed
(Power = 0.8 Difference = 10%)



Number of Sample Pairs Needed
(Power = 0.9 Difference = 10%)



Number of Sample Pairs Needed
(Power = 0.5 Difference = 25%)



Number of Sample Pairs Needed
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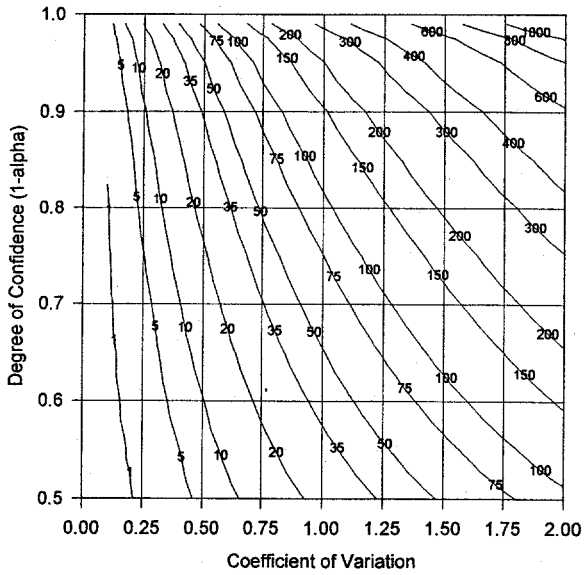
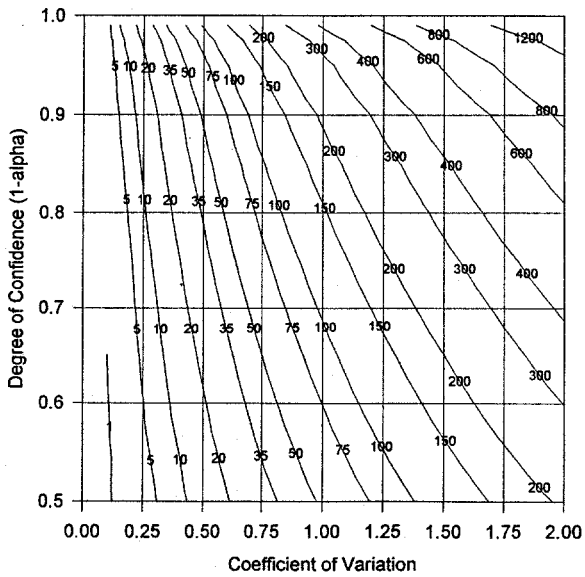
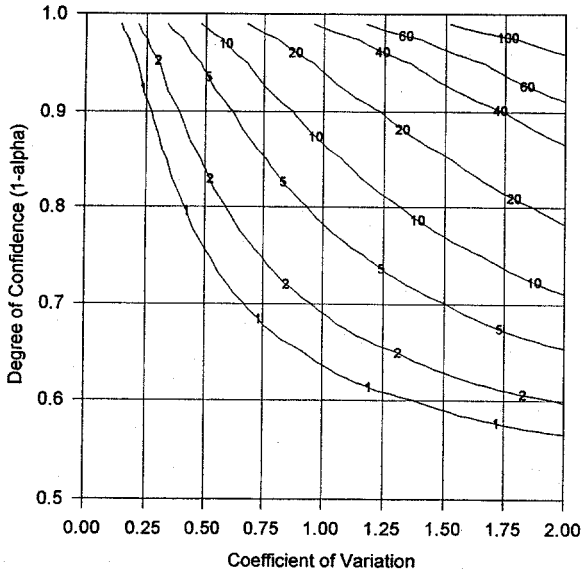


