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SAGE Model Documentation

For Further Information:

Copies of this documentation, source code for the model, and all publically available data are available at `https://www.epa.gov/environmental-economics/cge-modeling-regulatory-analysis`

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Contents

1 Introduction

SAGE is an applied general equilibrium model of the United States economy developed to aid in the analysis of environmental regulations and policies.[1] It is an intertemporal model with perfect foresight, resolved at the sub-national level. Each of the nine regions in the model, representing the nine census divisions, has five households reflective of national income quintiles. Each region has 23 representative firms, most of which are focused on the manufacturing and energy sectors that are often impacted by environmental policies. Production technologies are represented with nested CES functions, which may include natural resource inputs. Capital for these firms is represented in a partial putty-clay framework to aid in capturing transition dynamics. A single government agent levies taxes on labor earnings, capital earnings, production, and consumption. The United States is treated as a small open economy using the Armington framework governing both domestic and international trade. The baseline is calibrated to the Energy Information Administration's Annual Energy Outlook and the model is solved using the General Algebraic Modeling System (GAMS).

In the following section, technical details on the structure of the model are presented. Section 3 describes the model's calibration. Section 4 discusses the solution algorithm. Section 5 discusses potential options for representing regulations within the model. Section 6 provides a description of how to run the model and describes the verification checks run by the model to test the solution. For a more general description of the model and sensitivity analyses of the model's results we refer the interested reader to Marten et al. (2019).

2 Model Structure

SAGE solves for the set of relative prices that return the economy to equilibrium after the imposition of a policy or other shock, such that all markets clear. This section describes the model's basic structure by first defining the markets in the model, followed by how firms, households, and the government are represented. The section concludes by describing the market clearance conditions that are used to determine equilibrium, where supply equals demand in all markets, along with the closures applied in the model.

2.1 Trade

The United States is represented as a small open economy, with perfectly elastic demand for its international exports and perfectly elastic supply for international imports. Intra-national trade is pooled at the national level. That is, there exists a single market clearing price for commodities traded across regions, independent of the region of origin or destination.[2] This structure for intra-national trade is similar to other CGE models with subnational detail (e.g., Rausch et al. (2011);

[1] We use a recursive naming convention, where SAGE stands for SAGE is an Applied General Equilibrium model.

[2] The pooled approach for national trade is due to a lack of well established state-by-state bilateral trade data by commodity.

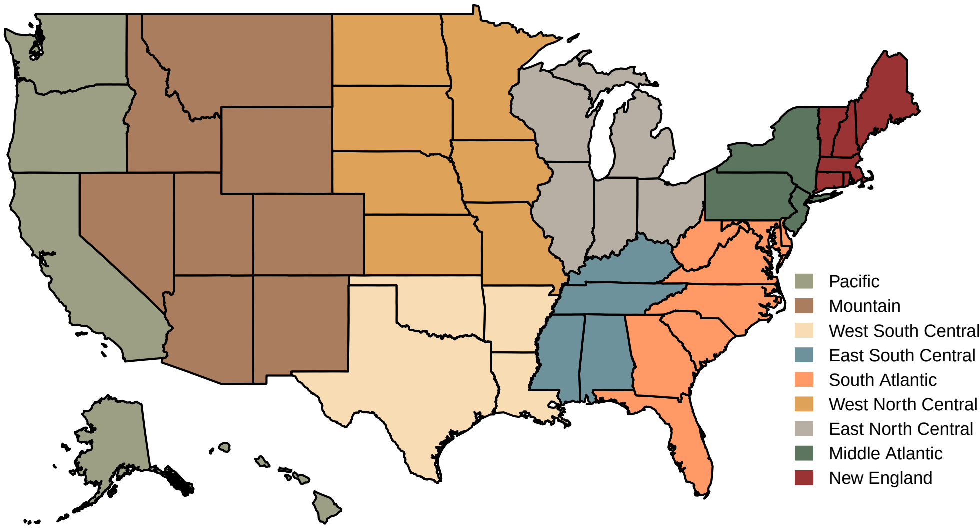


Figure 1: SAGE Regions

Ross (2014)).[3] There are nine subnational regions in the model matching the nine U.S. Census divisions (see Figure 1). Labor and natural resources are not mobile across regions. Capital once installed is not mobile across regions; however, investment is mobile across regions.

Within a region, goods from different origins markets (regional, intra-national imports, international imports) are aggregated using the Armington specification (Armington, 1969). The Armington aggregate is based on first bundling regional output with intra-national imports and then combining that bundle with international imports. A constant elasticity of transformation (CET) function is used to differentiate regional output between different destination markets (regional, intra-national exports, international exports). This structure is presented in Figure 2.

More specifically, the Armington aggregate is defined as

$$a_{t,r,s} = a0_{r,s}\left\{ cs\_nf_{r,s}\left(\frac{m_{t,r,s,ftrd}}{m0_{r,s,ftrd}}\right)^{\frac{se\_nf-1}{se\_nf}} + (1-cs\_nf_{r,s})\left[cs\_dn_{r,s}\left(\frac{m_{t,r,s,dtrd}}{m0_{r,s,dtrd}}\right)^{\frac{se\_dn-1}{se\_dn}} + (1-cs\_dn_{r,s})\left(\frac{d_{t,r,s}}{d0_{r,s}}\right)^{\frac{se\_dn-1}{se\_dn}}\right]^{\frac{(se\_nf-1)se\_dn}{se\_nf(se\_dn-1)}}\right\}^{\frac{se\_nf}{se\_nf-1}}, \tag{1}$$

where $a_{t,r,s}$ is the Armington composite in period t and region r for commodity s, $m_{t,r,s,trd}$ is imports from market trd, $d_{t,r,s}$ is domestic production consumed locally.[4] The national market

[3]However, we note that there are examples where estimates of state-by-state bilateral trade matrices have been applied (e.g., Balistreri and Rutherford (2001); Caron and Rausch (2013))

[4]Throughout this document a 0 trailing a variable name denotes the value in the benchmark year; benchmark cost

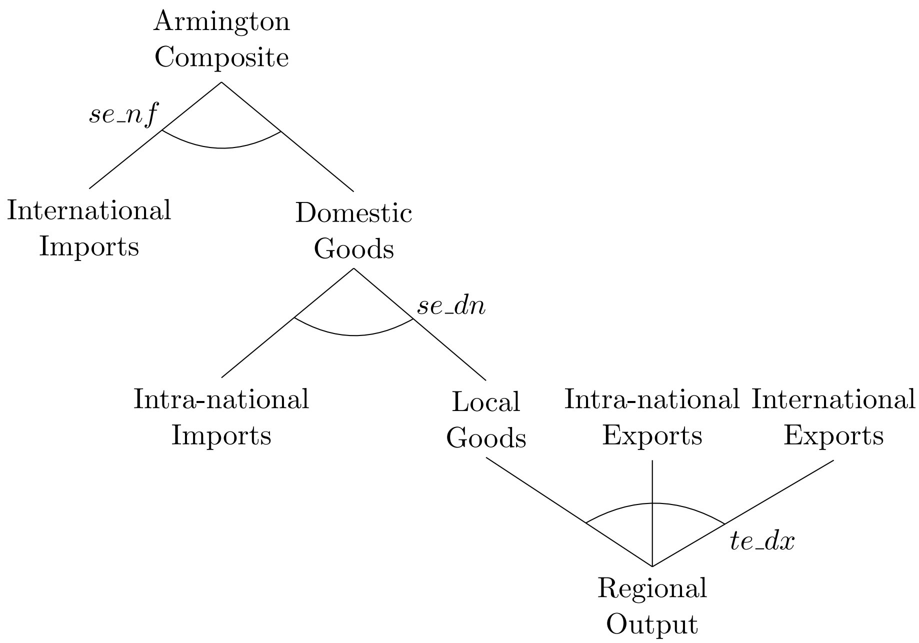


Figure 2: Armington Trade Specification

is denoted $dtrd$ and the international market is denoted $ftrd$. The parameter $cs\_nf_{r,s}$ represents the international imports share of the Armington composite, and $cs\_dn_{r,s}$ represents the share of national imports in the domestic-national composite. The substitution elasticity between international imports and the domestic-national composite is se_nf and the substitution elasticity between domestic production and national imports is se_dn. The inputs into the Armington aggregate are determined based on minimizing the price of the composite good, $pa_{t,r,s}$, given the price in the domestic market, $pd_{t,r,s}$, the price in the national market, $pn_{t,s}$, and the price of foreign exchange, pfx_t.

The CET function to differentiate domestic output across destination markets is defined as

$$
\begin{aligned}
y_{t,r,s} + y\_ex_{t,r,s} = y0_{r,s} \Bigg[& cs\_dx_{r,s,d} \left(\frac{d_{t,r,s}}{d0_{r,s}} \right)^{\frac{te\_dx-1}{te\_dx}} \\
& + cs\_dx_{r,s,dtrd} \left(\frac{x_{t,r,s,dtrd}}{x0_{r,s,dtrd}} \right)^{\frac{te\_dx-1}{te\_dx}} \\
& + cs\_dx_{r,s,ftrd} \left(\frac{x_{t,r,s,ftrd}}{x0_{r,s,ftrd}} \right)^{\frac{te\_dx-1}{te\_dx}} \Bigg]^{\frac{te\_dx}{te\_dx-1}},
\end{aligned} \tag{2}
$$

where $y_{t,r,s}$ is output from production with new capital, $y\_ex_{t,r,s}$ is output from production with extant capital, $x_{t,r,s,trd}$ is exports to market trd, $cs\_dx_{r,s,mkt}$ is the share of output destined for

shares have the prefix cs; and substitution elasticities have the prefix se. In the model, most substitution elasticities vary across sectors as discussed in further detail in Section 3. However, to simplify the exposition, in this document we forgo the sector subscript on substitution elasticities.

market mkt, and te_dx is the transformation elasticity. Within the production possibilities frontier represented by equation (2), firms select the shares of production destined for each market based on maximizing the price of output, $py_{t,r,s}$, given the price of the commodity in the different destination markets.

2.2 Production

Production in the model is aggregated to 23 sectors, with greater detail in manufacturing and energy. The sectors in the model and their associated NAICS codes are presented in Table 1. This default disaggregation represents sectors that have historically been the focus of environmental regulations. This set of sectors also maps nicely into the industrial sectors of the U.S. Energy Information Administration's (EIA) National Energy Model System (NEMS) that are used to inform the baseline calibration.

2.2.1 Manufacturing and Service Sectors

In SAGE, perfectly competitive firms maximize profits subject to market prices and a given production technology. Due to their parsimony and global regularity, nested constant elasticity of substitution (CES) production functions have become widely used in applied general equilibrium modeling (Brockway et al., 2017), and this is particularly true in the case of CGE models used to analyze energy and environmental policies. Similarly, SAGE makes use of nested CES functions (in calibrated share form) to define the production functions for the sectors represented. The policy response of CGE models based on nested CES production functions may be sensitive to the ordering of the nests, as this choice defines separability of the production functions amongst inputs (Lecca et al., 2011). Thus, there has been much discussion about the hierarchy for nested CES production functions, particularly with regards to capital, K, labor, L, and energy, E, inputs. Much of this discussion has been based on heuristics, although the empirical work of Van der Werf (2008) is a notable exception. Van der Werf (2008) studied the fit of different nesting structures given historical production data for 12 OECD countries between 1978 and 1996. Van der Werf (2008) finds that the nesting structure combining K and L in the lower nest and the KL bundle with E in the top nest, denoted $KL(E)$, provides a significantly better fit to the data compared to the other possible nesting structures. Furthermore, Van der Werf (2008) finds that the structure combining K and E in the lower nest provided the worst fit for the data, a finding that has been corroborated in other single country contexts (e.g., Dissou et al. (2015); Ha et al. (2012); Kemfert (1998)). Other multi- and single-country studies have found that the $KE(L)$ nesting structure may fit the data as well as the $KL(E)$ structure at the aggregate national level (e.g., Markandya and Pedroso-Galinato (2007); Su et al. (2012)). However, Kemfert (1998) finds that in cases where the $KE(L)$ nesting structure finds support at the aggregate national level the specification may actually provide a worse fit than the $KL(E)$ structure when disaggregated sectoral production functions are estimated. We use a structure that combines primary factors K and L in a lower nest, where that value-added bundle is then combined with an energy composite. At the top level of the production function the $KL(E)$

Table 1: Model Sectors

Abbreviation	Description	NAICS Codes	NEMS IDM Code
agf	Agriculture, forestry, fishing and hunting	11	1, 2
gas	Natural gas extraction and distribution	211,* 213111,* 213112,* 2212,	4*
cru	Crude oil extraction	211,* 213111,* 213112*	4*
col	Coal mining	2121, 213113	3
min	Metal ore and nonmetallic mineral mining	2122, 2123, 213114, 213115	5
ele	Electric power	2211	NA
wsu	Water, sewage, and waste	2213	NA
con	Construction	23	6
fbm	Food and beverage manufacturing	311, 312	7
wpm	Wood and paper product manufacturing	321, 322	8, 19
ref	Petroleum refineries	32411	NA
chm	Chemical manufacturing	325	9
prm	Plastics and rubber products manufacturing	326	20
cem	Cement	32731	22
pmm	Primary metal manufacturing	331	12, 13
fmm	Fabricated metal product manufacturing	332	14
cpu	Electronics and technology	334, 335	16, 18
tem	Transportation equipment manufacturing	336	17
bom	Balance of manufacturing	3122, 313, 314, 316, 323, 32412, 3271, 3272, 32732, 32733, 32739, 3274, 3279, 333, 337, 339	10, 15, 21, 23
trn	Non-Truck Transportation	481, 482, 483, 485, 486, 4869, 487, 488, 491, 492, 493	NA
ttn	Truck transportation	484	NA
srv	Services	42, 44, 45, 51, 52, 53, 54, 55, 56, 61, 624, 71, 72, 81	NA
hlt	Healthcare services	621, 622	NA

* Crude oil and natural gas extraction is included as a single sector in the benchmark data. However, we disaggregate this activity into separate sectors for crude oil and natural gas. Details are available in Section 3.1.1.

composite is combined with a Leontief composite of material inputs. This structure is similar to other CGE models used to analyze energy and environmental policies (e.g., Paltsev et al. (2005); Rausch et al. (2011); Capros et al. (2013); Cai et al. (2015)).

For the energy composite we also use a nested CES function to represent available production technologies. Initial work using energy-explicit CGE models typically combined all energy sources - including primary energy sources and electricity - in a single nest, commonly with a unit substitution elasticity (e.g., Borges and Goulder (1984)). Subsequent efforts separated electricity from other primary energy sources in a two-nest CES structure that defined the energy composite (e.g., Babiker et al. (1997); Paltsev et al. (2005); Rausch et al. (2011) Böhringer et al. (2018)). The assumption of weak seperability between primary energy inputs and electricity is representative of the primary energy choice across fuels for a sector being defined more by the production process or regional fuel supply characteristics than by the price of electricity. Some recent models have even gone a step further using a three-level CES nest to further disaggregate the primary energy composite in order to impose separability between some of the fossil-fuel use decisions in the cost-minimization problem (e.g., Burniaux and Truong (2002); Chateau et al. (2014); Ross (2014)). However, the three-level CES nesting structure has not been applied consistently across models, and evidence of weak separability in the data is lacking empirically (Serletis et al., 2010a).

SAGE applies the two-level energy nesting with the bottom level nest combining refined petroleum products (or by-products), coal, and natural gas. The second level nest combines the primary energy composite with electricity. This nesting structure is presented in Figure 3. More specifically, the production function for manufacturing goods and services produced with new capital is

$$y_{t,r,s} = y0_{r,s} \left[cs\_klem_{r,s} \left(\frac{mat_{t,r,s}}{mat0_{r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} + (1 - cs\_klem_{r,s}) \left(\frac{kle_{t,r,s}}{kle0_{r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} \right]^{\frac{se\_klem}{se\_klem-1}}, \quad (3)$$

where $mat_{t,r,s}$ is the materials bundle, which is defined as

$$mat_{t,r,s} = mat0_{r,s} \min \left(\frac{id_{t,r,agf,s}}{id0_{r,agf,s}}, \ldots, \frac{id_{t,r,srv,s}}{id0_{r,srv,s}} \right). \quad (4)$$

$id_{t,r,ss,s}$ is the demand for intermediate good ss, and $kle_{t,r,s}$ is the energy and value added composite, which is defined as

$$kle_{t,r,s} = kle0_{r,s} \left[cs\_kle_{r,s} \left(\frac{ene_{t,r,s}}{ene0_{r,s}} \right)^{\frac{se\_kle-1}{se\_kle}} + (1 - cs\_kle_{r,s}) \left(\frac{kl_{t,r,s}}{kl0_{r,s}} \right)^{\frac{se\_kle-1}{se\_kle}} \right]^{\frac{se\_kle}{se\_kle-1}}. \quad (5)$$

$ene_{t,r,s}$ is the electricity and primary energy composite, which is defined as

$$ene_{t,r,s} = ene0_{r,s} \left[cs\_ene_{r,s} \left(\frac{en_{t,r,s}}{en0_{r,s}} \right)^{\frac{se\_ene-1}{se\_ene}} + (1 - cs\_ene_{r,s}) \left(\frac{id_{t,r,ele,s}}{id0_{r,ele,s}} \right)^{\frac{se\_ene-1}{se\_ene}} \right]^{\frac{se\_ene}{se\_ene-1}}. \quad (6)$$

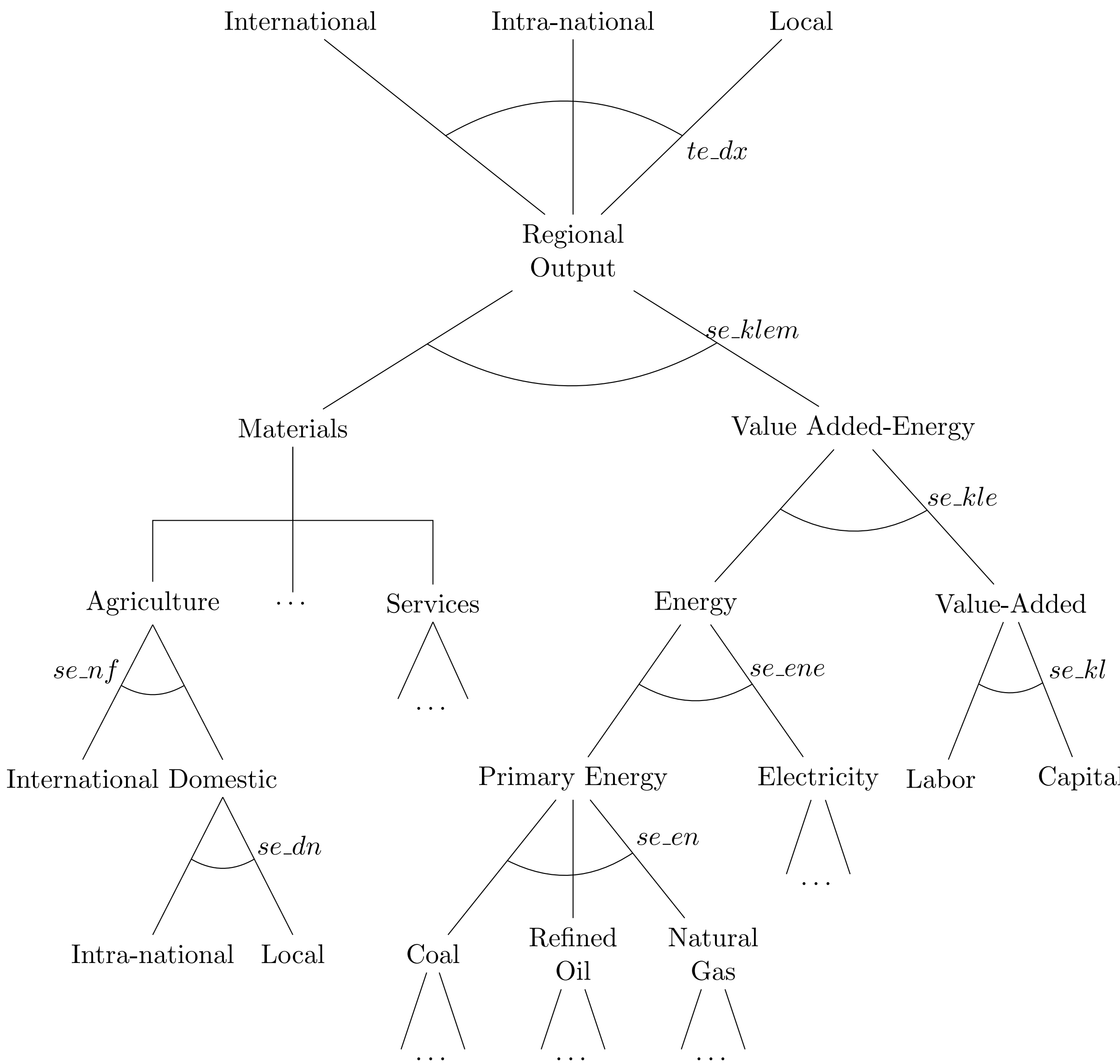


Figure 3: Manufacturing and Services Production Functions

$en_{t,r,s}$ is the primary energy composite, which is defined as

$$en_{t,r,s} = en0_{r,s} \left[cs\_en_{r,col,s} \left(\frac{id_{t,r,col,s}}{id0_{r,col,s}} \right)^{\frac{se\_en-1}{se\_en}} + cs\_en_{r,ref,s} \left(\frac{id_{t,r,ref,s}}{id0_{r,ref,s}} \right)^{\frac{se\_en-1}{se\_en}} \right. \\ \left. + cs\_en_{r,gas,s} \left(\frac{id_{t,r,gas,s}}{id0_{r,gas,s}} \right)^{\frac{se\_en-1}{se\_en}} \right]^{\frac{se\_en}{se\_en-1}}, \tag{7}$$

where $\sum_{ss} cs\_en_{r,ss,s} = 1$. Finally, $kl_{t,r,s}$ is the value added composite, which is defined as

$$kl_{t,r,s} = kl0_{r,s} \left[cs\_kl_{r,s} \left(\frac{kd_{t,r,s}}{kd0_{r,s}} \right)^{\frac{se\_kl-1}{se\_kl}} + (1 - cs\_kl_{r,s}) \left(\frac{ld_{t,r,s}}{ld0_{r,s}} \right)^{\frac{se\_kl-1}{se\_kl}} \right]^{\frac{se\_kl}{se\_kl-1}}, \tag{8}$$

where $kd_{t,r,s}$ is demand for new capital, and $ld_{t,r,s}$ is demand for labor. Parameters with the prefix cs are the relevant cost shares in the benchmark year, and parameters with the prefix se are the relevant substitution elasticities.

Markets are assumed to be perfectly competitive, such that firms are price takers. Given market prices, firms seek to maximize profits

$$(1 - ty_{t,r,s})\, py_{t,r,s} y_{t,r,s} - \sum_{ss} pa_{t,r,ss} id_{t,r,ss,s} - (1 + tk_{t,r})\, pr_{t,r} kd_{t,r,s} - pl_{t,r} ld_{t,r,s}, \tag{9}$$

where $py_{t,r,s}$ is the output price based on maximizing returns across destination markets per equation (2), $pa_{t,r,s}$ is the price of the Armington composite, $pr_{t,r}$ is the rental rate for new capital, $pl_{t,r}$ is the wage rate, and $ty_{t,r,s}$, and $tk_{t,r}$ are ad valorem taxes on output and capital income, respectively.[5]

2.2.2 Resource Extraction, Agriculture, and Forestry Sectors

The resource extraction sectors (crude oil, natural gas, coal, and mining) have an additional primary factor input, in this case representing the finite natural resource. In many cases, models have included this resource in a top-level nest with a bundle of non-resource inputs (e.g., Ross (2005); Paltsev et al. (2005); Sue Wing (2006); Rausch et al. (2011); Capros et al. (2013); Ross (2014); Böhringer et al. (2018)). While some models allow for substitution between materials, energy, and value-added in resource extraction sectors (e.g., Sue Wing et al. (2011); Capros et al. (2013)), other models treat energy, labor, and capital as Leontief inputs (e.g., Ross (2014)), although in most cases there is some substitutability allowed between labor and capital (e.g., Ross (2005); Paltsev et al. (2005); Sue Wing (2006); Rausch et al. (2011)). Recent empirical evidence suggests non-zero and statistically significant substitution elasticities between inputs in resource extraction industries (Young (2013); Koesler and Schymura (2015)). Therefore, we maintain the same structure as in

[5] Payroll taxes are included as part of households' tax rate on labor income to capture the limit on Old Age and Survivor's Insurance payments, which causes the marginal ad valorem tax rate to differ across employees based on income.

the standard production nesting albeit with the addition of a fixed resource. The structure of the production functions for the fossil fuel extraction sectors is presented in Figure 4.

We model the agriculture and forestry sectors using a similar production function with land as a fixed factor input. We recognize that there has been an ongoing discussion in the literature related to the degree of flexibility required by a production function to capture the separability, or lack thereof, observed in empirical studies of agricultural sectors (e.g., Higgs and Powell (1990); Zahniser et al. (2012); Simola (2015)). However, the decreasing returns to scale nature of production in the sector, as captured in Figure 4, is common among approaches, independent of the nesting structure applied.

For the resource extraction, agriculture, and forestry sectors the specific form of the production function is

$$y_{t,r,s} = y0_{r,s} \left[cs\_rklem_{r,s} \left(\frac{res_{t,r,s}}{res0_{r,s}} \right)^{\frac{se\_rklem-1}{se\_rklem}} + (1 - cs\_rklem_{r,s}) \left(\frac{klem_{t,r,s}}{klem0_{r,s}} \right)^{\frac{se\_rklem-1}{se\_rklem}} \right]^{\frac{se\_rklem}{se\_rklem-1}}, \tag{10}$$

where

$$klem_{t,r,s} = klem0_{r,s} \left[cs\_klem_{r,s} \left(\frac{mat_{t,r,s}}{mat0_{r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} + (1 - cs\_klem_{r,s}) \left(\frac{kle_{t,r,s}}{kle0_{r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} \right]^{\frac{se\_klem}{se\_klem-1}}, \tag{11}$$

and $mat_{t,r,s}$ and $kle_{t,r,s}$ are defined in (4)-(8). The fixed factors, $res_{t,r,s}$, are sector specific and in the baseline fixed at the benchmark level, $res_{t,r,s} = res0_{r,s}\ \forall t$.

The resource extraction, agriculture, and forestry markets are also assumed to be perfectly competitive, such that firms are price takers. Given market prices, firms seek to maximize profits

$$\begin{aligned} (1 - ty_{t,r,s})\, py_{t,r,s} y_{t,r,s} - \sum_{ss} pa_{t,r,ss} id_{t,r,ss,s} - (1 + tk_{t,r})\, pr_{t,r} kd_{t,r,s} - pl_{t,r} ld_{t,r,s} \\ - (1 + tk_{t,r})\, pres_{t,r,s} res_{t,r,s}, \end{aligned} \tag{12}$$

where $pres_{t,r,s}$ is the price of the fixed factor resource. It is assumed that returns to the fixed factor face the same ad valorem tax rate as income from physical capital.

2.3 Partial Putty-Clay Capital

To better represent limitations associated with transitioning existing capital stock between sectors or changing its production process, the model considers two capital vintages: existing stock in the benchmark year and new capital formed after the benchmark year. Production with new capital has the flexibility described in Figure 3 and 4. Production with extant capital has a Leontief production structure, as shown in Figure 5.[6] For a profit maximizing firm this means that output

[6] Given the Leontief structure of the production function with extant capital, the nesting pictured in Figure 5 is unnecessary but is retained to make the figure more readable.

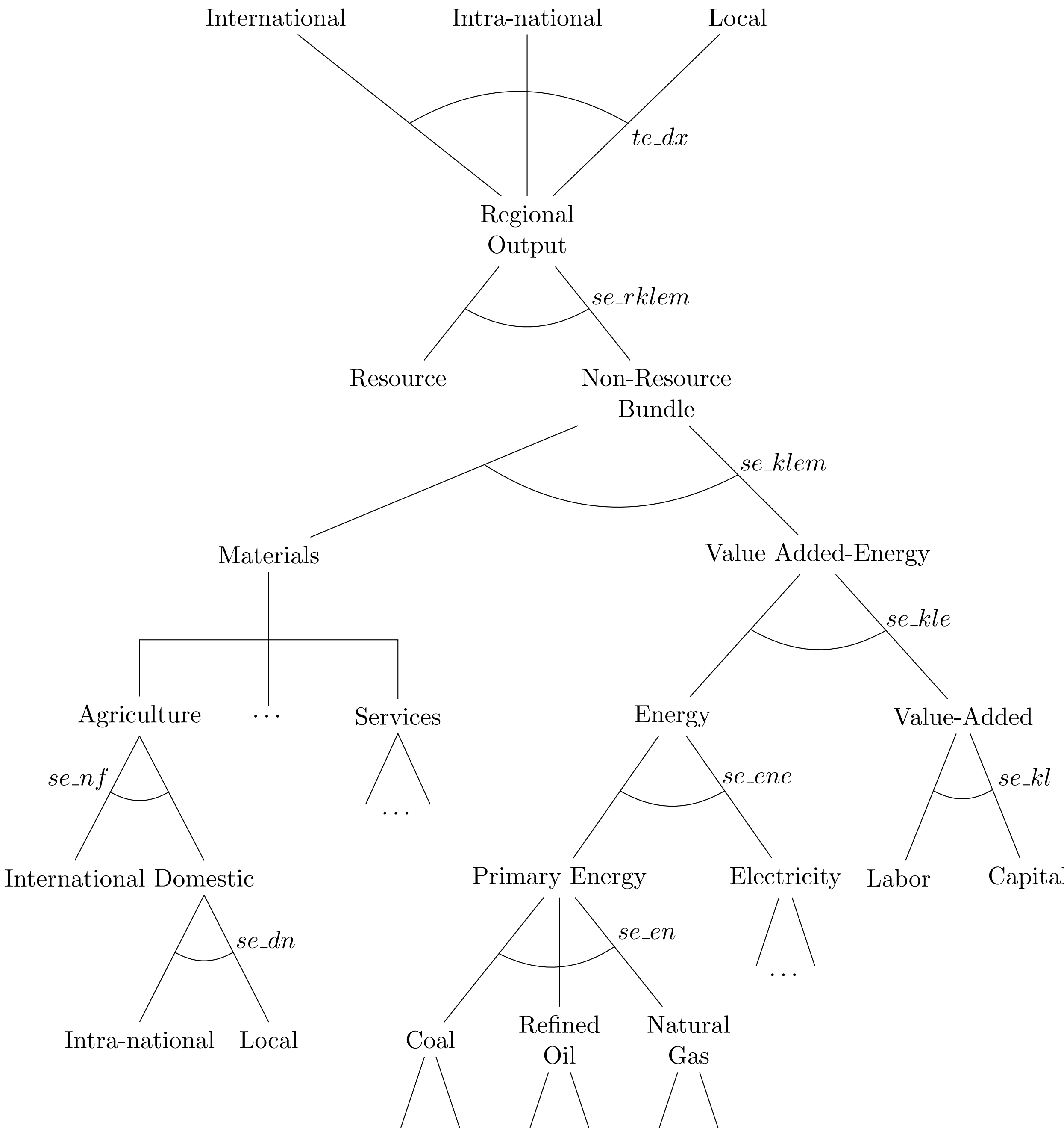


Figure 4: Resource Extraction, Agriculture, and Forestry Production Functions

of commodity s using extant capital is

$$y\_ex_{t,r,s} = y0_{r,s} \frac{kd\_ex_{t,r,s}}{kd0_{r,s}} \tag{13}$$

and demand for intermediate good ss, labor, and fixed factor resources to be used with extant capital will be

$$id\_ex_{t,r,ss,s} = id0_{r,ss,s} \frac{kd\_ex_{t,r,s}}{kd0_{r,s}}, \tag{14}$$

$$ld\_ex_{t,r,s} = ld0_{r,s} \frac{kd\_ex_{t,r,s}}{kd0_{r,s}}, \tag{15}$$

and

$$res\_ex_{t,r,s} = res0_{r,s} \frac{kd\_ex_{t,r,s}}{kd0_{r,s}}. \tag{16}$$

In our partial putty-clay specification, extant capital is primarily sector specific, although it allows a limited potential to shift extant capital across sectors at a cost. This feature is included to match observations that some extant capital (e.g., structures) can be transferred across sectors. To capture this characteristic, sector-specific extant capital, $kd\_ex_{t,r,s}$ is determined by a CET function that transforms a region's extant capital, $k\_ex_{t,r}$, with elasticity te_k_ex. More specifically, given the rental rates for sector-specific extant capital the returns to the stock of extant capital are maximized subject to the production possibilities frontier

$$k\_ex_{t,r} = k0_r \left[\sum_s cs\_kd\_ex_{r,s} \left(\frac{kd\_ex_{t,r,s}}{kd0_{r,s}} \right)^{\frac{te\_k\_ex-1}{te\_k\_ex}} \right]^{\frac{te\_k\_ex}{te\_k\_ex-1}}, \tag{17}$$

where $\sum_s cd\_kd\_ex_{r,s} = 1$.

Capital, regardless of vintage, is assumed to depreciate at rate δ. The law of motion for extant capital reflects this ongoing depreciation, such that

$$k\_ex_{t+1,r} = (1-\delta) k\_ex_{t,r}, \tag{18}$$

where $k\_ex_{0,r} = k0_r$. The law of motion for the new capital stock reflects both depreciation and new investment, such that

$$k_{t+1,r} = (1-\delta) k_{t,r} + inv_{t,r}, \tag{19}$$

where $inv_{t,r}$ is investment in region r in year t and $k_{0,r} = 0$. Formation of physical capital is assumed to be a Leontief process such that

$$inv_{t,r} = inv0_r \min_s \left(\frac{i_{t,r,s}}{i0_{r,s}} \right), \tag{20}$$

where $i_{t,r,s}$ is investment demand for commodity s.

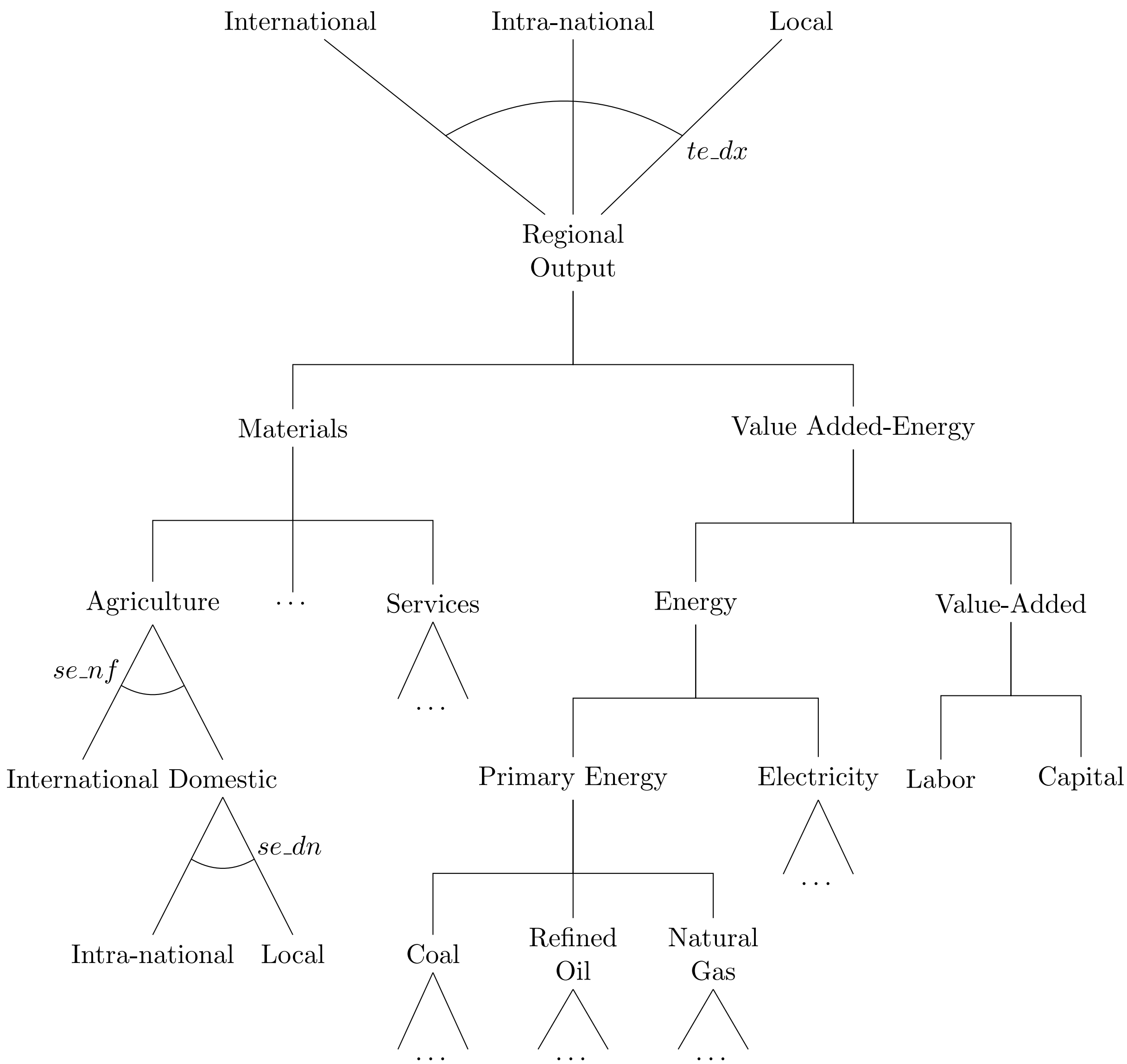


Figure 5: Manufacturing and Services Production Functions with Extant Capital

Table 3: SAGE Representative Households

Household	Benchmark Income
hh1	$\leq$ \$25,000
hh2	\$25,000-\$50,000
hh3	\$50,000-\$75,000
hh4	\$75,000-\$150,000
hh5	$\geq$ \$150,000

2.4 Households

Each region has 5 representative households differentiated by benchmark income. Benchmark incomes for the representative households are presented in Table 3. Based on the underlying economic data in our social accounting matrix, these represent the closest approximation to national income quintiles possible.

Each representative household seeks to maximize intertemporal welfare, which is defined for household h in region r as

$$W_{r,h} = \sum_{t=0}^{\infty} \beta^t n_{t,r,h} u \left(\frac{cl_{t,r,h}}{n_{t,r,h}} \right), \tag{21}$$

where β is the discount factor, $n_{t,r,h}$ are the number of households represented by this agent, $cl_{t,r,h}$ is the consumption-leisure composite, and $u(\cdot)$ is the intra-temporal utility function. The discount rate is defined as

$$\beta = \frac{1}{1+\rho}, \tag{22}$$

where ρ is the pure rate of time preference. Households seek to maximize welfare in (21) subject to a budget constraint

$$\begin{aligned} kh_{t+1,r,h} + pcl_{t,r,h} cl_{t,r,h} = & \left(1+r_t\right) kh_{t,r,h} + \left(1 - tl_{t,r,h} - tfica_{t,r,h}\right) pl_{t,r} te_{t,r,h} \\ & + pr\_ex\_agg_{t,r} kh\_ex_{t,r,h} + \sum_s pres_{t,r,s} rese_{t,r,s,h} \\ & + pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h} \\ & + pl_{t,r} tl\_refund_{t,r,h} \end{aligned}, \tag{23}$$

where $kh_{t,r,h}$ is household savings invested in new capital stock by household h, r_t is the after tax rate of return on that savings, $kh\_ex_{t,r}$ is their stock of extant capital, $rese_{t,r,s,h}$ is their endowment of fixed resource used by sector s, $bopdef_{t,r,h}$ is their share of changes in government or foreign debt, $tran_{t,r,h}$ are net government transfers, $pcl_{t,r,h}$ is the unit cost of full consumption (i.e., consumption and leisure) inclusive of any consumption taxes, $te_{t,r,h}$ is the household's effective time endowment, $pr\_ex\_agg_{t,r}$ is the value of extant capital stock, cpi_t is the consumer price index

$$cpi_t = \frac{\sum_{r,s,h} \left(1 + tc_{t,r}\right) pa_{t,r,s} cd0_{r,s,h}}{\sum_{r,s,h} \left(1 + tc0r\right) cd0_{r,s,h}}, \tag{24}$$

and $cd_{t,r,s,h}$ is demand for commodity s.[7]

There are two taxes collected from households on labor: personal labor income tax and Federal Insurance Contribution Act taxes, with ad valorem rates $tl_{t,r,h}$ and $tfica_{t,r,h}$, respectively. The FICA tax is collected in this manner, as opposed to a payroll tax on the firm side, to allow the effective FICA tax rate to incorporate the limit on the Old-Age, Survivors, and Disability Insurance tax. The last line in equation (23) represents transfers from government to households that are indexed by wages. The role of this transfer is to approximate the observed average income tax rate on labor, while still using marginal tax rates in the model to determine behavior. The parameter $tl\_refund_{t,r,h}$ represents the difference in the tax payment that would be collected by the government on labor income using the marginal tax rate for all labor income versus the average income tax rate. These values are returned to households and indexed by the wage rate, $pl_{t,r}$. In the absence of these factor price indexed transfers, the excess tax collected at the marginal rates would be returned through $tran_{t,r,h}$ and indexed based on the consumer price index, which can affect welfare estimates and incidence in particular.

The intra-temporal utility function is isoelastic, such that

$$u\left(cl_{t,r,h}\right) = \frac{cl_{t,r,h}^{1-\eta}}{1-\eta}, \tag{25}$$

where η represents the inverse of the intertemporal substitution elasticity of full consumption. Intra-temporal household preferences are defined by a nested CES utility function as presented in Figure 6.[8] Consumption of energy and non-energy goods are assumed to be weakly separable. The nesting structure within the composite energy good is similar to that used on the production side. The aggregate consumption bundle is then combined with leisure in the top-level nest of the utility function. More information about the inclusion of leisure and calibration of the substitution elasticity between consumption and leisure is presented in Section 3.3.4.