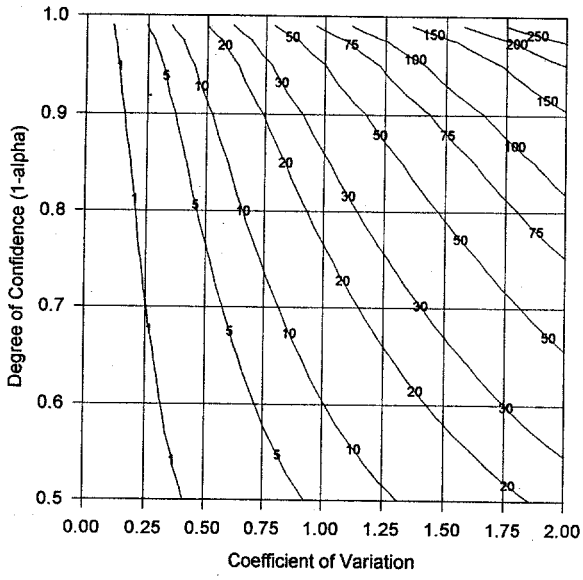
Number of Sample Pairs Needed
(Power = 0.9 Difference = 25%)



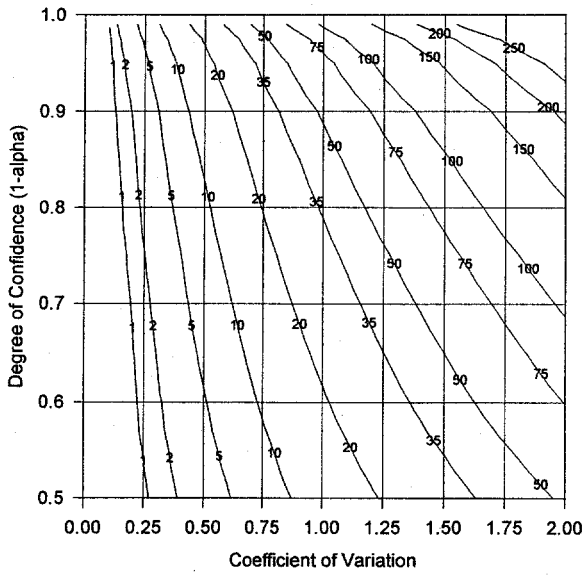
Number of Sample Pairs Needed
(Power = 0.5 Difference = 50%)



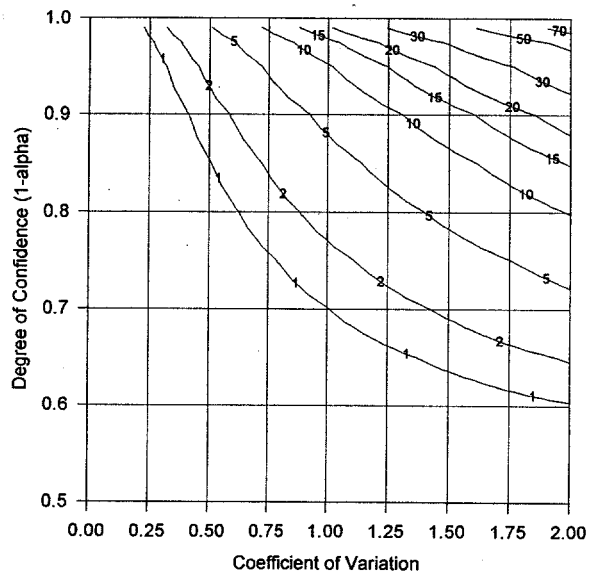
Number of Sample Pairs Needed
(Power = 0.8 Difference = 50%)



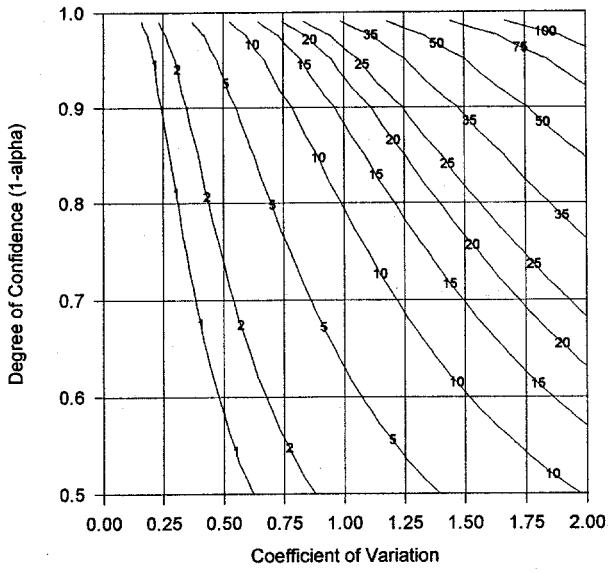
Number of Sample Pairs Needed
(Power = 0.9 Difference = 50%)