More specifically, intra-temporal household preferences over full consumption are defined as

$$cl_{t,r,h} = cl0_{r,h}\left[cs\_cl_{r,h}\left(\frac{c_{t,r,h}}{c0_{r,h}}\right)^{\frac{se\_cl-1}{se\_cl}} + \left(1-cs\_cl_{r,h}\right)\left(\frac{leis_{t,r,h}}{leis0_{r,h}}\right)^{\frac{se\_cl-1}{se\_cl}}\right]^{\frac{se\_cl}{se\_cl-1}}, \tag{26}$$

where $leis_{t,r,h}$ is leisure and $c_{t,r,h}$ is the final goods consumption composite. We define

$$c_{t,r,h} = c0_{r,h}\left[cs\_c_{r,h}\left(\frac{cm_{t,r,h}}{cm0_{r,h}}\right)^{\frac{se\_c-1}{se\_c}} + \left(1-cs\_c_{r,s}\right)\left(\frac{cene_{t,r,h}}{cene0_{r,h}}\right)^{\frac{se\_c-1}{se\_c}}\right]^{\frac{se\_c}{se\_c-1}}. \tag{27}$$

[7] To make the discussion easier to follow, $1 + r_t$ has been included in descriptions of the household's budget constraint in this documentation to explicitly denote the returns to household savings, such as in equation (23). However, in the model code the budget constraint is represented without this factor. The model solves for prices relative to the numeraire, which is defined in the initial period of the model. Therefore, all future prices are in present value terms and the returns to savings are implicitly defined based on the decline in the relative price of full consumption over time.

[8] Coal is included in Figure 6 and other discussions related to household consumption for completeness. However, there is no household consumption of coal represented in the model.

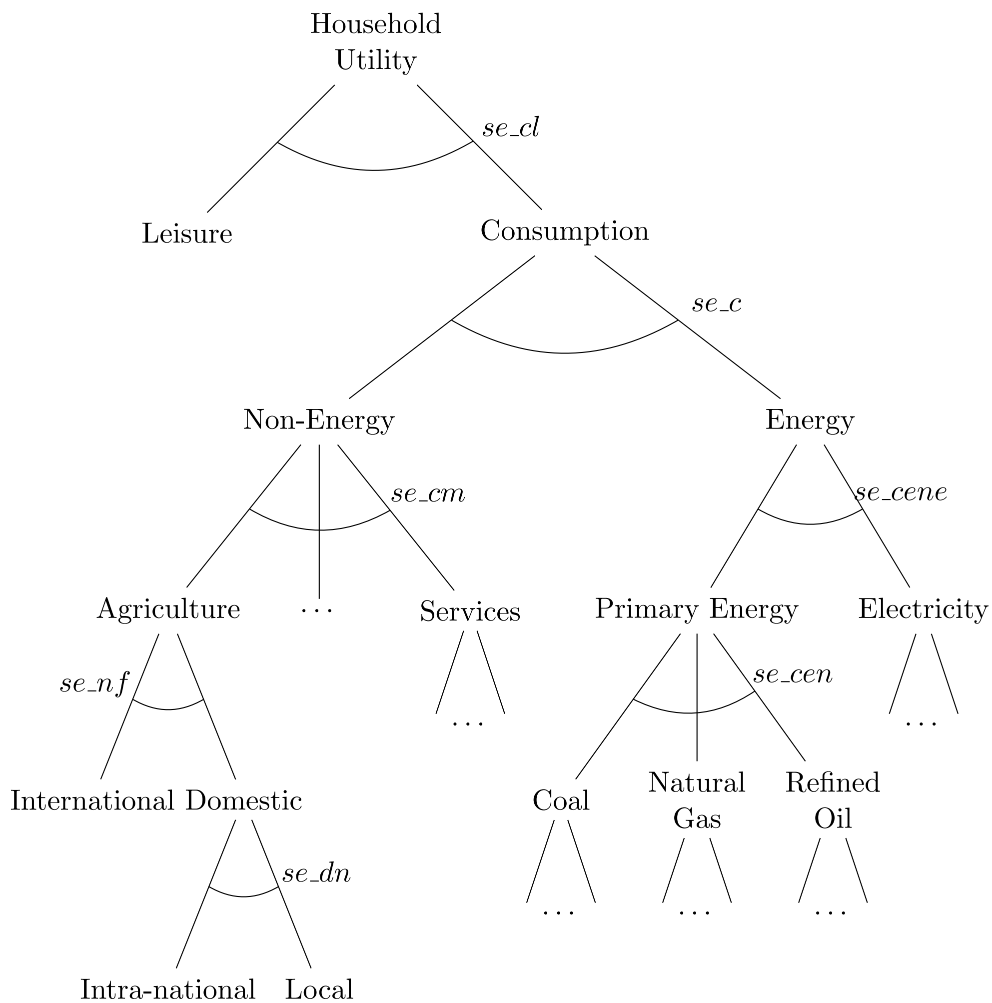


Figure 6: Household Consumption

$cm_{t,r,s}$ is the non-energy composite

$$cm_{t,r,h} = cm0_{r,h} \left[\sum_{s \in scm} cs\_cm_{r,s,h} \left(\frac{cd_{t,r,s,h}}{cd0_{r,s,h}} \right)^{\frac{se\_cm-1}{se\_cm}} \right]^{\frac{se\_cm}{se\_cm-1}}, \tag{28}$$

where scm is the set of non-energy commodities, $\sum_s cs\_cm_{r,s,h} = 1$, and $cene_{t,r,s}$ is the electricity and primary energy composite, which is defined as

$$cene_{t,r,h} = cene0_{r,h} \left[cs\_cene_{r,h} \left(\frac{cen_{t,r,h}}{cen0_{r,h}} \right)^{\frac{se\_cene-1}{se\_cene}} + (1 - cs\_cene_{r,h}) \left(\frac{cd_{t,r,ele,h}}{cd0_{r,ele,h}} \right)^{\frac{se\_cene-1}{se\_cene}} \right]^{\frac{se\_cene}{se\_cene-1}}. \tag{29}$$

Finally, $cen_{t,r,s}$ is the primary energy composite and is defined as

$$\begin{aligned} cen_{t,r,h} = cen0_{r,h} \Bigg[& cs\_cen_{r,h,col} \left(\frac{cd_{t,r,col,h}}{cd0_{r,col,h}} \right)^{\frac{se\_cen-1}{se\_cen}} + cs\_cen_{r,h,ref} \left(\frac{cd_{t,r,ref,h}}{cd0_{r,ref,h}} \right)^{\frac{se\_cen-1}{se\_cen}} \\ & + cs\_cen_{r,h,gas} \left(\frac{cd_{t,r,gas,h}}{cd0_{r,gas,h}} \right)^{\frac{se\_cen-1}{se\_cen}} \Bigg]^{\frac{se\_cen}{se\_cen-1}}, \end{aligned} \tag{30}$$

where $\sum_s cs\_cen_{r,h,s} = 1$.

Since households are assumed to "purchase" leisure at its opportunity cost (i.e., the wage rate), the household labor supply, $l_{t,r,h}$, will be determined according to the time endowment constraint

$$te_{t,r,h} = leis_{t,r,h} + l_{t,r,h}. \tag{31}$$

The population and the time endowment are assumed to grow at exogenous rates, as is discussed in further detail in Section 3.4.

2.5 Government and Taxes

There is a single national government in the model that imposes ad valorem taxes on capital income, production, wage income, and consumption, $tk_{t,r}$, $ty_{t,r,s}$, $tl_{t,r,h}$, and $tc_{t,r}$, respectively. The taxes are region specific, the production tax is also sector specific, and the labor income tax rates are household specific. While they remain constant over time in the baseline, we allow for the possibility of future changes in tax rates in the policy simulations.

Government purchases in region r are assumed to be Leontief, such that

$$gov_{t,r} = gov0_r \min_s \left(\frac{g_{t,r,s}}{g0_{r,s}} \right), \tag{32}$$

where $g_{t,r,s}$ is public demand for commodity s in region r, and $gov_{t,r}$ is the composite public consumption good. The government is assumed to keep real government expenditures per effective household in a region fixed, such that

$$gov_{t,r} = gov0_r \frac{\sum_h te_{t,r,h}}{\sum_h te0_{r,h}}. \tag{33}$$

The government's budget constraint is

$$\begin{aligned}
\sum_r pgov_{t,r} gov_{t,r} + \quad & \sum_h cpi_t tran_{t,r,h} + pl_{t,r} tl_refund_{t,r,h} \\
= \sum_r \sum_s \Bigg\{ \quad & ty_{t,r,s} py_{t,r,s} \left(y_{t,r,s} + y_ex_{t,r,s} \right) \\
& + tk_{t,r} \left[pr_{t,r} kd_{t,r,s} + pr_ex_{t,r,s} kd_ex_{t,r,s} + pres_{t,r,s} \left(res_{t,r,s} + res_ex_{t,r,s} \right) \right] \Bigg\} \\
+ \sum_r \sum_h & \left[\left(tl_{t,r,h} + tfica_{t,r,h} \right) pl_{t,r} l_{t,r,h} + tc_{t,r} pa_{t,r,s} cd_{t,r,s,h} \right],
\end{aligned} \tag{34}$$

where $pgov_{t,r}$ is the unit cost of government consumption based on (32).

The government's budget is balanced through lump sum transfers $incadj_t$, which are shared out to households based on their share of national consumption in the benchmark dataset. Therefore, net transfers to households are

$$tran_{t,r,h} = tran0_{r,h} \frac{te_{t,r,h}}{te0_{r,h}} + incadj_t \frac{c0_{r,h}}{\sum_{r,h} c0_{r,h}}, \tag{35}$$

such that other real transfer payments per effective capita remain constant in steady-state.

2.6 Market Clearance

Given firm, household, and government behavior, along with the capital dynamics described in the preceding sections, prices in equilibrium are assumed to clear all markets.

The price of the Armington aggregate, $pa_{t,r,s}$, clears the goods market, such that

$$a_{t,r,s} = \sum_{ss} id_{t,r,s,ss} + id\_ex_{t,r,s,ss} + \sum_h cd_{t,r,s,h} + i_{t,r,s} + g_{t,r,s}. \tag{36}$$

The price of domestic output consumed domestically, $pd_{t,r,s}$, clears the domestic market, such that

$$\frac{y\_ex_{t,r,s} + y_{t,r,s}}{y0_{r,s}} \left(\frac{pd_{t,r,s}}{py_{t,r,s}} \right)^{te\_dx} = \frac{d_{t,r,s}}{d0_{r,s}}, \tag{37}$$

where the left hand side defines the optimal share of output supplied to the domestic market based on the output transformation function in (2). The price of labor, $pl_{t,r}$, (i.e., the wage rate) clears the labor market, such that

$$\sum_h l_{t,r,h} = \sum_s ld_{t,r,s} + ld\_ex_{t,r,s}. \tag{38}$$

The rental rate for sector specific extant capital, $pr\_ex_{t,r,s}$, clears the market for extant capital, such that

$$\frac{k\_ex_{t,r}}{k0_r}\left(\frac{pr\_ex_{t,r,s}}{pr\_ex\_agg_{t,r}}\right)^{te\_k\_ex}=\frac{kd\_ex_{t,r,s}}{kd0_{r,s}}, \tag{39}$$

where the left hand side defines the optimal share of extant capital supplied to sector s based on the extant transformation function in (17). The rental rate for new capital, $pr_{t,r}$, clears the market for new capital, such that

$$k_{t,r}=\sum_s kd_{t,r,s}. \tag{40}$$

The price of new capital, $pk_{t,r}$, clears the investment market, such that

$$k_{t-1,r}\left(1-\delta\right)+inv_{t-1,r}=k_{t,r}. \tag{41}$$

The price of foreign exchange, pfx_t, clears the foreign exchange market, such that

$$\sum_{r,s} x_{t,r,s,ftrd}+\sum_{r,h} bopdef_{t,r,h}=\sum_{r,s} m_{t,r,s,ftrd}. \tag{42}$$

The price of commodities on the national market, $pn_{t,s}$, clears the market for national trade, such that

$$\sum_r x_{t,r,s,dtrd}=\sum_r m_{t,r,s,dtrd}. \tag{43}$$

Finally, the rental rate for sector-specific fixed factors, $pres_{t,r,s}$, clears the market for sector-specific fixed factors, such that

$$\sum_h rese_{t,r,s,h}=res_{t,r,s}+res\_ex_{t,r,s}. \tag{44}$$

Given that the CES and CET functions defining much of the model's structure are homothetic, the prices for composite goods (e.g., $py_{t,r,s}$ and $pcl_{t,r,h}$) are defined by their unit cost.

2.7 Closures

This section summarizes the main model closures, which are needed to ensure the model is well specified and that there are enough equations to solve for the endogenous variables in the model. These include the government account, trade accounts, intertemporal no-arbitrage condition, and the terminal condition for the finite time horizon model. While some of these are presented above, they are repeated here to provide a complete accounting in one section.

The government budget constraint in (34) is balanced through lump-sum transfers with households, where the endogenous transfers are distributed according to shares of benchmark consumption per equation (35). The government budget is balanced via lump-sum transfers is to avoid altering the marginal incentives in the model through the speculative choice of which tax(es) to adjust.

The domestic trade account is closed each period with a single national price per commodity, per the market clearance condition in (43). Each region's overall balance of payments (across all commodities) in the domestic trade market is not required to be zero in a given period. Deviations from zero are therefore, indicative of whether investment is flowing in or out of the region.

The foreign trade account, across all commodities, is closed each period by the price of foreign exchange, per the market clearance condition in (42). The balance of payments is exogenously specified and is further described in equation (48).

To ensure that the model does not allow for intertemporal arbitrage opportunities, the following constraint is placed on the price of capital

$$pk_{t,r} = pr_{t,r} + pk_{t+1,t}(1-\delta). \tag{45}$$

In other words, the price of capital in period t must equal the return it receives in period t plus the present value of the depreciated asset in period $t+1$. This is equivalent to an equilibrium price of capital that is equal to the present value of returns it will earn over its lifetime.

To close the finite approximation to the infinite time problem we follow Lau et al. (2002). The capital stock in the post-terminal period, kt_r, is introduced as an endogenous variable with associated price pkt_r. The post-terminal capital stock is determined by requiring that investment is growing at the rate of aggregate consumption growth, such that

$$\frac{inv_{T,r}}{inv_{T-1,r}} = \frac{\sum_h c_{T,r,h}}{\sum_h c_{T-1,r,h}}, \tag{46}$$

where T is the terminal period. The price of terminal capital stock is determined by requiring the law of motion for capital to hold, such that

$$k_{T,r}\,(1-\delta) + inv_{T,r} = kt_r, \tag{47}$$

where households' share of the post-terminal capital stock is assumed to be equivalent to their benchmark shares of the capital stock.

3 Calibration and Data

There are multiple sets of data and parameters that define the calibration of the model. The benchmark social accounting matrix; the substitution and transformation elasticities in the model's production and utility functions; parameters defining the transformation and depreciation of capital stocks; tax rates; and the parameters defining the baseline projection. This section describes the sources of each of these in turn.

3.1 Benchmark Data

The benchmark data is based on IMPLAN's 2016 database of the U.S. economy aggregated up to the 23 sectors in Table 1 for each of the nine regions in Figure 1, five representative households in

Table 3, and a single government.[9] The data are used to define the benchmark year values and cost shares. In this section we describe transformations and modifications made to the database to conform to the structure of our model. Smaller transformations include:

- Household exports, which are primarily purchases by foreign tourists, are shared out across commodities based on final good consumption shares and transferred from households to sector-specific foreign exports.
- Government production (make and use) is integrated with private sector production.
- Investment demand, $i0_{r,s}$, is determined as the residual that would lead the goods market clearance condition in (36) to hold.
- Balance of payments are shared out to households based on their share of final goods consumption,

$$bopdef0_{r,h} = \left(\sum_{s,rr} m0_{rr,s,ftrd} - x0_{rr,s,ftrd} \right) \frac{\sum_s cd0_{r,s,h}}{\sum_{s,rr,hh} cd0_{rr,s,hh}}. \tag{48}$$

3.1.1 Crude Oil and Natural Gas Extraction Disaggregation

The underlying IMPLAN data does not distinguish between crude oil and natural gas extraction. Therefore, we disaggregate the single IMPLAN oil and gas extraction sector into separate natural gas extraction and crude oil extraction sectors. To determine the natural gas share of consumption/use we assume that crude oil serves as an intermediate input only to the petroleum refining sector and that natural gas is the only intermediate input (between the two) to all other sectors. We make the same assumptions for household and government consumption and investment demand. In the IMPLAN data, some of the intermediate inputs to the petroleum refining sector are natural gas. To determine that share and the natural gas share of production and trade we minimize the sum of squared deviations for those shares from observed values or assumed shares conditional on market clearance conditions and the assumption of weakly positive domestic use of production. The observed or assumed shares we try to match are derived as follows:

1. The observed share of natural gas production by region is defined using EIA data on crude oil and natural gas production by state aggregated up to the regional level. To arrive at a value share we multiply state-level production quantities by EIA data on state-level wellhead prices for crude oil and city gate natural gas prices as a proxy for natural gas wellhead prices (which are not available).
2. The shares of natural gas international imports and exports by region are defined using census data on state-level international imports and exports of crude oil and natural gas aggregated to the regional level.

[9]IMPLAN Group, LLC, 16740 Birkdale Commons Parkway, Suite 206, Huntersville, NC 28078, www.IMPLAN.com

3. A region's intra-national import share of natural gas is assumed to be similar to the region's share of natural gas use relative to the region's total crude oil and natural gas use. A region's intra-national export share of natural gas is assumed to be similar to the share of natural gas production in the region.

4. The observed share of natural gas used as an intermediate input in the refining sector is estimated based on national annual averages of crude oil and natural gas inputs to the sector collected by EIA and converted to values using the Brent and Henry Hub average annual prices as reported by EIA.

3.1.2 Filtering and Balancing Benchmark

To improve the computational performance of the model we filter out small values and rebalance the SAM. We remove any value less than $.5 \times 10^{-4}$ and any intermediate input whose cost share is less than $.5 \times 10^{-4}$.

After filtering small values the SAM is rebalanced by minimizing the squared percent deviation from the original values weighted by the original values. Specifically we solve for new values of intermediate input demand, $id0_{r,ss,s}$, labor demand, $ld0_{r,s}$, capital demand, $kd0_{r,s}$, imports, $m0_{r,s,trd}$, exports, $x0_{r,s,trd}$, household consumption, $cd0_{r,s,h}$, government spending, $g0_{r,s}$, investment, $i0_{r,s}$, capital endowment, labor endowment, household savings, and lump sum government transfers, $tran0_{r,h}$. This optimization is subject to the market clearance conditions in (36), (38), (40), and (43), the budget constraints in (23) and (34), the balance of payment sharing in (48), the zero profit condition

$$(1 - ty_{r,s})y0_{r,s} = \sum_{s} id0_{r,ss,s} + ld0_{r,s} + (1 + tk_r)\, kd0_{r,s}, \tag{49}$$

the requirement that regional investment equals household savings

$$\sum_{s} i0_{r,s} = \sum_{h} kh0_{t+1,r,h} - kh0_{t,r,h}, \tag{50}$$

consistent with the original data set, weakly positive domestic own use

$$y0_{r,s} > \sum_{trd} x0_{r,s,trd}, \tag{51}$$

and where household savings is consistent with steady-state growth. The balancing occurs prior to distinguishing between types of capital: new, extant, and that of fixed factor resources, as covered in the next section. Therefore, the notation is somewhat simpler.

3.1.3 Natural Resources

Capital returns in the benchmark SAM are disaggregated into returns on man-made capital and natural resources. The disaggregation is based on estimates of the returns to natural resources as a share of gross surplus for those sectors. As described in greater detail below, these shares are assumted to be approximately 25% for the oil and natural gas extraction sectors, 40% for the coal mining sector, 40% for the agricultural and forestry sectors, and 40% for other mining sectors.

Through 2009, the U.S. Energy Information Administration (EIA) collected information on the performance of major U.S. energy-producing companies. Based on the most recent survey, they estimated that the total upstream costs (lifting costs plus finding costs) between 2007 and 2009 for crude oil and natural gas companies included in the survey was $33.76 per barrel of oil equivalent (EIA, 2011). EIA reports that the U.S. produced 1.95 billion barrels of crude oil[10] and 3.67 billion barrels of oil equivalent of natural gas[11] in 2009.[12] The U.S. Bureau of Economic Analysis (BEA) estimates that in 2009 the output value for the oil and natural gas extraction sectors was $220 billion with gross operating expenditures of $123 billion.[13] Combined, these estimates suggest that the output value of the sector exceeded the upstream costs by $30 billion, which is 25% of the gross operating surplus.

An alternative approach, to defining the share of gross operating surplus due to rents paid to natural resource ownership, is to consider royalty payments. The United States has widespread private ownership of minerals, including crude oil and natural gas. In 2012 an estimated 77% of onshore crude oil and natural gas production revenue was associated with privately owned minerals for which $22 billion in private royalties were paid (Fitzgerald and Rucker, 2016). In 2012 $8.5 billion in federal royalty payments were collected from onshore and offshore oil and gas production according to the U.S. Department of Interior's Natural Resources Revenue Data.[14] The BEA estimates that in 2012 value added for the crude oil and natural gas extraction sectors, less employee compensation and production taxes, was $157 billion.[15] Private and federal royalties represented approximately 19% of this remaining value added. Brown et al. (2016) found evidence that private royalty rates may not represent full rent on the natural resource, potentially due to monopsony power and long-term contracts. Similarly, government royalty rates may not represent the full rent associated with the nonrenewable resource. Therefore, 19% likely represents a lower bound on the rents associated with crude oil and natural gas resources.

Sue Wing (2001), based on BEA rent estimates from 1994[16] and before the growth in shale production, estimated resource rents to be approximately 45% for crude oil and natural gas production. Technological progress such as horizontal drilling and hydraulic fracturing likely placed downward pressure on the resource rents (e.g., Farzin (1992); Lin and Wagner (2007)). Given the breadth of technical progress in these markets, 45% therefore may be a reasonable upper bound.

The share of gross surplus associated with coal resources is approximated using average ex-

[10] https://www.eia.gov/dnav/pet/PET_CRD_CRPDN_ADC_MBBLPD_A.htm

[11] https://www.eia.gov/dnav/ng/hist/n9070us2A.htm

[12] Natural gas was converted to equivalent barrels of oil at 0.178 barrels per thousand cubic feet following EIA (2011).

[13] https://www.bea.gov/industry/input-output-accounts-data

[14] https://revenuedata.doi.gov/explore/#federal-revenue

[15] https://www.bea.gov/industry/input-output-accounts-data

[16] https://apps.bea.gov/scb/account_articles/national/0494od2/maintext.htm

traction cost estimates from Jordan et al. (2018) along with additional information on operation costs for coal companies. Jordan et al. (2018) estimate average per ton extraction costs for coal by region based on 10-K filings from large publically traded coal companies (Figure 1 from their paper). Based on there estimates of extraction costs and regional coal production levels, the extraction costs for the industry in 2012 were approximately \$37 billion. This value does not include consumption of fixed capital, sales, or general administrative costs. Using the 10-K fillings for the same publicly traded coal companies evaluated in Jordan et al. (2018), these additional costs were on average 20% of extraction operating costs in 2012. BEA estimates that in 2012 the total output value for the coal mining sector was \$52 billion with \$19 billion in gross operating surplus.[17] Estimating the payments to the resource as the difference between the total output value and extraction costs scaled to include other costs yields \$8 billion, which is approximately 40% of gross surplus for the sector. Notably, extraction operating costs may include some royalty payments, which may represent returns to the resource, leading to an underestimated share of gross surplus associated with resource payments. Conversely, the estimates of variable input costs do not include expenditures associated with mine closures, which can be large, leading to an overestimate of the share of gross surplus associated with resource payments. We note that, while based on data from the early 1990s, Sue Wing (2001) similarly estimated resource payments to be 40% of gross surplus in the coal sector.

Remaining mineral and metal mining activity is aggregated into another mining sector (*min*). Approximately two-thirds of the output value from the sector is attributable to stone mining and quarrying (NAICS 21231) or sand and gravel mining (NAICS 21232). Of the remaining third of the sector's output value, copper ore mining (NAICS 212234) accounts for approximately half. Due to a lack of recent data that would facilitate an exercise similar to those conducted for the other mining sectors we assume the share of gross surplus attributable to the resource is 40% following the coal sector.

Disaggregating the returns to agricultural and forestry land as 40% of gross operating surplus is consistent with rental data from the U.S. Department of Agriculture (USDA). The USDA estimates that the rental value in 2016 for cropland and pastureland is \$136 and \$13 per acre, respectively,[18] and that there is approximately 245 million acres of cropland[19] and 528 million acres of pastureland.[20] The BEA estimates gross surplus in 2016 for the agricultural sectors to be \$103 billion. Using the USDA estimates to compute the total rent paid to agricultural land and dividing by the BEA gross surplus estimate, suggests that land rental values are up to 40% of gross surplus for the sector. It is worth noting that the USDA rent per acre estimates may include the returns to some structures, potentially making them an overestimate of the returns to land. Since the agriculture and forestry sectors are combined in the default aggregation of SAGE this assumption

[17] `https://www.bea.gov/industry/input-output-accounts-data`

[18] `https://quickstats.nass.usda.gov/results/ABF12C63-5DDA-3745-A0B3-C91279A860D1`

[19] `https://www.fsa.usda.gov/Assets/USDA-FSA-Public/usdafiles/NewsRoom/eFOIA/crop-acre-data/zips/2016-crop-acre-data/2016_fsa_acres_data_aug2016_dr6.zip`

[20] `https://www.nrcs.usda.gov/wps/portal/nrcs/detail/national/landuse/rangepasture/?cid=nrcsdev11_001074`

Table 4: Tax/Subsidy Rates on Production

	nen	mat	enc	wnc	sat	esc	wsc	mnt	pac
agf	0.03	0.02	0.01	0.01	0.00	0.00	0.01	0.00	0.00
cru	0.00	0.05	0.03	0.19	0.03	0.09	0.12	0.15	0.18
col	0.00	0.03	0.07	0.24	0.08	0.05	0.07	0.10	0.10
min	0.03	0.02	0.02	0.04	0.03	0.02	0.02	0.04	0.02
ele	0.08	0.10	0.09	0.06	0.10	0.05	0.07	0.07	0.09
gas	0.03	0.05	0.05	0.08	0.04	0.05	0.11	0.11	0.13
wsu	-0.07	-0.10	-0.06	-0.02	-0.02	-0.01	0.00	-0.01	-0.03
con	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
fbm	0.02	0.05	0.01	0.01	0.05	0.04	0.01	0.01	0.03
wpm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ref	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
chm	0.02	0.02	0.02	0.01	0.01	0.02	0.01	0.01	0.02
prm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
cem	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
pmm	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
fmm	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
cpu	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
tem	0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00
bom	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.01
trn	0.02	0.03	0.04	0.03	0.04	0.02	0.03	0.05	0.04
ttn	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.01
srv	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.05	0.05
hlt	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

is also implicitly applied to the returns to land for the forestry sector, which accounts for less than 9% of the gross surplus for the combined sector.

3.2 Taxes

As previously noted, the model explicitly includes business taxes/subsidies, $ty_{r,s}$, personal labor income taxes, $tl_{r,h}$, and capital income taxes, tk_r.The taxes are introduced into the dataset prior to aggregation to the model's regions. When aggregating the dataset, taxes are set to keep the tax revenue constant between the disaggregated and aggregated datasets. Production taxes net of any subsidies, $ty_{r,s}$, are based on the average rate observed in the IMPLAN database. The production tax rates are presented in Table 4. Based on the design of the IMPLAN database these values also include sales and excise taxes. A placeholder exists for consumption taxes, tc_r, in the model's code to allow for future development work that may move the sales and excise taxes out of production taxes. In the current version of the model, explicit consumption taxes are set to zero. Therefore, as it currently stands, sales and excise taxes are applied on the supply side of the market as opposed to the demand side and are associated with the sector that submits the tax payment and not necessarily the sector that produces the taxed commodity.

Personal income taxes on labor are differentiated across regions and households. Effective marginal Federal Insurance Contribution Act (FICA) taxes are also differentiated across regions and households. This allows the payroll tax rates to capture the annual limit on Old Age and Survivor's Insurance (OASI) taxes, which would not be possible if the payroll taxes were collected on the firm side due to the model's structure. Data from the U.S. Census Bureau's Current Population Survey (CPS) Annual Social and Economic Supplement (ASEC) is used to create a representative sample of tax returns. These sample returns are then run through NBER's Taxsim model version 27 to estimate marginal tax rates for wage income and FICA for each sample return (Feenberg and Coutts, 1993).[21] For each region and household we compute the weighted average effective marginal tax rate from the sample returns by weighting the Taxsim results by the CPS ASEC earned income and applying the supplement weights.

From the CPS, the filing status variable (filestat) and the dependent status variable (dep_stat) are used to distinguish between single/head of household taxpayers and dependent taxpayers. All married taxpayers are assumed to file jointly, and the person records for each couple are identified using the a_spouse variable. The dep_row variable in the CPS is used to assign non-filing dependents to taxpayers, along with the ages of the dependents. This information is used to populate the Taxsim variables used to assess personal exemptions, the Dependent Care Credit, the Child Credit, and the Earned Income Tax Credit.

The income variables in the CPS ASEC are mapped to the Taxsim variables as described in Table 5. For married couples, all income values entered into Taxsim are the joint earnings, except in the case of wage and salary income, which are kept separate. Dividend income reported in the CPS is split between qualified and ordinary dividends based on the aggregate share of dividends that are qualified, $qual\_frac$, from the IRS individual income tax returns line item totals.[22] The CPS no longer includes imputed capital gains; therefore they are omitted from the submission to Taxsim. This limitation may bias the weighted average effective marginal tax rates downwards for the household representing the top income quintile (where nearly all capital gains accrue) if the inclusion would cause some households to be in a higher tax bracket.

The implicit deductions for each filer are computed as the difference between adjusted gross income (agi) and taxable income (tax_inc) as reported in the CPS minus personal exemption deductions accounting for the phase out. From this value, we subtract property and state taxes. We submit either this value or zero, whichever is higher, to Taxsim as potential sources of itemized deductions. Property taxes in the CPS ASEC (prop_tax) are associated with household records so we divide those taxes equally amongst all tax filing units in a household.

For each representative filer, Taxsim returns the effective marginal tax rate for primary earner wage income. Using the CPS ASEC person weights and primary earner wages, a weighted average of the effective marginal tax rates for wage income are computed for each region and representative household in the model. Primary earner wages are used as the weight because each married couple

[21] `http://users.nber.org/~taxsim/taxsim27/`

[22] `https://www.irs.gov/pub/irs-soi/16inlinecount.pdf`

Table 5: CPS to NBER Taxsim Income Mapping

Taxsim Variable	Description	CPS Variable(s)*
pwage	Wage and salary income of primary taxpayer	ws_val, semp_val, frse_val
swage	Wage and salary income of spouse	ws_val, semp_val, frse_val
dividends	Qualified dividend income	qual_frac×div_val
stcg	Short term capital gains or losses	NA**
ltcg	Long term capital gains or losses	NA**
otherprop	Other property income	rnt_val, (1 − qual_frac) ×div_val
nonprop	Other non-property income	oi_val, ed_val
pensions	Taxable pensions and IRA distributions	rtm_val
gssi	Gross social security benefits	ss_val, ssi_val, srvs_val, dsab_val
ui	Unemployment compensation	uc_val
transfers	Other non-taxable transfer income	paw_val, wc_val, vet_val, csp_val, fin_val

* Except for the primary and spouse wage and salary income, for married taxpayers each Taxsim variable is the sum of the CPS variables for both the primary taxpayer and their spouse.
** The CPS ASEC does not include information on imputed capital gains after 2010.

has two returns in our sample that are the same except for switching the primary and secondary earner.

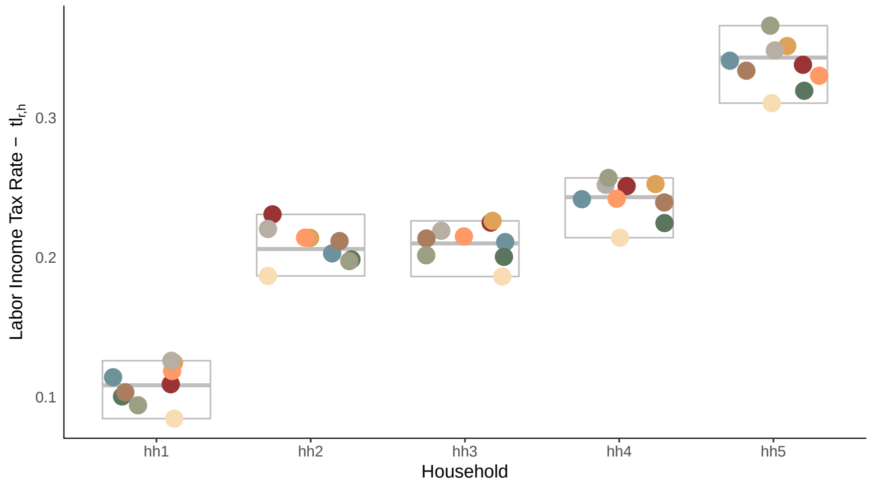
The personal labor income tax rates by region and household are presented in Figure 7a, and the FICA tax rates are presented in 7b. The crossbars represent the national income weighted average effective marginal tax rate for the representative household.

To calibrate the factor price indexed transfer from government to households, $tl\_refund_{t,r,h}$, that allows the model to match the average tax rate, we use estimates of the average individual income tax rate, $tl\_avg0_{r,h}$. The average individual income tax payment for household h in region r is computed following the same procedure outlined above for the effective marginal individual income tax rate on labor income. However, in this case the rate used is the average individual income tax rate calculated by Taxsim. The average individual income tax rates by region and household are presented in Figure 7c. These estimates are consistent with recent estimates by the U.S. Congressional Budget Office, noting that the estimates for SAGE are slightly higher due to the inclusion of state income taxes (CBO, 2018). Based on these estimates, the "refund" provided to households at the wage rate is

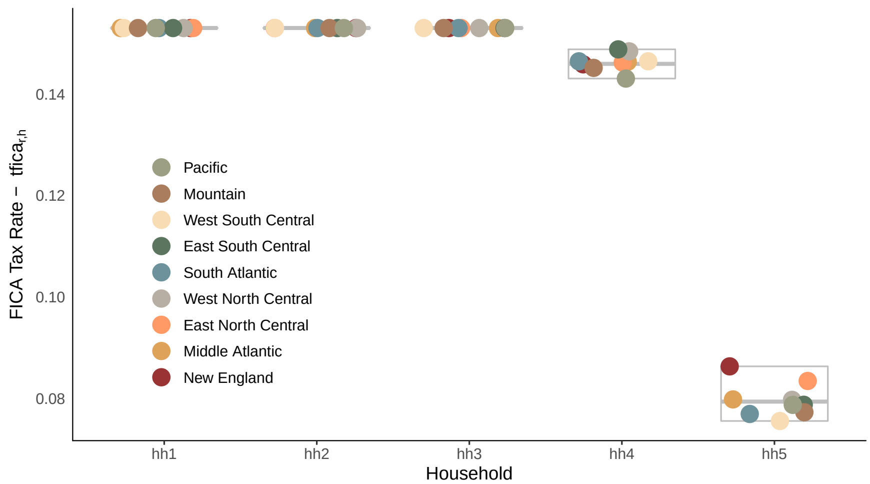
$$tl\_refund0_{t,r,h} = tl0_{r,h} l0_{r,h} - \max\left(tl\_avg0_{r,h} l0_{r,h}, 0\right), \tag{52}$$

which reflects any overpayment that would be made at the marginal tax rate (noting that the benchmark wage rate is normalized to unity).

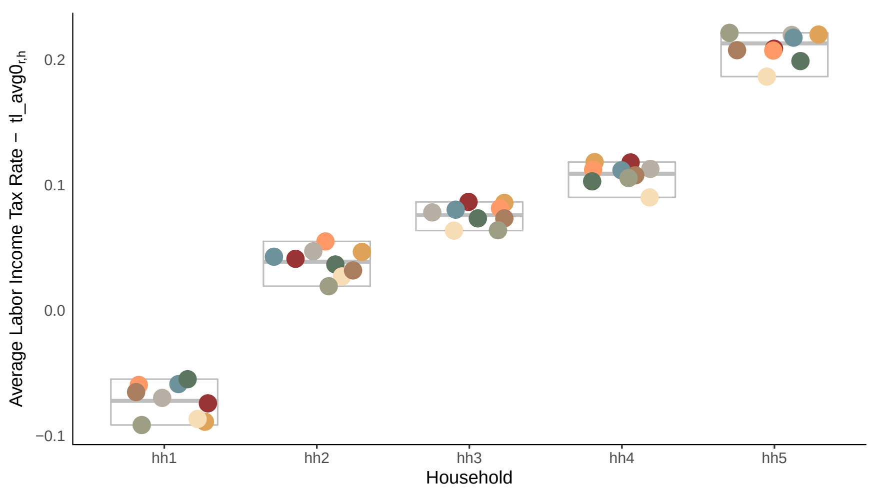
The effective marginal tax rate on capital income is calculated as a weighted average of corporate and personal income tax rates. The exercise described above for determining personal labor income



(a) Labor Income Effective Marginal Tax Rate by Household and Region



(b) FICA Effective Marginal Tax Rate by Household and Region



(c) Average Individual Income Tax Rate by Household and Region

Figure 7: Household Effective Marginal Labor Tax Rates

Table 6: Tax Rates on Capital Income

Region	tk
nen	0.33
mat	0.33
enc	0.33
wnc	0.33
sat	0.33
esc	0.33
wsc	0.33
mnt	0.33
pac	0.33

tax rates is replicated for qualified dividends, interest income, and other business income, such as ordinary dividends and income from sole proprietorships and partnerships In these cases, a national weighted average for the effective marginal tax rate is calculated with weights based on the income category being considered. The corporate income tax is based on an assessment of the average effective marginal corporate income tax rate by the U.S. Congressional Budget Office (CBO, 2017). Specifically the average effective marginal tax corporate income rate is set to 0.186. It is assumed that capital income passed on to households in the form of interest payments or dividends are subject to both the effective corporate income tax rate and the effective personal income tax rate for those types of income. In contrast, capital returns associated with sole proprietorships and partnerships is assumed be only subject to the effective personal income tax rate on those types of income. Based on those assumptions, the effective marginal tax rate on capital income, tk_r, is calculated as a weighted average of the effective marginal tax rates on capital income distributed as interest, qualified dividends, ordinary dividends, and other business income, where the weights are the IRS individual income tax returns line item totals for the types of capital income.[23] The values of the capital income tax rate based on these calculations are presented in Table 6.

3.3 Substitution Elasticities

In the calibrated CES and CET functions, the input-output data are used to define the benchmark value shares, and the free parameters are defined by the substitution elasticity parameters. The list of substitution elasticities included in the model is presented in Table 7.

3.3.1 Armington Elasticities

The sector-specific Armington elasticities between national and foreign goods, se_nf, are based on the estimates included in the GTAP database (Hertel et al., 2008). The GTAP elasticities are based on econometrically estimated substitution elasticities between imports across foreign sources, se_m, by Hertel et al. (2007) and using the "rule of two." The rule, first proposed by Jomini et al.

[23] `https://www.irs.gov/pub/irs-soi/16inlinecount.pdf`

Table 7: Elasticity Parameters

Parameter	Description
Standard Production	
se_klem	Substitution elasticity between material inputs and energy-value-added
se_kle	Substitution elasticity between energy and value added
se_kl	Substitution elasticity between capital and labor
se_ene	Substitution elasticity between electricity and primary energy
se_en	Substitution elasticity among primary energy sources
Resource Extraction, Agriculture, and Forestry Specific	
se_rklem	Substitution elasticity between resource and materials-energy-value-added
Trade	
se_nf	Elasticity of substitution between national and foreign goods
se_dn	Elasticity of substitution between domestic goods and national imports
te_dx	Transformation elasticity between domestically consumed and exported goods
Putty-Clay Capital	
te_k_ex	Transformation elasticity of sector differentiated extant capital
Household	
se_cl	Substitution elasticity between consumption bundle and leisure
se_c	Substitution elasticity between non-energy and energy consumption goods
se_cm	Substitution elasticity between non-energy consumption goods
se_cen	Substitution elasticity between primary energy consumption
se_cene	Substitution elasticity between electricity and primary energy consumption
eta	Inverse intertemporal substitution elasticity of consumption

(1991) and applied widely in CGE modeling, suggests that the elasticity of substitution across foreign sources is twice as large as the elasticity of substitution between domestic and imported commodities[24] such that,

$$se\_nf = \frac{se\_m}{2}. \tag{53}$$

In cases where more than one of the 57 GTAP sectors map into one of the sectors in SAGE, we use value-weighted averages based on GTAP v9 imports by the United States at world prices (Narayanan et al., 2016).

To define the elasticity of substitution between domestic goods and intra-national imports we follow the work of Caron and Rausch (2013). They provide a framework for estimating U.S. intra-national trade elasticities of substitution based on empirical estimates of international and domestic border effects. Specifically, they note that the relative strength of the intra-national and international border effects, α, is defined by the ratio of one minus the substitution elasticities between intra-national sources, se_d, and international sources, se_m, such that

$$\alpha = \frac{1 - se\_d}{1 - se\_m}. \tag{54}$$

Given an estimate for α and se_m, this relationship may be used to solve for the substitution elasticity across domestic sources, se_d. We follow Caron and Rausch (2013) and apply the rule of two to calibrate the substitution elasticity between locally produced goods in the region and intra-national imports, such that $se\_dn = se\_d/2$. Given this relationship, along with (53) and (54), we can solve for the substitution elasticity between locally produced goods

$$se\_dn = \frac{1}{2} - \alpha\left(\frac{1}{2} - se\_nf\right). \tag{55}$$

Coughlin and Novy (2013) estimate both intra-national and international border effects for the U.S. Based on their results we assume that α is 1.868. The SAGE values for se_nf and se_dn are presented in Table 8.

We also follow Caron and Rausch (2013) in setting the transformation elasticity of output between domestic use, national exports, and international exports, te_dx, to 2.

3.3.2 Production Elasticities of Substitution

Koesler and Schymura (2015) provide empirical estimates of the capital-labor substitution elasticities (se_kl), (capital-labor)-energy substitution elasticities (se_kle), and (capital-labor-energy)-materials substitution elasticities (se_klem) at the industry level using a CES nesting structure that is consistent with our standard production structure in Figure 3 and the resource dependent sectors production structure in Figure 4. The estimates are calculated with a panel dataset, covering 1995 to 2007, allowing for the estimation of long-run elasticities, which have been previously applied to

[24] Using a back-casting experiment, Liu et al. (2004) found no evidence to reject the rule of two, providing additional support for its continued use.