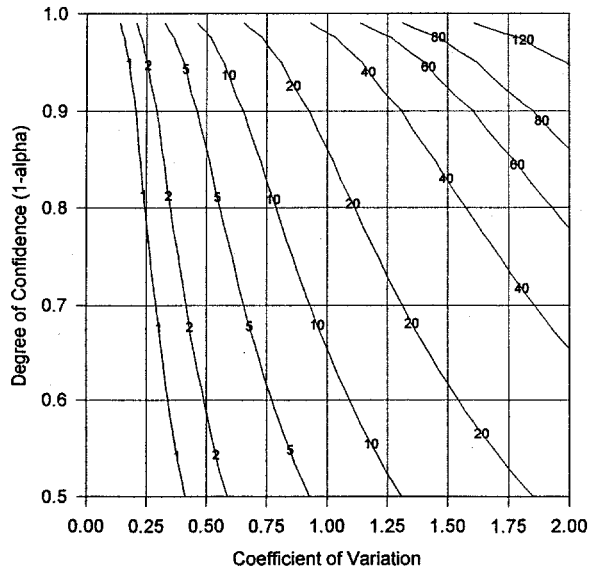
Number of Sample Pairs Needed
(Power = 0.5 Difference = 75%)



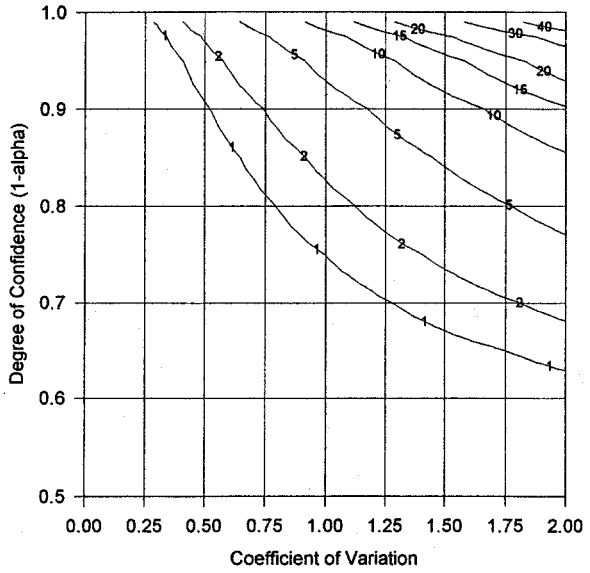
Number of Sample Pairs Needed
(Power = 0.8 Difference = 75%)



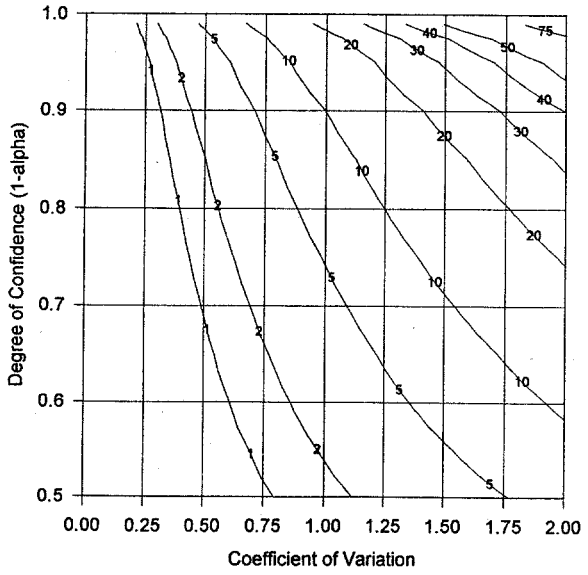
Number of Sample Pairs Needed
(Power = 0.9 Difference = 75%)