Table 8: SAGE Elasticities

Sector	se_kl	se_kle	se_klem	se_ene	se_en	se_nf	se_dn
agf	1.07	0.40	0.98	0.69	0.33	2.45	4.13
bom	0.36	0.19	0.56	0.69	0.33	4.01	7.06
cem	0.20	0.25	0.81	0.69	0.33	2.90	4.98
chm	0.24	0.72	0.94	0.69	0.33	3.30	5.73
col	0.79	0.42	0.22	0.69	0.33	3.05	5.26
con	0.17	0.15	0.61	0.69	0.33	1.90	3.12
cpu	0.10	1.06	0.64	0.69	0.33	4.40	7.79
cru	0.79	0.42	0.22	0.69	0.33	7.30	13.20
ele	1.00	0.46	0.68	0.01	0.23	2.80	4.80
fbm	0.22	0.19	0.63	0.69	0.33	2.66	4.53
fmm	0.18	1.01	0.11	0.69	0.33	3.75	6.57
gas	0.79	0.42	0.22	0.69	0.33	2.80	4.80
hlt	0.58	0.16	0.80	0.77	0.10	1.90	3.12
min	0.79	0.42	0.22	0.69	0.33	0.90	1.25
pmm	0.18	1.01	0.11	0.69	0.33	3.74	6.56
prm	0.12	0.18	0.68	0.69	0.33	3.30	5.73
ref	0.73	0.38	0.42	0.69	0.33	2.10	3.49
srv	0.31	0.27	0.66	0.77	0.10	1.90	3.12
tem	0.18	0.16	0.38	0.69	0.33	3.46	6.02
trn	0.54	0.46	0.73	0.25	0.25	1.90	3.12
ttn	0.14	0.42	0.22	0.25	0.25	1.90	3.12
wpm	0.12	0.24	0.67	0.69	0.33	3.06	5.28
wsu	1.00	0.46	0.68	0.69	0.33	2.80	4.80

CGE modeling (e.g., Böhringer et al. (2016)). The 34 sectors estimated by Koesler and Schymura (2015) are roughly consistent with our default aggregation, though notably they have more detail in the service sectors and less detail in the resource extraction sectors. For cases where a one-to-one mapping between their sectors and SAGE's sectors is not possible we use a weighted average of the Koesler and Schymura (2015) elasticities, where the weighting is by the U.S. sectoral output value in the last year of their dataset. For some sectors, the estimation routine of Koesler and Schymura (2015) returned non-finite values for *se_kl*. Therefore, for the electricity and refining sectors we use values from the recent study by Young (2013), which estimates sector-specific value-added substitution elasticities for the United States[25] Koesler and Schymura (2015) also reported a non-finite value for *se_kle* in the refining sector, in which case we apply the total industry value. The SAGE values for *se_kl*, *se_kle*, and *se_klem* are presented in Table 8. In general, a larger value for the substitution elasticity suggests a greater degree of substitutability between the inputs.

The interfuel substitution elasticities are based on estimates from Serletis et al. (2010a), which provide the most recent estimates for the United States based on contemporary data disaggregated across the industrial, commercial, electricity, and residential sectors. For the primary energy substitution elasticity, *se_en*, in the industrial sectors we use the Allen elasticity across refined petroleum and natural gas, as coal expenditures represent a small share of overall energy expenditures in those sectors. For the electricity sector (ele), the primary energy substitution elasticity is set equal to the estimate of the Allen substitution elasticity between coal and natural gas, as refined petroleum inputs represent a very small share. The results of Serletis et al. (2010a) suggest there are few substitution possibilities between refined petroleum and natural gas in the commercial sectors, so the substitution elasticity in the services and healthcare sectors (srv and hlt) is set to be commensurate with that finding. The substitution elasticity between the primary energy composite and electricity, *se_ene*, is a weighted average of the Allen substitution elasticity estimates for electricity and primary fuels from Serletis et al. (2010a). The weights represent the sector's national primary fuel expenditures in the model's benchmark year based on EIA's State Energy Data System.[26] We assign values from the industrial sector to the manufacturing and resource extraction sectors in the model.[27] We assign values from the commercial sector to the services and healthcare sectors (srv and hlt). For the electricity sector we assume that the nest combining electricity and primary energy inputs is essentially Leontief. For the transportation sectors we base the substitution elasticities on the estimates of Serletis et al. (2010b) for high-income countries. The SAGE values for *se_en* and *se_ene* are presented in Table 8.

3.3.3 Resource Extraction, Agriculture, and Forestry

In sectors with a fixed factor input, including the resource extraction sectors and the agriculture and forestry sectors, the elasticity of substitution between the fixed factor resource and other inputs,

[25] We use the non-normalized generalized method of moments estimates from Young (2013).

[26] `https://www.eia.gov/state/seds/`

[27] Following this same procedure but using the meta-analysis results of Stern (2012) for the industrial sector produces similar values for *se_en* and *se_ene*.

se_rklem, is calibrated to match a long-run supply elasticity based on the benchmark conditions, similar to Balistreri and Rutherford (2001). In partial equilibrium, with fixed prices for all non-resource inputs and a fixed quantity for the resource, the elasticity of supply for a given sector is given by

$$\eta = -\sigma_{res}, \tag{56}$$

where σ_{res} is the Allen own-price elasticity of substitution (Hertel and Tsigas, 2002). In the nesting structure for sectors with a fixed factor, as depicted in Figure 4, the Allen own price elasticity for sector s in region r is

$$\sigma_{res} = -se\_rklem_{r,s}\left(\theta^{-1}_{r,s,res} - 1\right), \tag{57}$$

where $\theta_{r,s,res}$ is the benchmark resource cost share of total costs (Keller, 1976). Combining (56) and (57) provides the calibrated substitution elasticity for a given elasticity of supply

$$se\_rklem_{r,s} = \frac{\eta}{\theta^{-1}_{r,s,res} - 1}. \tag{58}$$

The endogenous supply elasticity in the model is a function of the share of production from new capital in the sector and the endogenously determined value shares, which differ from $\theta_{r,s,res}$. As production with extant capital becomes a smaller share of total production over time, the endogenous supply elasticity increases towards the long-run value to which the function is calibrated. However, as demand for the sector's commodity expands over time the value share of production from variable inputs increases (akin to a stock effect on marginal extraction costs), which in the case of the CES production function places downward pressure on the endogenous supply elasticity.

Arora (2014) examines the natural gas supply elasticity in the United States before and after the expansion of shale gas production through hydraulic fracturing, finding evidence of more elastic supply in recent years. Based on these estimates, Arora and Cai (2014) suggest a long-run supply elasticity of 0.5 for natural gas production as a reference case in CGE modeling. We apply a long-run supply elasticity of 0.5 for the natural gas extraction sector (*gas*).

U.S. oil supply is also considered to be inelastic. Huntington (1992) reviewed expectations of U.S. crude oil supply elasticities through the elasticities implicitly used in energy modeling systems of the time and found an average long-run elasticity of 0.40. There is evidence that in recent decades, the oil supply has been more inelastic than those implied expectations (Greene and Liu, 2015). Krichene (2002) estimates the long-run world crude oil supply elasticity to be 0.25 over the period 1918-1999, with a lower elasticity estimates of 0.10 when the sample was restricted to the later years. This is relatively consistent with recent estimates of short-run world crude oil supply of 0.10 by Caldara et al. (2018) and 0.15 by Baumeister and Hamilton (2019). Caldara et al. (2018) provides evidence that short-run supply elasticities may be lower in non-OPEC nations relative to the world value. However, Bjørnland et al. (2017) finds that supply elasticity for shale wells in the U.S. (which are responsible for around 60% of U.S. oil production[28]) may be notably larger in the

[28] `https://www.eia.gov/tools/faqs/faq.php?id=847&t=6`

range of 0.3 to 0.9 depending on well characteristics. Finally, using a long-run supply elasticity of 0.25, Beckman et al. (2011) find that the GTAP-E model was able to adequately capture the variance of oil price responses to supply and demand shocks based on historical observations. Based on this evidence, we apply a long-run supply elasticity of 0.15 for the crude oil extraction sector (*cru*).

The supply of coal in the United States is generally thought to be elastic. For example, Balistreri and Rutherford (2001) use a long-run supply elasticity to 1.9 to calibrate an energy detailed CGE model. This value is consistent with the long-run supply elasticity in other previous modeling exercises (Golombek et al. (1995); Brown and Huntington (2003)). Empirical elasticities of coal supply elasticities are limited. Dahl and Duggan (1996) survey the literature and find a range of estimates between 0.05 and 7.9 for the United States, with a median value of 0.79. However, data used in the included studies all end in the early 1970s. In a study of coal supply in Australia, Beck et al. (1991) find a long-run supply elasticity of 1.9. Econometric analyses conducted by EIA staff (EIA (2001)) find coal supply elasticities in the range of 1.5 to 3.0. Haggerty et al. (2015) calculate an average supply elasticity of 2.4 from the results of econometric analyses underlying recent versions of EIA's National Energy Modeling System. Based on this evidence, we apply a long-run supply elasticity of 2.4 for the coal mining sector (*col*).

The long-run supply elasticity for the aggregate other mineral and metal mining sector (*min*) is also likely to be elastic.[29] Empirical estimates of supply elasticities for stone, sand, and gravel mining are extremely limited. However, past investigations by the U.S. International Trade Commission (ITC) found the short-run supply elasticity for cement and clinker to be between 2 and 4, suggesting the supply of stone inputs is likely to be fairly elastic (ITC, 2014a). There appear to be no recent estimates of the supply elasticity for copper, however, older estimates suggest that the supply is elastic. For example, Foley and Clark (1981) estimate the long-run supply elasticity of copper in the United States to be 6. While refractory minerals represent a smaller share of the sector, a recent ITC investigation concluded the supply elasticity to be in the range of 5 to 7 (ITC, 2014b). Similarly the ITC found that pure magnesium and alloy magnesium have a short-run supply elasticity of 1.5 to 3 and 3 to 5, respectively (ITC, 2011). Based on this evidence, we apply a long-run supply elasticity of 5 for the other mineral and metal mining sector (*min*).

The agriculture and forestry sector is dominated by crop and livestock production and therefore, we focus on empirical estimates of long-run supply elasticities in those areas. The majority of U.S. cropland is associated with the production of grains, with corn and soybeans the dominant crops. Kim and Moschini (2018) estimate the long-run supply elasticity of corn and soybeans in the United States to be 0.4 and 0.3, respectively. These results are consistent with those of Hendricks et al. (2014), who find a long-run supply elasticity for both corn and soybeans in the United States of 0.3. While older studies also find a long-run supply elasticity for corn of 0.3, the estimate for soybeans is higher at 1.6 (Shideed and White, 1989). Iqbal and Babcock (2018) find global long-

[29] As reported previously, approximately two-thirds of the output value from the sector is attributable to stone mining and quarrying (NAICS 21231) or sand and gravel mining (NAICS 21232). Of the remaining third of the sector's output value, copper ore mining (NAICS 212234) accounts for approximately half.

Table 9: Household Substitution Elasticities

Parameter	Value
se_cl	Calibrated
se_c	0.25
se_cm	0.25
se_cene	0.67
se_cen	0.24
eta	1.66

run supply elasticity estimates of 0.2 and 0.6, respectively. However, Roberts and Schlenker (2013) find a slightly lower global supply elasticity for corn of around 0.1. For non-grain U.S. agricultural production, a significant portion of production value in attributed to California. Russo et al. (2008) study long-run supply elasticities of Californian horticulture and generally find estimates of less than 1, with values of 0.7 for almonds, 0.2 for walnuts, and 0.4 for tomatoes.

For elasticities in livestock production, Kaiser (2012) estimates a long-run elasticity of hog supply of 0.3. Boetel et al. (2007) estimate a long-run supply elasticity of breeding stock with respect to the hog price of 0.6. Marsh (2003) and Sarmiento and Allen (2000) estimate long-run cattle supply elasticities of 0.6 to 2.8 and 0.3 to 2.9, respectively. These ranges are roughly consistent with previous estimates of cattle supply elasticities (e.g., Rucker et al. (1984) and Buhr and Kim (1997)). Little empirical evidence exists for the long-run supply elasticity of poultry (e.g., broilers) in the United States. Kapombe and Colyer (1998), Holt and Aradhyula (1998), and Holt and McKenzie (2003) all find evidence of a short-run supply elasticity of 0.1. Based on this evidence, we apply a long-run supply elasticity of 0.5 for the agriculture and forestry sector (*agf*).

3.3.4 Consumption Elasticities

For the elasticity of substitution across the consumption of non-energy goods, *se_cm*, and the elasticity of substitution across consumption of non-energy goods and energy goods, *se_c*, we follow the specification of Rausch et al. (2011). To calibrate the elasticity of substitution across consumption of primary energy sources, *se_cen*, we use a small value commensurate with the finding of Serletis et al. (2010a), that there are few substitution possibilities between refined petroleum and natural gas consumption in the residential sector. For the substitution elasticity between primary energy and electricity consumption, *se_cene*, we apply the same approach used in Section 3.3.2 based on the empirical estimates of Serletis et al. (2010a). These values are presented in Table 9.

In the households' welfare maximization problem the additively separable nature of the intertemporal welfare function in (21) and the isoelastic form of the intra-temporal utility function (25) mean the elasticity of intertemporal substitution will be $1/\eta$. In a recent review of over 1,400 estimates of the elasticity of intertemporal substitution for the United States, Havranek et al. (2015) find a mean value of 0.6. Based on this evidence, we set the value of η to 1.66.

The consumption-leisure substitution elasticity is determined jointly with the time endowment in the model to match observed estimates of the compensated and uncompensated labor supply elasticities in a static setting. Consider the demand system in (26) and the simplified budget constraint

$$pcl_{t,r,h}cl_{t,r,h} = (1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h} + \pi_{t,r,h}, \tag{59}$$

where $\pi_{t,r,h}$ represents non-labor income net of savings. Assuming labor income taxes are constant over time, $tl_{t,r,h} + tfica_{t,r,h} = tl_{t+1,r,h} + tfica_{t+1,r,h}\ \forall\ t$, the Marshallian demand for leisure is

$$\begin{aligned} leis_{t,r,h} =& leis0_{r,h} \left(\frac{\pi_{t,r,h} + (1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h}}{\pi 0_{r,h} + (1 - tl_{t,r,h} - tfica_{t,r,h} + pl0_{r}tl\_refund0_{r,h})\, pl0_{r}te0_{r,h}} \right) \left(\frac{pl_{t,r}}{pl0_{r}} \right)^{-se\_cl} \\ & \times \left[cs\_cl_{r,h} \left(\frac{pc_{t,r,h}}{pc0_{r,h}} \right)^{1-se\_cl} + (1 - cs\_cl_{r,h}) \left(\frac{pl_{t,r}}{pl0_{r}} \right)^{1-se\_cl} \right]^{-1}. \end{aligned} \tag{60}$$

The uncompensated price elasticity of leisure demand, μ_l, may be obtained from (60), such that

$$\begin{aligned} \mu^{leis}_{t,r,h} \equiv& \frac{\partial leis_{t,r,h}}{\partial pl_{t,r}} \frac{pl_{t,r}}{leis_{t,r,h}} \\ =& \frac{(1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h}}{\pi_{t,r,h} + (1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h}} \\ & - (1 - cs\_cl_{r,h}) \left(\frac{pl0_{r}}{pl_{t,r}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} \\ & + se\_cl \left[(1 - cs\_cl_{r,h}) \left(\frac{pl0_{r}}{pl_{t,r,h}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} - 1 \right], \end{aligned} \tag{61}$$

where

$$e\left(pc_{t,r,h}, pl_{t,r}\right) = \left[cs\_cl_{r,h} \left(\frac{pc_{t,r,h}}{pc0_{r,h}} \right)^{1-se\_cl} + (1 - cs\_cl_{r,h}) \left(\frac{pl_{t,r}}{pl0_{r}} \right)^{1-se\_cl} \right]^{\frac{1}{se\_cl-1}}. \tag{62}$$

The first two components of (61) define the income elasticity of leisure,

$$\begin{aligned} \mu^{I}_{t,r,h} =& \frac{(1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h}}{\pi_{t,r,h} + (1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r}te_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h}} \\ & - (1 - cs\_cl_{r,h}) \left(\frac{pl0_{r}}{pl_{t,r}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl}, \end{aligned} \tag{63}$$

and the third component represents the substitution effect, or the compensated price elasticity of leisure demand,

$$\mu^{leis|\bar{cl}}_{t,r,h} = se\_cl \left[(1 - cs\_cl_{r,h}) \left(\frac{pl0_{r}}{pl_{t,r,h}} \right)^{se\_cl-1} e\left(pc_{t,r,h}, pl_{t,r}\right)^{1-se\_cl} - 1 \right] \tag{64}$$

This may be verified through the Hicksian demand function via the Slutsky equation. Given the definition of labor supply, $te_{t,r,h} - leis_{t,r,h}$, the compensated labor supply elasticity, or substitution effect, is

$$\epsilon^{l|\bar{cl}}_{t,r,h} = -\mu^{leis|\bar{cl}}_{t,r,h} \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}. \tag{65}$$

And the uncompensated labor supply elasticity is

$$\epsilon^{l}_{t,r,h} = -\mu^{leis}_{t,r,h} \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}, \tag{66}$$

which, may be written as

$$\epsilon^{l}_{t,r,h} = -\left(\mu^{I}_{t,r,h} + \mu^{leis|\bar{cl}}_{t,r,h}\right) \frac{leis_{t,r,h}}{te_{t,r,h} - leis_{t,r,h}}. \tag{67}$$

We define the share of the time endowment spent on leisure as $\phi_{t,r,h} = leis_{t,r,h}/te_{t,r,h}$ and rewrite (65) and (67) as

$$\epsilon^{l|\bar{cl}}_{t,r,h} = \frac{-\phi_{t,r,h}}{1 - \phi_{t,r,h}} \mu^{leis|\bar{cl}}_{t,r,h} \tag{68}$$

and

$$\epsilon^{l}_{t,r,h} = \frac{-\phi_{t,r,h}}{1 - \phi_{t,r,h}} \left(\mu^{I}_{t,r,h} + \mu^{leis|\bar{cl}}_{t,r,h}\right). \tag{69}$$

Substituting (68) into (69) yields

$$\epsilon^{l}_{t,r,h} = \frac{-\phi_{t,r,h}}{1 - \phi_{t,r,h}} \mu^{I}_{t,r,h} + \epsilon^{l|\bar{cl}}_{t,r,h}. \tag{70}$$

From (63), the benchmark year income elasticity of leisure is

$$\mu^{I}_{0,r,h} = \frac{(1 - tl_{0,r,h} - tfica_{0,r,h})\, pl0_r te0_{r,h} + pl0_r tl\_refund0_{r,h}}{\pi 0_{r,h} + (1 - tl_{0,r,h} - tfica_{0,r,h})\, pl0_r te0_{r,h} + pl0_r tl\_refund0_{r,h}} - (1 - cs\_cl_{r,h}). \tag{71}$$

Assuming that in the benchmark prices are normalized to unity such that the effective labor price is $(1 - tl_{0,r,h} - tfica_{0,r,h})$ and given the definition of $cs\_cl_{r,h}$ and an estimate of the income elasticity of labor, $\hat{\epsilon}^I$, (59) and (71) may be substituted into (70) to yield the calibrated benchmark value of leisure

$$leis0_{r,h} = -\frac{c0_{r,h}\hat{\epsilon}^I}{(1 - tl_{0,r,h} - tfica_{0,r,h})\,(1 + \hat{\epsilon}^I) + \frac{tl\_refund0_{r,h}}{l0_{r,h}}}. \tag{72}$$

From (64), the benchmark uncompensated leisure demand elasticity is

$$\mu^{l\bar{e}is|\bar{cl}}_{0,r,h} = -se\_cl \cdot cs\_cl_{r,h}. \tag{73}$$

Substituting (73) into (68) yields the calibrated version of the elasticity of substitution between

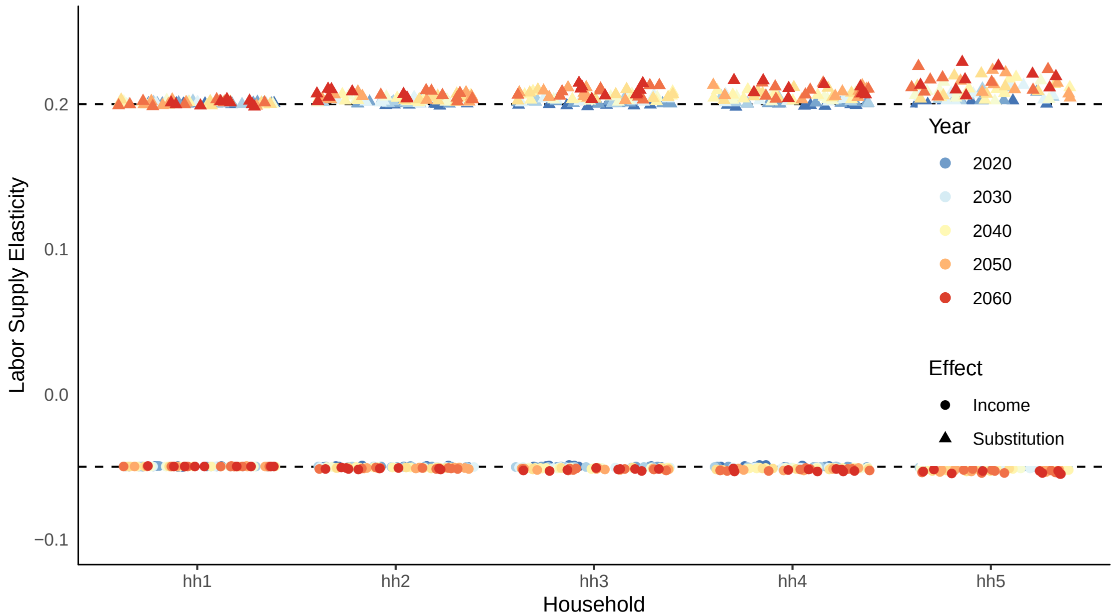


Figure 8: Calibrated Labor Supply Elasticities

consumption and leisure,

$$se\_cl = \frac{\hat{\epsilon}^{l|\bar{cl}} cl0_{r,h} l0_{r,h}}{leis0_{r,h} c0_{r,h}}, \tag{74}$$

where $\hat{\epsilon}^{l|\bar{cl}}$ is the empirical estimate of the substitution elasticity. The observed labor earnings are combined with the calibrated benchmark value of leisure in (72) to determine the time endowment $te0_{r,h} = l0_{r,h} + leis0_{r,h}$.

To calibrate the time endowment and the substitution elasticity between consumption and leisure, we use the conclusions from the literature review by McClelland and Mok (2012) on estimates of the income and substitution effects for the United States. Specifically, they conclude that estimates on the order of $\hat{\epsilon}^I = -0.05$ and $\hat{\epsilon}^{l|\bar{cl}} = 0.20$ are representative of the most recent empirical evidence. Given the dynamic nature of the model and the baseline calibration that deviates from the assumptions in the simplified static household problem above, the model's endogenous labor supply elasticities differ slightly from the calibration points. Figure 8 presents the substitution and income effects implicit in the model's baseline.

3.4 Dynamic Baseline

The foundation for the model's baseline is a neoclassical balanced growth reference path with population growth and Harrod neutral technological progress. The baseline augments the reference path to capture ways in which the baseline may deviate from balanced growth that are potentially relevant for estimating the equilibrium impacts of some environmental regulations. These additions include the presence of fixed factors in some sectors (as discussed in Section 2.2.2) and calibrating the energy intensity of future production technologies and consumption to be consistent with EIA's

Annual Energy Outlook (AEO) forecast.[30]

The steady-state interest rate along the balanced growth path, $rbar$, is set to 0.045. The interest rate reflects the average after-tax rate of return on private capital. Given the capital tax in Section 3.2 the social return on private capital in the model is approximately 0.07, which is consistent with the average pre-tax rate of return on capital observed between 1960 and 2014 (CEA, 2017). The depreciation rate, δ, is set to 0.05, which is the average U.S. capital depreciation rate from 1950 to 2014 as estimated by Feenstra et al. (2015). This rate is applied to both new and extant capital.

Population is assumed to grow at rate γ, such that

$$n_{t,r,h} = n0_{r,h} \left(1 + \gamma\right)^t , \tag{75}$$

where γ is set to the average annual population growth rate in the AEO, 0.006. Technological progress is assumed to be Harrod neutral (i.e., labor augmenting). Labor productivity growth, ω, is assumed to be 0.016, which is the average annual labor productivity growth in the AEO. Labor productivity growth is implemented through the effective time endowment, such that

$$te_{t,r,h} = te0_{r,h} \left(1 + \gamma + \omega\right)^t . \tag{76}$$

Based on the isoelastic form of the intra-temporal utility function, the pure rate of time preference, ρ, in (22) is defined as

$$\rho = \frac{1 + rbar}{\left(\frac{1+\gamma+\omega}{1+\gamma}\right)^\eta} - 1. \tag{77}$$

The presence of the population growth rate, γ, adjusts for cases with larger time steps or larger population growth rates. If both of those are small, (77) is well approximated by the more common form

$$\rho = \frac{1 + rbar}{(1 + \omega)^\eta} - 1. \tag{78}$$

3.4.1 Baseline Energy Use

We calibrate the cost shares in the production functions to capture expected technological change in the energy intensity of production based on the AEO forecasts. To get the unit energy consumption (UEC) we divide the total energy consumption in the AEO by the real value of shipments for the sectors. The National Energy Modeling System (NEMS) used for the AEO only allows for limited fuel switching within the industrial sectors, so changes in the UEC over time predominately represent exogenous forecasts regarding technological change in energy efficiency. We use the average growth rate of the UEC over the AEO time horizon, denoted as ene_growth_s, to calibrate the cost shares in the production function.

The change in energy efficiency is assumed to be capital embodied. Therefore, the change is

[30]The calibration is conducted with the most recent AEO forecast that includes a representation of the benchmark year for SAGE.

represented as a shift from energy use to capital such that the "benchmark" values for intermediate and capital inputs as well as the cost shares are time dependent. The partial putty-clay framework needs to be accounted for to ensure that the overall UEC trend in SAGE is consistent with AEO, since only production with new capital is associated with the improvements and the goal is to match the overall UEC trend in AEO. The energy-related intermediate inputs and capital benchmark values for production with new capital are calibrated, such that

$$id0_{t,r,ss,s} = ene\_factor_{t,s} id0_{0,r,ss,s} \;\; ss \in sene \tag{79}$$

and

$$kd0_{t,r,s} = kd0_{0,r,s} + \frac{(1 - ene\_factor_{t,s}) \sum_{ss \in sene} id0_{0,r,ss,s}}{1 + tk0_r}, \tag{80}$$

where

$$ene\_factor_{t,s} = \frac{(1 + ene\_growth_s)^t (1 + \gamma + \omega)^t - (1 - \delta)^t}{(1 + \gamma + \omega)^t - (1 - \delta)^t} \tag{81}$$

and $sene \in (col, gas, ref, ele)$ is the set of primary energy commodities plus electricity. The relevant cost shares, cs_kle and cs_kl, become time dependent and are adjusted to be consistent with (79) and (80).[31]

The mapping from the AEO sectors to the SAGE sectors, along with the UEC growth parameters, are presented in Table 10. For the non-truck transportation sector, trn, the UEC growth rate is based on the average growth rate of air transportation fuel efficiency as forecast by the AEO, since this represents a large share of the energy consumption for the sector. For the truck transportation sector, ttn, the UEC growth rate is based on the average growth rate of truck freight transportation fuel efficiency as forecast by the AEO. No changes in the energy intensity of the electricity sector are assumed.

Household and government energy consumption shares are assumed to change over time to match the energy intensity forecasts in AEO. Consumption shares of electricity and natural gas are assumed to grow at the same average rate as in the AEO forecast, $cd\_ene\_growth_{ele}$ and $cd\_ene\_growth_{gas}$, respectively. The consumption share of refined petroleum is assumed to grow based on the average consumption share growth rate of light duty vehicle fuel expenditures, $cd\_ene\_growth_{ref}$. This is assumed to represent a shift towards other consumption goods in proportion to their benchmark consumption shares, such that

$$cd0_{t,r,h,s} = (1 + cd\_ene\_growth_s)^t cd0_{0,r,h,s} \;\; s \in sene \tag{82}$$

[31] Outside of this section we exclude the time subscript on the benchmark values and cost shares to simplify the exposition.

Table 10: Unit Energy Consumption Growth Rates

SAGE Sector	AEO Sectors	*ene_growth*
agf	agg	-0.0054
col	ming	-0.0051
min	ming	-0.0051
ele		
gas	ming	-0.0051
cru	ming	-0.0051
wsu	bmf	-0.0109
con	cns	-0.0013
fbm	fdp	-0.0044
wpm	ppm, wdp	-0.0075
ref	ref	-3e-04
chm	bch	-0.0069
prm	pli	-0.0141
cem	cem	-0.017
pmm	ism, aap	-0.0038
fmm	fbp	-0.0134
cpu	cmpr, eei	-0.011
tem	teq	-0.0131
bom	bmf, ggr, mchi	-0.0134
trn		-0.0062
ttn		-0.0094
srv	comm	-0.0167
hlt	comm	-0.0167

Table 11: Energy Consumption Share Growth Rates

Commodity	*cd_ene_growth*
ele	-0.0189
gas	-0.0204
ref	-0.0316
col	0

and

$$cd0_{t,r,h,s} = cd0_{0,r,h,s} + \left\{ \sum_{ss \in sene} \left[1 - (1 + cd\_ene\_growth_{ss})^t\right] cd0_{0,r,h,ss} \right\} \frac{cd0_{0,r,h,s}}{\sum_{ss \notin sene} cd0_{0,r,h,ss}} \quad s \notin sene. \tag{83}$$

The values for the growth rates are presented in Table 11. Government consumption is subject to the same treatment.

The growth of natural gas consumed per unit of electricity produced in the baseline is roughly consistent with forecasts from AEO. However, the growth of coal consumed per unit of electricity produced, absent any adjustment, would be higher than AEO forecasts due to regulatory and market changes. Therefore, we adjust the cost share of coal in electricity production by the average growth rate in the share of electricity generated from coal in the AEO forecast, col_ele_growth. The reduction in the cost share is offset by an increase in the cost share of capital and labor, which would be associated with the alternative non-fossil fuel sources of generation growing in the AEO forecasts. Specifically, the intermediate, capital, and labor inputs are adjusted over time, such that

$$id0_{t,r,col,ele} = col\_ele\_factor_t id0_{0,r,col,ele}, \tag{84}$$

$$kd0_{t,r,ele} = kd0_{0,r,ele} + \left[1 - col\_ele\_factor_t id0_{0,r,col,ele}\right] \frac{kd0_{0,r,ele}}{kl0_{r,ele}}, \tag{85}$$

and

$$ld0_{t,r,ele} = ld0_{0,r,ele} + \left[1 - col\_ele\_factor_t id0_{0,r,col,ele}\right] \frac{ld0_{0,r,ele}}{kl0_{r,ele}}, \tag{86}$$

where

$$col\_ele\_factor_{t,s} = \frac{(1 + col\_ele\_growth_s)^t (1 + \gamma + \omega)^t - (1 - \delta)^t}{(1 + \gamma + \omega)^t - (1 - \delta)^t}. \tag{87}$$

The cost shares cs_en, cs_ene, and cs_kle are also adjusted accordingly.

4 Solution

To solve the model, the primal version of the problem in Section 2 is converted to a series of non-linear equations that define profit maximizing firm behavior, welfare maximizing household behavior, market clearance, balanced budgets, and perfect competition following Mathiesen (1985) and Rutherford (1999).

Given the assumption of constant returns to scale, one can solve for the constant unit cost function of producing good z denoted as $C^{z}_{t,r,z}$. Perfect competition may then be represented along with profit maximization by zero-profit conditions that assume the unit cost function under optimal behavior is at least as great as the price for the good. If it is the case that the unit cost function is greater than the price such that profits are negative, it must be the case that the quantity produced is zero, providing the complementarity condition. This will hold for production with both new and extant capital and provision of the Armington aggregate, government goods, and investment. The zero-profit conditions associated with these activities are

$$C^{y}_{t,r,s}\left(pa_{t,r,agf},\ldots,pa_{t,r,srv},pr_{t,r},pres_{t,r,s},pl_{t,r},tk_{t,r},ty_{t,r,s}\right)\geq py_{t,r,i}\quad\perp\quad y_{t,r,i}\geq 0,\tag{88}$$

$$C^{y\_ex}_{t,r,i}\left(pa_{t,r,agf},\ldots,pa_{t,r,srv},pr\_ex_{t,r,s},pres_{t,r,s},pl_{t,r},tk_{t,r},ty_{t,r,s}\right)\geq py_{t,r,i}\quad\perp\quad y\_ex_{t,r,i}\geq 0,\tag{89}$$

$$C^{a}_{t,r,i}\left(pd_{t,r,i},pn_{t,i},pfx_{t}\right)\geq pa_{t,r,i}\quad\perp\quad a_{t,r,i}\geq 0,\tag{90}$$

$$C^{g}_{t,r}\left(pa_{t,r,agf},\ldots,pa_{t,r,srv}\right)\geq pgov_{t,r}\quad\perp\quad gov_{t,r}\geq 0,\tag{91}$$

and

$$C^{i}_{t,r}\left(pa_{t,r,agf},\ldots,pa_{t,r,srv}\right)\geq pinv_{t,r}\quad\perp\quad inv_{t,r}\geq 0.\tag{92}$$

where $C^{y}_{t,r,s}$ is the unit cost function for production of s using new capital based on (3) and (10), $C^{y\_ex}_{t,r,s}$ is the unit cost function for production of s using extant capital based on (13), $C^{a}_{t,r,s}$ is the unit cost function for the Armington aggregate based on (1), $C^{g}_{t,r}$ is the unit cost function for the government good based on (32), and $C^{i}_{t,r,s}$ is the unit cost function for the investment good based on (20). A similar condition can be established for the "price" of full consumption

$$e_{t,r,h}\left(pa_{t,r,agf},\ldots,pa_{t,r,srv},tl_{t,r,h},tfica_{t,r,h},tc_{t,r}\right)\geq pcl_{t,r,h}\quad\perp\quad cl_{t,r,h}\geq 0,\tag{93}$$

where $e_{t,r,h}$ is the unit expenditure function for full consumption based on the intra-temporal preferences in (26). Following Section 2.7, the final zero-profit condition requires that for households to hold capital the price must equal the present value of returns, such that

$$pk_{t,r}\geq pr_{t,r}+(1-\delta)\,pk_{t+1,r}\quad\perp\quad k_{t,r}.\tag{94}$$

From Shepard's lemma the Hicksian demands for each input is the partial derivative of the unit cost function with respect to the price of the input times the level of the activity. As such, the input demands for profit maximizing firms using new capital, conditional on the equilibrium level of production, are

$$id_{t,r,ss,s}=\frac{\partial C^{y}_{t,r,s}}{\partial pa_{t,r,ss}}y_{t,r,s},\tag{95}$$

$$kd_{t,r,s}=\frac{\partial C^{y}_{t,r,s}}{\partial pr_{t,r}}y_{t,r,s},\tag{96}$$

$$ld_{t,r,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pl_{t,r}} y_{t,r,s}, \tag{97}$$

and

$$res_{t,r,s} = \frac{\partial C^{y}_{t,r,s}}{\partial pres_{t,r,s}} y_{t,r,s}. \tag{98}$$

Similarly inputs to production using extant capital are defined as

$$id\_ex_{t,r,ss,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pa_{t,r,ss}} y\_ex_{t,r,s}, \tag{99}$$

$$kd\_ex_{t,r,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pr\_ex_{t,r,s}} y\_ex_{t,r,s}, \tag{100}$$

$$ld\_ex_{t,r,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pl_{t,r}} y\_ex_{t,r,s} \tag{101}$$

and

$$res\_ex_{t,r,s} = \frac{\partial C^{y\_ex}_{t,r,s}}{\partial pres_{t,r,s}} y\_ex_{t,r,s}. \tag{102}$$

The inputs to the formation of capital and government consumption may be similarly defined as

$$g_{t,r,s} = \frac{\partial C^{g}_{t,r}}{\partial pa_{t,r,s}} gov_{t,r} \tag{103}$$

and

$$i_{t,r,s} = \frac{\partial C^{i}_{t,r}}{\partial pa_{t,r,s}} inv_{t,r}. \tag{104}$$

Given the equilibrium level of full consumption, the demands for final consumption goods are

$$cd_{t,r,h,s} = \frac{\partial e_{t,r,h}}{\partial pa_{t,r,s}} cl_{t,r,h}, \tag{105}$$

where leisure demand can be similarly defined as

$$leis_{t,r,h} = \frac{\partial e_{t,r,h}}{\partial pl_{t,r}} cl_{t,r,h}. \tag{106}$$

Imports and domestically-sourced use are defined conditional on the equilibrium level of the Armington aggregate as

$$d_{t,r,s} = \frac{\partial C^{a}_{t,r,s}}{\partial pd_{t,r,s}} a_{t,r,s}, \tag{107}$$

$$m_{t,r,s,dtrd} = \frac{\partial C^{a}_{t,r,s}}{\partial pn_{t,r}} a_{t,r,s}, \tag{108}$$

and

$$m_{t,r,s,ftrd} = \frac{\partial C^{a}_{t,r,s}}{\partial pfx_{t}} a_{t,r,s}. \tag{109}$$

Exports are determined from the CET function in (2), such that

$$x_{t,r,s,dtrd} = \frac{y\_ex_{t,r,s} + y_{t,r,s}}{y0_{r,s}} \left(\frac{pn_{t,r}}{py_{t,r,s}}\right)^{te\_dx} \tag{110}$$

and

$$x_{t,r,s,ftrd} = \frac{y\_ex_{t,r,s} + y_{t,r,s}}{y0_{r,s}} \left(\frac{pfx_{t}}{py_{t,r,s}}\right)^{te\_dx}, \tag{111}$$

where the right hand side defines the optimal share of output supplied to the export markets based on the output transformation function in (2).

Given the Hicksian demands conditional on equilibrium activity levels, the market clearance conditions in Section 2.6 can be defined. If any of the conditions in (36)-(44) holds with strict inequality it would imply that supply exceeds demand in equilibrium, such that the price of that activity's output must be zero. This leads to a series of complementarity conditions, which define the market clearance conditions. The price of the Armington aggregate, $pa_{t,r,s}$, clears the goods market

$$a_{t,r,s} \geq \sum_{ss} id_{t,r,s,ss} + id\_ex_{t,r,s,ss} + \sum_{h} cd_{t,r,s,h} + i_{t,r,s} + g_{t,r,s} \quad \perp \quad pa_{t,r,s} \geq 0. \tag{112}$$

The price of domestic output consumed domestically, $pd_{t,r,s}$, clears the domestic market

$$\frac{y\_ex_{t,r,s} + y_{t,r,s}}{y0_{r,s}} \left(\frac{pd_{t,r,s}}{py_{t,r,s}}\right)^{te\_dx} \geq \frac{d_{t,r,s}}{d0_{r,s}} \quad \perp \quad pd_{t,r,s} \geq 0, \tag{113}$$

where the left hand side defines the optimal share of output supplied to the domestic market based on the output transformation function in (2). The price of labor, $pl_{t,r}$, (i.e., the wage rate) clears the labor market

$$\sum_{h} l_{t,r,h} \geq \sum_{s} ld_{t,r,s} + ld\_ex_{t,r,s} \quad \perp \quad pl_{t,r} \geq 0. \tag{114}$$

The rental rate for sector specific extant capital, $pr\_ex_{t,r,s}$, clears the market for extant capital

$$\frac{k\_ex_{t,r}}{k0_{r}} \left(\frac{pr\_ex_{t,r,s}}{pr\_ex\_agg_{t,r}}\right)^{te\_k\_ex} \geq \frac{kd\_ex_{t,r,s}}{kd0_{r,s}} \quad \perp \quad pr\_ex_{t,r,s} \geq 0 \tag{115}$$

where the left hand side defines the optimal share of extant capital supplied to sector s based on the extant transformation function in (17). The rental rate for new capital, $pr_{t,r}$, clears the market for new capital

$$k_{t,r} \geq \sum_{s} kd_{t,r,s} \quad \perp \quad pr_{t,r} \geq 0. \tag{116}$$

The price of new capital, $pk_{t,r}$, clears the investment market

$$k_{t-1,r}\left(1-\delta\right) + inv_{t-1,r} \geq k_{t,r} \quad \perp \quad pk_{t,r} \geq 0. \tag{117}$$

The price of foreign exchange, pfx_t, clears the foreign exchange market

$$\sum_{r,s} m_{t,r,s,ftrd} \geq \sum_{r,s} x_{t,r,s,ftrd} + \sum_{r,h} bopdef_{t,r,h} \quad \perp \quad pfx_t \geq 0. \tag{118}$$

The price of commodities on the national market, $pn_{t,s}$, clears the market for national trade

$$\sum_{r} x_{t,r,s,dtrd} \geq \sum_{r} m_{t,r,s,dtrd} \quad \perp \quad pn_{t,s} \geq 0. \tag{119}$$

The rental rate for sector specific fixed factors, $pres_{t,r,s}$, clears the market for sector specific fixed factors

$$\sum_{h} rese_{t,r,s,h} \geq res_{t,r,s} + res\_ex_{t,r,s} \quad \perp \quad pres_{t,r,s} \geq 0. \tag{120}$$

Equilibrium also requires that aggregate household holdings of new capital, $kh_{t,r,h}$, across all households and regions equal the aggregate level of new capital, $k_{t,r}$, across all regions. However, due to Walras law one of the constraints is redundant and we choose to omit this capital aggregation constraint.