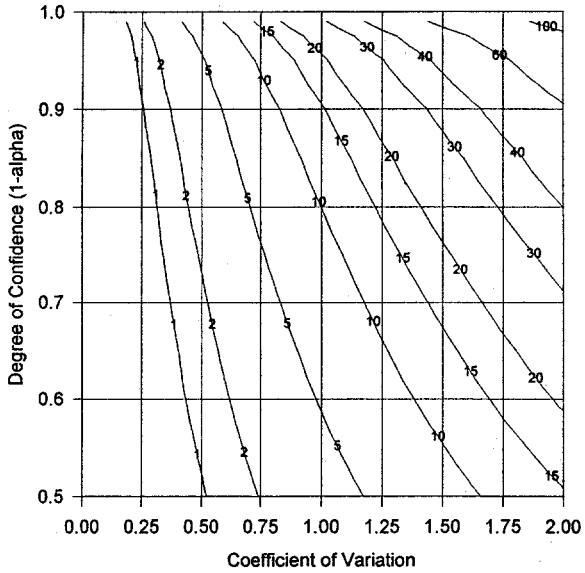
Number of Sample Pairs Needed
(Power = 0.5 Difference = 95%)



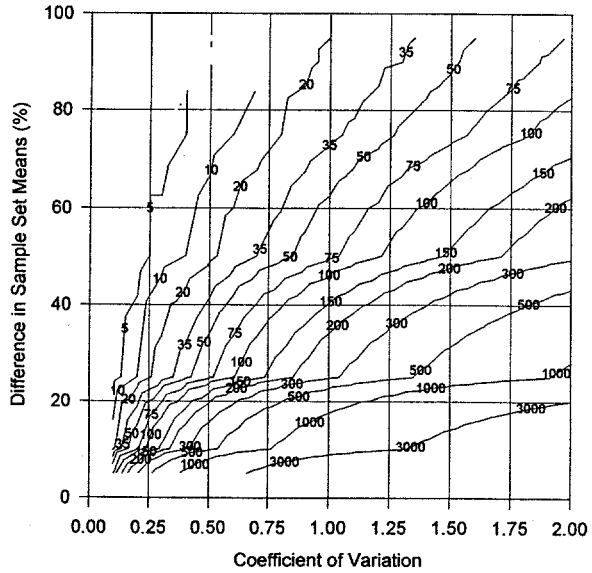
Number of Sample Pairs Needed
(Power = 0.8 Difference = 95%)

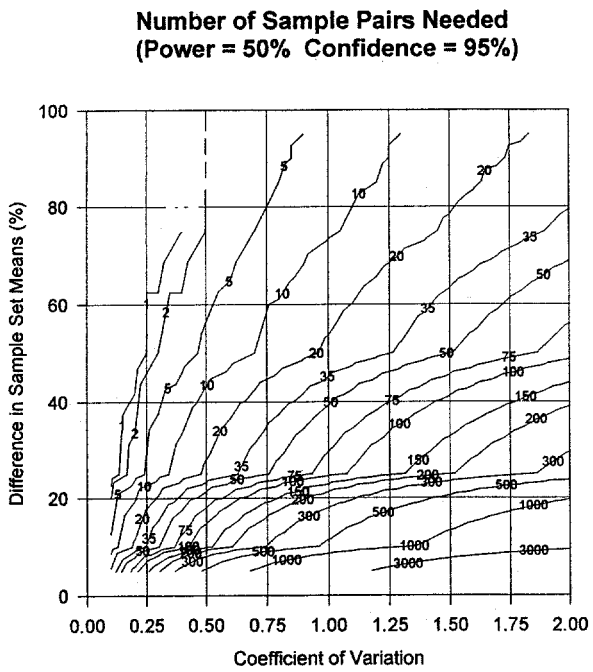


Number of Sample Pairs Needed
(Power = 0.9 Difference = 95%)