In addition, the problem requires that households maximize intertemporal welfare in (21). The Karush-Kuhn-Tucker conditions for the welfare maximization problem are

$$\left(\frac{cl_{t,r,h}}{n_{t,r,h}}\right)^{-\eta} \geq \lambda_{t,r,h} pcl_{t,r,h} \quad \perp \quad cl_{t,r,h} \geq 0, \tag{121}$$

$$\beta_{t+1,r,h}\lambda_{t+1,r,h} \geq \beta_{t,r,h}\lambda_{t,r,h} \quad \perp \quad kh_{t+1,r,h} \geq 0, \tag{122}$$

and

$$\begin{aligned} kh_{t+1,r,h} + pcl_{t,r,h}cl_{t,r,h} \geq (1+r_t)\, kh_{t,r,h} &+ (1 - tl_{t,r,h} - tfica_{t,r,h})\, pl_{t,r} te_{t,r,h} \\ &+ pr\_ex\_agg_{t,r} kh\_ex_{t,r,h} + \sum_s pres_{t,r,s} rese_{t,r,s,h} \\ &+ pfx_t bopdef_{t,r,h} + cpi_t tran_{t,r,h} \\ &+ pl_{t,r} tl\_refund_{t,r,h} \\ &\perp \quad \lambda_{t,r,h} \geq 0. \end{aligned} \tag{123}$$

where the level of labor supply is determined by the time constraint, such that

$$te_{t,r,h} \geq leis_{t,r,h} + l_{t,r,h} \quad \perp \quad l_{t,r,h} \geq 0. \tag{124}$$

The problem requires that the government budget constraint holds, as described in Section 2.5, such that

$$
\begin{aligned}
\sum_r pgov_{t,r}gov_{t,r}+ \quad & \sum_h cpi_t tran_{t,r,h} + pl_{t,r}tl\_refund_{t,r,h} \\
& \geq \sum_r \sum_s \Bigg\{ \quad ty_{t,r,s}py_{t,r,s}\left(y_{t,r,s} + y\_ex_{t,r,s}\right) \\
& \qquad + tk_{t,r}\left[pr_{t,r}kd_{t,r,s} + pr\_ex_{t,r,s}kd\_ex_{t,r,s} + pres_{t,r,s}\left(res_{t,r,s} + res\_ex_{t,r,s}\right)\right] \Bigg\} \\
& \quad + \sum_r \sum_h \left[\left(tl_{t,r,h} + tfica_{t,r,h}\right) pl_{t,r}l_{t,r,h} + tc_{t,r}pa_{t,r,s}cd_{t,r,s,h} \right]. \\
& \qquad \perp \quad incadj_t \geq 0.
\end{aligned}
\tag{125}
$$

Finally, we include the conditions to close the finite time approximation to the infinite time problem. As noted in Section 2.7, the post-terminal capital stock is determined by requiring that investment grows at the rate of aggregate consumption growth, such that

$$
\frac{inv_{T,r}}{inv_{T-1,r}} \geq \frac{\sum_h c_{T,r,h}}{\sum_h c_{T-1,r,h}} \quad \perp \quad kt \geq 0. \tag{126}
$$

The price is determined based on the law of motion for capital, such that

$$
k_{T,r}\left(1-\delta\right) + inv_{T,r} \geq kt_r \quad \perp \quad pkt \geq 0, \tag{127}
$$

where households' share of the post-terminal capital stock is assumed to be equivalent to their benchmark shares of the capital stock.

The equations (88)-(127) define the equilibrium conditions of the model. The problem is formulated in the General Algebraic Modeling System (GAMS).[32] The model is solved using the PATH solver (Ferris and Munson, 2000). We set the numeraire to the price of foreign exchange, pfx_0, in the initial period.

In this documentation all variables are defined in levels for ease of exposition and interpretation. The model in the code is mathematically equivalent to that which is laid out in this documentation. However, in the implementation most variables are defined as indices relative to the benchmark value instead of in levels. This provides for a fairly well scaled problem with only limited need for scaling of equations and variables prior to the solve. This implementation does not affect the model solution, but does mean that some of the equations as implemented in the code may differ slightly from what is laid out in the documentation.

[32] GAMS Development Corporation. General Algebraic Modeling System (GAMS) Washington, DC, USA, 2014.

4.1 Calculating Welfare Effects

Households' willingness to pay to avoid the costs of the policy requirements, that is the social costs associated with the policy, are estimated using equivalent variation (EV). EV is estimated as the amount of income households could forgo under baseline prices and still achieve the same level of welfare as simulated in the policy case. More specifically, EV is calculated as the difference between expenditures (on all goods including leisure) in the baseline and the alternative case where households face baseline prices but are constrained to the welfare level achieved in the policy case.

The households' optimization problem described by (21)-(31) yields optimal levels of consumption and leisure given prices, taxes, transfers, and shadow prices on their budget constraint, $\lambda_{t,r,h}$. For simplicity of exposition, let $\mathbf{z}_{r,h}^{sim}$ be a vector of all prices, taxes, and transfers faced by household h in region r, where sim denotes the given simulation: $base$ for baseline and pol for policy. The optimal levels of consumption and leisure are then given by

$$cd_{t,r,s,h}^{sim} = cd_{t,r,s,h}\left(\mathbf{z}_{r,h}^{sim}, \boldsymbol{\lambda}_{r,h}^{sim}\right) \tag{128}$$

and

$$leis_{t,r,h}^{sim} = leis_{t,r,h}\left(\mathbf{z}_{r,h}^{sim}, \boldsymbol{\lambda}_{r,h}^{sim}\right), \tag{129}$$

where $\boldsymbol{\lambda}_{r,h}^{sim}$ is a vector of shadow prices over the simulation's time horizon. Households' expenditures on full consumption are then defined as

$$expenditure\left(\mathbf{z}_{r,h}^{sim}, \boldsymbol{\lambda}_{r,h}^{sim}\right) = \sum_t \left(1 - tl_{t,r,h}^{sim} - tfica_{t,r,h}^{sim}\right) pl_{t,r}^{sim} leis_{t,r,h}^{sim} + \sum_s \left(1 + tc_{t,r}^{sim}\right) pa_{t,r,s}^{sim} cd_{t,r,s,h}^{sim}, \tag{130}$$

noting that since the prices are relative to the numeraire in the initial period they are already in present value terms.

From the first order conditions to the household optimization problem in (122), the evolution of the shadow price is defined as

$$\beta\lambda_{t+1,r,h}^{sim} = \lambda_{t,r,h}^{sim}, \tag{131}$$

where β is the discount factor defined in (22). Therefore, given a terminal value $\lambda_{T,r,h}$, the sequences of shadow prices can be determined. Given a vector of prices, taxes, and transfers and a vector of shadow prices, (128) and (129) define the paths of consumption and leisure. Based on those paths, (21) defines the households' welfare. Computing EV is therefore reduced to a problem of finding a value $\lambda_{T,r,h}^{ev}$ that, along with $\mathbf{z}_{r,h}^{base}$, leads to a level of welfare equal to the level in the policy simulation. Given this value, define

$$cd_{t,r,s,h}^{ev} = cd_{t,r,s,h}\left(\mathbf{z}_{r,h}^{base}, \boldsymbol{\lambda}_{r,h}^{ev}\right) \tag{132}$$

and

$$leis_{t,r,h}^{ev} = leis_{t,r,h}\left(\mathbf{z}_{r,h}^{base}, \boldsymbol{\lambda}_{r,h}^{ev}\right). \tag{133}$$

EV is then defined as

$$EV_{r,h} = expenditure\left(\mathbf{z}_{r,h}^{base}, \boldsymbol{\lambda}_{r,h}^{base}\right) - expenditure\left(\mathbf{z}_{r,h}^{base}, \boldsymbol{\lambda}_{r,h}^{ev}\right). \tag{134}$$

The value of EV defined in (134) is reported in the model output under the name `ev` and is based on the simulation years in the model and a finite time horizon. Two additional measures of EV are also standard outputs for a policy simulation. These include `ev_annual`, which linearly interpolates (130) between simulation years to estimate EV over all years covered by the policy simulation, and `ev_inf`, which extends `ev_annual` from a finite to an infinite time horizon based on the assumption that quantities and prices follow their steady state paths after the terminal period in the model.

5 Modeling Regulatory Requirements

Environmental regulations can vary over many dimensions and therefore, the appropriate approach to introduce the requirements of a regulation into the model will depend on the specific details of the policy. For example, EPA (2015) describe four categories of regulations commonly promulgated to address air pollution: single sector emission rate limits or technology standards; regional or state-implemented emission targets; multi-sector boiler or engine-level emission limits or technology standards; and federal product standards. Each of these categories has unique characteristics that may affect how the compliance requirements of the regulations and incentives created by the regulation should be modeled. Environmental regulations addressing additional pathways for pollution (e.g., land, water) have many similarities with the aforementioned categories but also have additional attributes that may be relevant for how they are modeled.

In practice, with the exception of federal product standards or prohibitions, environmental regulations are typically source-level technology standards, performance-based emission-rate limits, or workplace standards. In each case, sources are required to undertake abatement and monitoring activities in addition to their regular production activities. As a starting point for reflecting heterogeneity across regulatory approaches, the default version of the model has two built-in approaches for simulating abatement requirements on producers that may be calibrated to engineering or partial equilibrium estimates of compliance costs: a productivity shock and an explicit abatement activity. Each approach is discussed below, followed by an example that highlights the differences between them.

5.1 Compliance Requirements as a Productivity Shock

The production functions in the model, as described in equations (3)-(8), are of the calibrated share form in which the inputs are entered relative to the benchmark values. Or, in other words, the production function in (3)-(8) can be described as

$$
\begin{aligned}
y_{t,r,s,v} = f_y\Bigg(& f_{mat}\left(\frac{id_{t,r,agf,s,v}}{id0_{r,agf,s}}, \ldots, \frac{id_{t,r,srv,s,v}}{id0_{r,srv,s}}\right), \\
& f_{kle}\Bigg(f_{ene}\Bigg(\frac{id_{t,r,ele,s,v}}{id0_{r,ele,s}}, \\
& \qquad f_{en}\left(\frac{id_{t,r,col,s,v}}{id0_{r,col,s}}, \ldots, \frac{id_{t,r,gas,s,v}}{id0_{r,gas,s}},\right)\Bigg), \\
& \quad f_{kl}\left(\frac{kd_{t,r,s,v}}{kd0_{r,s}}, \frac{ld_{t,r,s,v}}{ld0_{r,s}}\right)\Bigg)\Bigg).
\end{aligned} \tag{135}
$$

To generalize this discussion, (135) introduces the index $v \in (new, extant)$ to describe the vintage of capital used in the production function. Under the case $v = extant$, (135) represents the Leontief production function for production with extant capital implicitly described by (13)-(16).

In the code an additional parameter, $prod\_ind_{t,r,z,s,v}$, is introduced to allow for modeling of a productivity shock on input z. The implementation essentially redefines (135) as

$$
\begin{aligned}
y_{t,r,s,v} = f_y\Bigg(& f_{mat}\left(\frac{id_{t,r,agf,s,v}}{prod\_ind_{t,r,agf,s,v}id0_{r,agf,s}}, \ldots, \frac{id_{t,r,srv,s,v}}{prod\_ind_{t,r,srv,s,v}id0_{r,srv,s}}\right), \\
& f_{kle}\Bigg(f_{ene}\Bigg(\frac{id_{t,r,ele,s,v}}{prod\_ind_{t,r,ele,s,v}id0_{r,ele,s}}, \\
& \qquad f_{en}\left(\frac{id_{t,r,col,s,v}}{prod\_ind_{t,r,col,s,v}id0_{r,col,s}}, \ldots, \frac{id_{t,r,gas,s,v}}{prod\_ind_{t,r,gas,s,v}id0_{r,gas,s}}\right)\Bigg), \\
& \quad f_{kl}\left(\frac{kd_{t,r,s,v}}{prod\_ind_{t,r,k,s,v}kd0_{r,s}}, \frac{ld_{t,r,s,v}}{prod\_ind_{t,r,l,s,v}ld0_{r,s}}\right)\Bigg)\Bigg),
\end{aligned} \tag{136}
$$

where in the baseline $prod\_ind_{t,r,z,s,v} = 1$, in which case (135) and (136) are equivalent.

The interpretation of this additional parameter is that increasing *prod_ind* for a specific input from 1 to $1 + \Delta$ and holding all other inputs fixed would require a $\Delta \times 100\%$ increase in the affected input to continue producing the baseline level. Based on this interpretation one can define an approach to calibrating the value of *prod_ind* to reflect the compliance requirements associated with a regulation. For example, suppose an engineering cost analysis estimates that a regulation impacting sector s will require additional expenditures on input z of $cost_{t,r}$ in year t and region r to produce the baseline level of output at new production sources, $y_{t,r,s,new}$. This may be represented by

$$
prod\_ind_{t,r,z,s,new} = 1 + \frac{cost_{t,r}}{(1+\tau_z)\, z0_{r,s}} \frac{y0_{r,s}}{y_{t,r,s,new}}, \tag{137}
$$

where $z0_{r,s}$ is the benchmark value of input z and τ_z represents any potential ad valorem tax on input z paid by producers (e.g., taxes on capital returns). This calibration would yield a situation where holding the output level and all other inputs fixed at their baseline levels, consistent with the setup in most engineering cost analyses, would require additional expenditures (gross of taxes) of $cost_{t,r}$ on input z. However, it should be noted that, after implementing the shock, firms may substitute away from the now less productive input towards other inputs. Based on the nature of the productivity shock these implicit substitution possibilities in the compliance activity are defined by the substitution elasticities in the regulated sector's production function.

5.2 Modeling Explicit Compliance Requirements

The model also allows for the explicit specification of input requirements for regulatory compliance. This is accomplished by extending the nesting structure of the production function depicted in Figures 3-5 to include a top level Leontief nest that combines production of saleable goods and services with pollution abatement activities. For the standard manufacturing and services production functions with new capital, this extended production function is presented in Figure 9. Production then requires both the traditional production activity and an abatement activity, which is itself a Leontief function of inputs used in regulatory compliance.

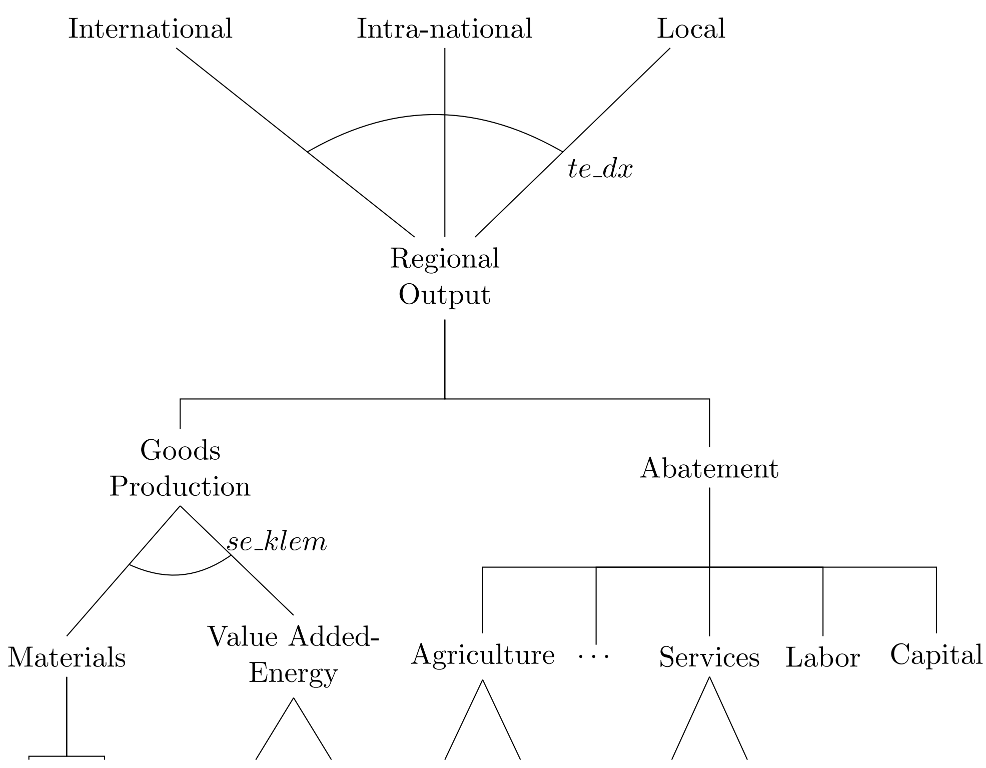


Figure 9: Manufacturing and Services Production Functions with Abatement

With the implementation of the extended production function to account for potential compliance activities, output in the manufacturing and service sectors that is not associated with a fixed factor resource is defined as

$$y_{t,r,s} = y0_{r,s} \min \left(\frac{klem_{t,r,s}}{klem0_{t,r,s}}, \frac{abate_{t,r,s}}{abate0_{t,r,s}} \right), \tag{138}$$

where $klem_{t,r,s}$ represents the traditional production activity defined in Section 2.2.1, such that

$$\begin{aligned} klem_{t,r,s} = klem0_{r,s} \Bigg[& cs\_klem_{r,s} \left(\frac{mat_{t,r,s}}{mat0_{t,r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} \\ & + (1 - cs\_klem_{r,s}) \left(\frac{kle_{t,r,s}}{kle0_{t,r,s}} \right)^{\frac{se\_klem-1}{se\_klem}} \Bigg]^{\frac{se\_klem}{se\_klem-1}}. \end{aligned} \tag{139}$$

The abatement activity is defined as a Leontief function of intermediate inputs, labor, and new capital, such that

$$abate_{t,r,s} = abate0_{t,r,s} \min \left(\frac{id\_abate_{t,r,agf,s}}{id\_abate0_{t,r,agf,s}}, \ldots, \frac{id\_abate_{t,r,srv,s}}{id\_abate0_{t,r,srv,s}}, \frac{ld\_abate_{t,r,s}}{ld\_abate0_{t,r,s}}, \frac{kd\_abate_{t,r,s}}{kd\_abate0_{t,r,s}} \right), \tag{140}$$

where $id\_abate_{t,r,ss,s}$, $ld\_abate_{t,r,s}$, and $kd\_abate_{t,r,s}$ are inputs of commodity ss, labor, and capital for abatement activities, respectively. The "benchmark" values in (140) include time subscripts because the required level of abatement activities or the inputs associated with the abatement activity may change over time, for example due to a phase in of the regulation. With the extended production function that includes abatement activities, firms are assumed to maximize profits inclusive of the abatement inputs,

$$\begin{aligned} & (1 - ty_{t,r,s})\, py_{t,r,s} y_{t,r,s} - \sum_{ss} pa_{t,r,ss} \left(id_{t,r,ss,s} + id\_abate_{t,r,ss,s} \right) \\ & - (1 + tk_{t,r})\, pr_{t,r} \left(kd_{t,r,s} + kd\_abate_{t,r,s} \right) - pl_{t,r} \left(ld_{t,r,s} + ld\_abate_{t,r,s} \right), \end{aligned} \tag{141}$$

subject to the production function defined by (138)-(140) and (4)-(8). Similar extensions are implemented for production with extant capital and sectors associated with fixed factor resources. In the case of production associated with extant capital, abatement activities are still assumed to make use of new capital in (140).

The solution approach outlined in Section 4 is easily extended to accommodate the expanded production structure inclusive of abatement activities. While the default implementation represents abatement activities as Leontief technologies, alternative functional forms can be adopted if warranted. It is also possible to represent abatement activities as substitutes for emissions in a CES function where the elasticity is calibrated to match available estimates of marginal abatement cost curves for emissions from the sector under study following Kiuila and Rutherford (2013), allowing more complex regulatory designs to be modeled.

5.3 Difference Between Productivity Shock and Explicit Compliance Requirements

There are two main differences between modeling compliance requirements as a productivity shock versus a nesting structure that explicitly represents the abatement activity. First, the substitution possibilities allowed between inputs for compliance differ. The productivity shock implicitly assumes that compliance inputs have the same substitution elasticities as the underlying production technology for the regulated sector. Alternatively, explicit representation of abatement requirements, at least as defined in Section 5.2 and the default version of the model, does not allow flexibility in how the abatement requirements are met. Second, the explicit abatement requirement assumes that any capital inputs for compliance activities are always new capital investments regardless of whether the regulation affects new or existing sources of production. For the productivity shock, capital requirements associated with compliance activities at existing sources are implicitly assumed to be repurposed extant capital.[33]

To highlight the main differences between these approaches to modeling regulatory requirements the example `examples/regulatory_modeling_approach.R` simulates an identically-specified regulation under both approaches. For this example, as well as the others presented in Section 6, we use a hypothetical regulation in the primary metal manufacturing (*pmm*) sector loosely calibrated to an initial round of regulations that were promulgated about 20 years ago under section 112 of the Clean Air Act. Section 112 of the Clean Air Act (CAA) requires the EPA to list industrial categories of major sources of one or more hazardous air pollutants (HAPs) and to then establish a national emissions standard for those categories (also referred to as a NESHAP). Major sources of HAPs are defined as new or existing facilities that emit 10 tons or more annually of any single HAP or 25 tons or more annually of a combination of HAPs. A NESHAP is typically based on an assessment of the degree to which emission reductions have been achieved at the best performing facilities in a particular source category using existing abatement control techniques. This standard is referred to as a Maximum Achievable Control Technology or MACT floor because it specifies the minimum level of HAPs control required. Specifically, the Clean Air Act requires the NESHAP to reflect the maximum degree of reduction in HAP emissions that is achievable, taking into consideration the cost of achieving the emission reductions (as well as a few other factors). For existing sources, the MACT floor is the average emission rate of the least-emitting 12 percent of facilities within that industry at the time of promulgation.

For primary metal manufacturing, it was estimated that the abatement technology available to meet the initial emission limits for integrated iron and steel manufacturing and primary and secondary aluminum manufacturing would require capital investments equivalent to approximately 0.4% of those sector's capital stock and annual operating costs equivalent to approximately 2.0% of those sector's labor expenditures at the time. Since the goal is to provide a hypothetical scenario to test the behavior of the model and not to develop quantitative impact estimates for a specific policy, we make many simplifying assumptions to keep the example as clear as possible. For example, costs

[33] For example, where part of an existing structure must be repurposed for compliance activities.