Number of Sample Pairs Needed
(Power = 90% Confidence = 95%)




Number of Sample Pairs Needed
(Power = 50% Confidence = 95%)
Difference in Sample Set Means (%)
Coefficient of Variation
0
20
40
60
80
100
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

APPENDIX E

DERIVATION OF THE NUMBER OF SAMPLES REQUIRED TO MEASURE A STATISTICAL DIFFERENCE IN POPULATION MEANS

Define: COV = $\sigma / \overline{C}$

$$\% \text{ removal} = \left(\overline{C}_{in} - \overline{C}_{out}\right) \Big/ \overline{C}_{in}$$

Setting the lower boundary of the influent confidence interval to the upper boundary of the effluent confidence interval gives:

$$\overline{C}_{in} - Z_{\alpha/2} \frac{\sigma_{in}}{\sqrt{n}} = \overline{C}_{out} + Z_{\alpha/2} \frac{\sigma_{out}}{\sqrt{n}}$$

The COV is substituted for the σ in the above equation. While the σ of a BMP effluent is almost certainly less than the σ of the BMP influent, the assumption that $COV_{in} = COV_{out}$ is a more reasonable one. In most instances the COV of the BMP effluent would be less than the influent. Ample data are available for estimating the COV for influent flows to stormwater BMPs, such as the ASCE database; this is not the case for effluent flows. It is also assumed that n is the same for the influent and effluent ($n_{in} = n_{out}$). These assumptions simplify the equation.

Substituting $\sigma_{in} = COV \times \overline{C}_{in}$ and $\sigma_{out} = COV \times \overline{C}_{out}$, where $COV_{in} = COV_{out}$ yield:

$$\overline{C}_{in} - Z_{\alpha/2} \frac{COV \times \overline{C}_{in}}{\sqrt{n}} = \overline{C}_{out} + Z_{\alpha/2} \frac{COV \times \overline{C}_{out}}{\sqrt{n}}$$

rearranging:

$$\overline{C}_{in} - \overline{C}_{out} = COV \times Z_{\alpha/2} \left(\frac{\overline{C}_{in} + \overline{C}_{out}}{\sqrt{n}} \right)$$

Substituting for $\overline{C}_{out} = \overline{C}_{in} - \overline{C}_{in}(\%removal)$ gives:

$$\overline{C}_{in} \times \%removal = COV \times Z_{\alpha/2} \left(\frac{2 \times \overline{C}_{in} - \%removal \times \overline{C}_{in}}{\sqrt{n}} \right)$$

Dividing both sides by $\overline{C}_{in}$ and solving for n yields:

$$n = \left[\frac{Z_{\alpha/2} \times COV \times (2 - \%removal)}{\%removal} \right]^2$$

The above approach considers the number of samples required for a power of 50%. For an arbitrary power the equation becomes:

$$n = \left[\frac{\left(Z_{\alpha/2} + Z_{\beta/2} \right) \times COV \times \left(2 - \%removal \right)}{\%removal} \right]^2$$

where,

$Z_{\beta/2}$: false negative rate (1-β is the power. If used, a value of β of 0.2 is common, but it is frequently ignored, corresponding to a β of 0.5.)

APPENDIX F
RELATIONSHIPS OF LOG-NORMAL DISTRIBUTIONS

Table F.1

T = EXP (U)	S = M * CV
M = EXP (U + 0.5 * W^2)	W = SQRT (LN (1 + CV^2)
M = T * SQRT (1 + CV^2)	U = LN (M/EXP (O.5 * W^2))
CV = SQRT (EXP (W^2) - 1)	U = LN (M/SQRT (1 + CV^2)

Parameter designations are defined as:

	Arithmetic	**Logarithmic**
MEAN	M	U
STD DEVIATION	S	W
COEF OF VARIATION	CV	
MEDIAN	T	

LN(x) designates the base e logarithm of the value x
SQRT(x) designates the square root of the value x
EXP(x) designates e to the power x