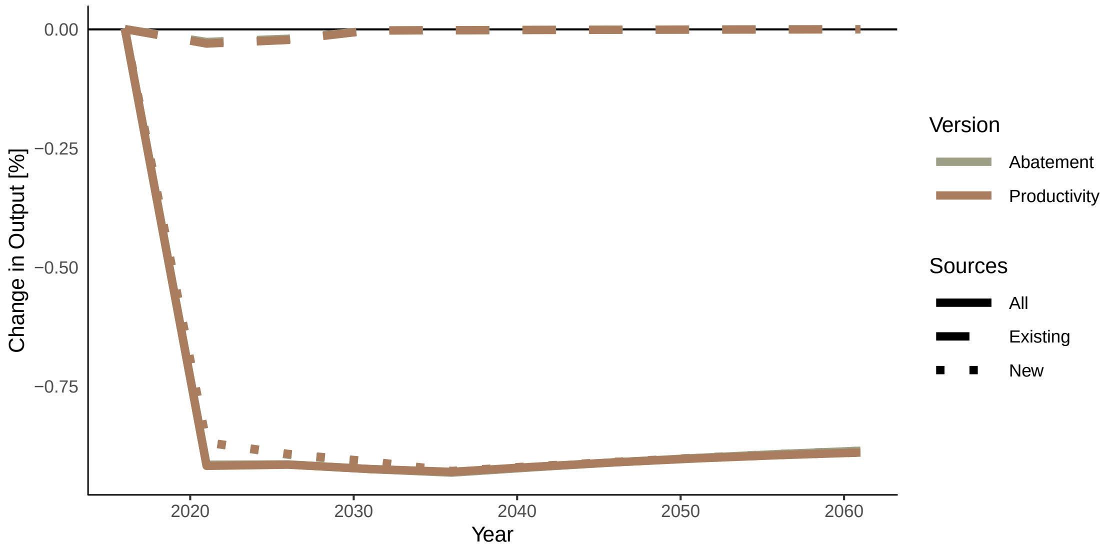
as a share of benchmark capital and labor inputs are assumed to be uniform across regions; costs scale with output over time; operating costs are assumed to be associated with labor only; new and existing sources are assumed to face the same compliance costs; the other primary metal production activities included in the default aggregated *pmm* sector are assumed to face similar compliance costs for abating HAP emissions; and the policy is assumed to begin in the second modeling period. The details of how this scenario is run are presented in Section 6.

For each modeling approach we consider three policy simulations in which the regulatory requirements apply to 1) all sources of production; 2) only production associated with extant capital; and 3) only production associated with new capital. As previously noted, there are two potential differences between the approaches to modeling abatement requirements: differences in the substitution possibilities in the abatement activity, and the vintage of capital required for compliance. When only production associated with new capital is subject to the regulatory requirements, then new capital is required for compliance in both cases, and differences between the two approaches are driven by varying assumptions about substitution possibilities in the abatement activity. On the other hand, because production associated with extant capital is modeled as Leontief, if only production associated with extant capital is subject to the regulatory requirements, then neither modeling approach provides any substitution possibilities in the abatement activity and differences across the two approaches are due to the vintage of capital required for compliance.

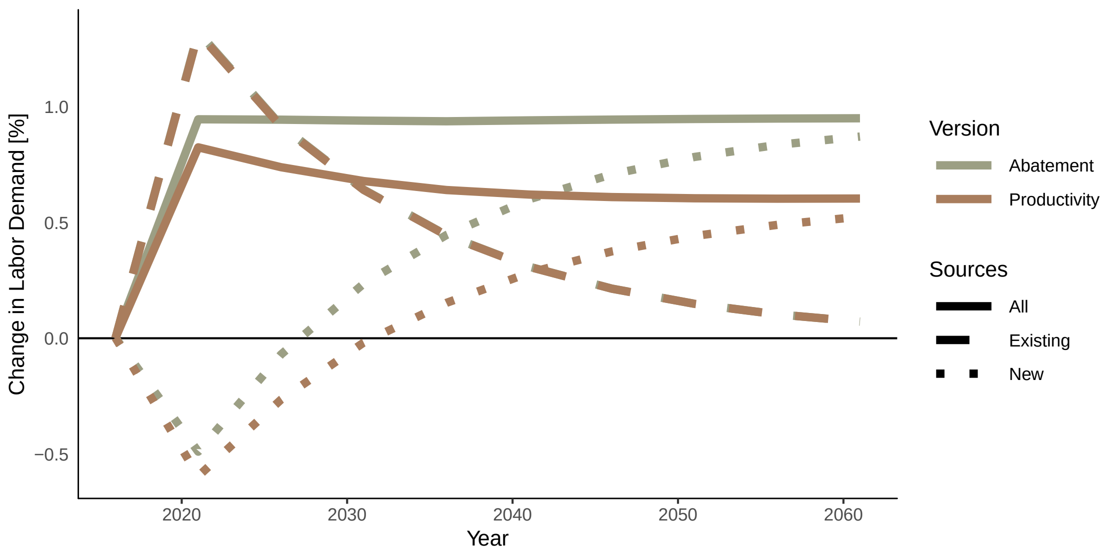
Figure 10 presents the simulated percent change in output and labor demand (inclusive of abatement requirements) for the regulated sector under the different scenarios. There is little difference in output between the two modeling approaches independent of which sources of production are subject to the abatement requirements. When the abatement requirements fall on existing sources and there is no difference in the assumptions about substitution possibilities in the abatement activity, the change in labor demand is approximately the same. In the other cases the labor intensity of production is lower in the case of the productivity shock relative to the explicit abatement requirement as firms substitute away from labor under this approach.

In a first best setting, we expect that the additional compliance flexibility assumed under the productivity shock approach would lower the social costs of the regulation, but in a second best setting the differential effect on the real wage rate leaves the direction of the difference ambiguous a priori. Table 12 presents estimates of EV under both approaches to representing the hypothetical regulation on the *pmm* sector (i.e., productivity shock and explicit abatement activity), varying which sources are affected (i.e., all, existing sources only, or new sources only).[34] For this example, regardless of which sources are affected, the EV for the two approaches to representing abatement requirements are within 0.5% of each other.

[34] The values presented represent the infinite time horizon approximation of EV discussed in Section 4.1 and contained in the output variable `ev_inf`.



(a) Change in Output in Regulated Sector



(b) Change in Labor Demand in Regulated Sector

Figure 10: Effect of Approach to Modeling Abatement Requirements

6 Using the Model

The core SAGE package is composed of 1) a build routine for constructing the model's database and 2) the modeling files for performing simulations. The programs are written to allow flexibility in how the datasets are constructed and provide options for including different modeling assumptions. The build routine constructs a consistent set of value shares based on IMPLAN data and compiles all other exogenous data parameters (including elasticities, growth rates, population totals, tax rates, and oil and gas production data) to form the necessary inputs to run the model. The SAGE model is run in a sequence and is intended to be used to compare the impacts of a policy shock

Table 12: EV Comparison Across Modeling Approaches [Billion $]

Affected Sources	Productivity Shock	Abatement Requirement
All	33.6	33.7
Existing	9.2	9.3
New	23.8	23.9

against a specified reference case. The user must run the model to calculate the baseline level of all model variables, design a policy shock that alters the reference equilibrium point, and rerun the model to compare the resulting equilibrium with baseline values for computing the economy-wide impacts.

6.1 Directory Structure

The build routine, model, and all examples are designed to be run from the package's top directory level. The package is composed of the subdirectories in Table 13.

Table 13: Directory Structure

Subdirectory	Description
`build`	Subdirectory contains data and files for constructing the benchmark dataset. The build routine relies on a mixture of GAMS and R routines. The launching program is called `build_default_datasets.R`
`utilities`	Subdirectory for custom R routines and functions for compiling external data sources and running model code. This code is referenced throughout the build routine and modeling examples.
`data`	Subdirectory containing reconciled benchmark data and exogenous parameter files.
`model`	Subdirectory containing the core SAGE modeling file.
`examples`	Subdirectory holding examples using the model.
`output`	Place holder subdirectory for generated model results.
`documentation`	Subdirectory containing documentation for the model. The documentation source is available in the latex file `documentation.tex`, while a typeset pdf version of the documentation is available in `documentation.pdf`.

6.2 Building the Dataset

The data compilation routines in the build stream includes programs written in both R and GAMS.[35] Some custom R routines are included (see the `utilities` subdirectory) to automate the download of external data sources and facilitate their subsequent compilation. External R packages used by SAGE but not currently available on the the system are automatically installed when the build stream is run, or may be installed separately by running `utilities/install_R_packages.R`.[36]

The build routine is controlled through the launch program, `build_default_datasets.R`, in the `build` subdirectory. The build script is designed to be run for the top-level directory of the SAGE package. Note that an internet connection is required for running the routine as it relies on external data sources for calibration.

The major steps in the build stream are as follow:

1. Initially, `build_parameters.R` compiles all parameters outside of the SAM (elasticities and growth rates) from included and downloaded data files and creates `data/parameters.gms`.

2. Additional external data sets needed for the creation of the SAM are downloaded and processed by `get_oil_and_gas_data.R` and `get_population_data.R`.

3. Data on effective marginal tax rates are processed by `get_tax_data.R`. Note that aside from reconciling data from the Current Population Survey, `get_tax_data.R` submits the compiled data to the NBER TAXSIM model to derive weighted marginal tax rates.

4. IMPLAN data is extracted using `build/data/implan` by `read_implan_state.gms`, which partitions each state data file into its submatrix components.[37]

5. Resulting state-level GDX outputs are merged and fed into `build_benchmark.gms` to create the SAM and disaggregate the oil and gas extraction sectors.

6. The SAM is aggregated to the requested levels in `aggregate_benchmark.gms` based on the aggregation defined in `aggregation_file` at the top of the launching program.

7. The SAM is filtered and rebalanced in `balance_benchmark.gms`, imposing microconsistency on the dataset (i.e. data satisfying all needed accounting identities in the modeling framework) as well as other calibration assumptions on the dynamic structure of the model. The aggregated dataset is balanced and filtered using a least squares optimization framework with options as listed in Table 14.

---

[35] R and GAMS must be included in the PATH environment variable. The current build stream was tested with GAMS 24.9 and R 3.5.

[36] Note that by default the package "fiftystater" is not installed, though this package is necessary for using the utility to create state choropleths with results or Figure 1. Instructions for how to install this package are included in `utilities/install_R_packages.R`.

[37] The process for constructing a SAM from the IMPLAN dataset is based on the IMPLANinGAMS package (Rausch and Rutherford, 2009). The original version of this software can be found at: `http://www.mpsge.org/implan98.htm`.

The build routine relies on both IMPLAN data and data from external sources (e.g., EIA and U.S. Census Bureau). The build stream requires that the state-level IMPLAN data files (`*.gms`) are stored in `build/data/implan` at the time of compilation.[38] All other data files are included with the SAGE package in `build/data` or downloaded from the internet throughout the routine. Following the successful completion of these routines, the file `build/data/satellite_data_versions.csv` is generated, which includes versioning information for downloaded data from API (Application Programming Interface) requests, the current population survey, and the TAXSIM model. Important options for controlling the build stream are located in the launch program and the SAM filtering and rebalancing script (`build/balance_benchmark.gms`). These options are described in Table 14 and are listed at the top of the associated programs. Once the build stream is finished, the resulting balanced dataset and generated parameters file containing all elasticities and assumed dynamic parameters are stored in the `data` subdirectory at the top level of the package's directory structure.

6.3 Running the Model

The model itself is written in GAMS and located in `model/sage.gms`. The model is designed to be run from the package's top directory and requires the PATH solver to be installed and licensed. The model is designed to run a single scenario, either a baseline scenario or a policy scenario. Therefore, the general process will be to first solve the model for the baseline solution and then rerun the model to solve for the counterfacutal policy solution. `sage.gms` is written to minimize the need for user adjustments to core model code (i.e. equations and data declarations) when solving counterfactual scenarios. Instead, the model offers multiple points during the execution where additional code can be included to change the specification or behavior of the model (examples below).

The model can be run through the GAMS IDE, but is designed to be run from the command line to take advantage of command line arguments to specify options including additional code to be included during counterfactual simulations. Running the model is done with a command line call to GAMS: `gams model/sage.gms`. The available command line options are presented in Table 15 and may be applied when running the model with the syntax `gams model/sage.gms --option1=choice1 --option2=choice2 ...`.

After each simulation all model variables are saved in both `.csv` and `.gdx` format. These files are written to the `output/` subdirectory. While these files are very similar in their contents, the CSV file contains additional output based on post-processing of the solution results, such as GDP, EV, etc. It is also worth noting that the quantity/activity variables in the GDX file are indices relative to the benchmark levels, while in the CSV file these indices have already been multiplied by the benchmark levels for the convenience of working with the output.

[38]IMPLAN is a proprietary data set and is therefore, not included in the publically available version of SAGE. Given a licensed version of IMPLAN for SAGE's benchmark year, to build the SAGE data sets first follow the instructions in `build/data/implan/implan_data_instructions.txt` to add the necessary IMPLAN data files into the SAGE directory structure.

Table 14: Selected Options in Data Set Build Stream

Place	Option	Description	Default Value
Launch	`aggregation_file`	Name of the mapping file that characterizes the level of sector, region, and household aggregation. Mapping files are located in `build/aggregation_map`. Alternative mappings can be used to modify the dimensionality of the dataset.	default_aggregation.gms
	`aeo.year`	Year of the EIA's AEO to use in the calibration.	Most recent AEO that includes benchmark year
	`aeo.scenario`	AEO scenario to use in the calibration.	`paste0("REF",aeo.year)`
	`balanced_growth`	Binary flag to calibrate benchmark investment levels consistent with a balanced growth path.	1
Matrix Balancing	`filter_small`	Binary flag to filter out small numbers.	1
	`include_taxes`	Binary flag to allow tax rate adjustments in balancing.	0
	`threshold`	Filter threshold for smallest value allowed in benchmark dataset.	5e-4
	`frac_deviations`	Binary flag to minimize percent deviations rather than absolute deviations.	1

Table 15: Command Line Options for the SAGE Model

Option	Description	Default Level
`benchmark_file`	File containing the benchmark dataset. The default aggregation is specified in Section 2.	`data/default_aggregation.gdx`
`putty_clay`	Binary flag for enabling the partial putty-clay specification. With a value of 0, capital is fully malleable.	1
`parameter_file`	File containing the exogenous assumptions including, elasticities, time steps, and baseline assumptions.	`data/parameters.gms`
`gdx_baseline_file`	A gdx file containing the results of a previous model solve. May be used to set the starting values and/or define baseline prices to calculate equivalent variation.	
`balanced_start_values`	Binary flag to set the starting values based on a balanced growth path solution independent of whether a baseline file was provided.	0
`policy_file`	Optional file containing GAMS code to define the policy changes in the model. If NULL the baseline is run.	
`gdx_save`	Binary flag for saving model results in a GDX file. The resulting GDX file is stored in the file specified by the environment variable `gdx_results_file`.	1
`gdx_results_file`	Provides the location and output name of the GDX file where the model results will be stored.	`output/results.gdx`
`output_file`	Provides the location and output name of the CSV file where the model results will be stored.	`output/results.csv`
`prologue`	Optional file containing GAMS code to be included *before* any data preprocessing and the model declaration. Useful for adjusting parameters in a sensitivity analysis.	
`epilogue`	Optional file containing GAMS code to be included *after* the model solution and any data post-processing. Useful for conducting additional post-processing of results.	
`perturb_start`	Debugging option to additively and uniformly perturb initial starting values on $y_{t,r,s}$. Value specifies the size of the perturbation.	0

6.4 Solution Checks

After each simulation the model performs a set of verification checks on the solution. The results of these post-solve diagnostics are reported in the GAMS listing file. This set of diagnostics includes checks that:

1. Nominal gross domestic product is the same when calculated based on expenditures and value added;
2. Accounting identities hold in the post-solve social accounting matrix; and
3. Aggregate household ownership of capital equals the installed capital stock (SAGE's excluded market clearance condition).

Following a simulation, a new social accounting matrix is constructed based on the computed post-policy equilibrium. This constructed matrix serves to verify that all of the accounting closures hold. These accounting closures include a check on commodities (the value of production and imports less exports must equal demand), activities (the value of production must equal the costs of labor, capital, intermediate inputs, and tax obligations), households (the value of consumption, investment, and tax payments must equal factor income and transfers), government (the value of government purchases less transfers equals tax income), and the rest of the world (the value of imports equals exports plus an exogenously defined balance of payments deficit). While the listing file contains the numerical values for each of these checks, for convenience it also reports a given check has "PASSED" or "FAILED" based on a selected tolerance, which has a default value of 10^{-4} (i.e., \$100,000).

6.4.1 Example of a Hypothetical Regulation

The file `examples/sample_abatement_requirement.gms` contains a representation of the hypothetical regulation in the primary metal manufacturing (*pmm*) sector that is described in Section 5.3. As previously stated, the hypothetical scenario assumes compliance with the regulation requires capital investments equivalent to approximately 0.4% of the regulated sector's capital stock and annual operating costs equivalent to approximately 2.0% of the regulated sector's labor expenditures; cost shares are uniform across regions; costs scale with output across time; operating costs are associated with labor only; and new and existing sources face the same compliance costs.

As noted in Table 15, the model provides options for including GAMS code in the model at multiple points during its compilation. The `policy_file` command line option allows the user to define a file containing GAMS code that will be included right before the model's solve statement. The file `examples/sample_abatement_requirement.gms` (see Listing 1) defines compliance requirements for the hypothetical regulation in the *pmm* sector based on the explicit abatement requirement approach of Section 5.2 and is intended to be used with the `policy_file` option.[39]

[39]For an example that implements the compliance requirements as a productivity shock, see `examples/regulatory_modeling_approch.R`.

Listing 1: `examples/sample_abatement_requirement.gms`

```
* the hypothetical regulation is assumed to affect the primary metal
* manufacturing sector and have an engineering cost estimate that compliance
* will require an additional 0.4% of baseline capital expenditures and 2% of
* labor expenditures. the requirements are assumed to begin in the second time
* period of the model. the requirements are assumed to be the same for
* production with new and extant capital.

ld_abate0(t,r,"pmm",v)$(ord(t) gt 1 or ord(t) eq card(t)) = 0.020*ld0(r,"pmm");
kd_abate0(t,r,"pmm",v)$(ord(t) gt 1 or ord(t) eq card(t)) = 0.004*kd0(r,"pmm");
```

The variables `ld_abate0(t,r,s,v)` and `kd_abate0(t,r,s,v)` define the labor expenditures and capital stock required for compliance when producing the benchmark level of output, `y0(r,s)`, in period t, region r, and sector s with capital of vintage v. There is an analogous variable for intermediate inputs of commodity ss for compliance, `id_abate0(t,r,ss,s,v)`. Care needs to be taken to ensure that the abatement costs are entered in the correct format, that is, the compliance costs at sources of vintage v when output from those sources is at the benchmark level.

In this example, the average compliance expenditures per unit of output in a region are assumed to remain constant over time. In addition, the compliance requirements are assumed to begin in the second period, hence the conditional `ord(t) gt 1`. The second part of the conditional is to ensure that the example will work with the static version of the model, which by definition only has one time period.

To analyze the hypothetical regulation the baseline is first calculated, after which the model is run with the hypothetical regulation. This can be accomplished from the command line using the commands presented in Listing 2.

Listing 2: Running the Sample Abatement Requirement from the Command Line

```
gams model/sage.gms --gdx_results_file=output/baseline.gdx
                    --output_file=output/baseline_results.csv

gams model/sage.gms --gdx_baseline_file=output/baseline.gdx
                    --policy_file=examples/sample_abatement_requirement.gms
                    --output_file=output/regulation_results.csv
```

In calculating the baseline in the first model run, the command line option `gdx_results_file` defines the GDX file where the results of the model solve will be saved. The command line option `output_file` defines a CSV file where the baseline results will be stored. The GDX file is used to define the baseline in the policy run using the `gdx_baseline_file` command line option. In this case, the baseline is used to both set the starting values and provides the baseline prices for calculating EV. The `output_file` command line option defines a CSV file where the results of the model run with the abatement requirement will be saved. The two output files may be used to calculate the changes in variables between the two simulations. Policy impacts should only be compared to their corresponding baseline.

Once the post policy equilibrium solution is determined, the SAGE listing file (`sage.lst`) will include the diagnostic checks described above (and can be found by searching the listing file for "Solution Check Results"). The diagnostics help determine if the model solution satisfies the necessary closures. Listing 3 presents part of the reported model diagnostics near the end of SAGE listing file. This represents an annual (here, 2016) snippet of the full set of diagnostic checks. Should the model fail to satisfy any of these requirements, "FAILED" will be listed next to the associated item, which means that the code included in the model was not properly specified. Passing the solution checks is a necessary but not sufficient condition to determine that the policy file has been properly specified.

Listing 3: Example diagnostics (`sage.lst`)

```
2016

----   2651 GDP check                     PASSED

----   2655 Commodity account check       PASSED

----   2659 Activity account check        PASSED

----   2663 Household account check       PASSED

----   2667 Government account check      PASSED

----   2671 Rest of World account check   PASSED

----   2675 Capital account check         PASSED
```

The model's use of command line options and compile time code inclusions allows the model to be easily run from scripts. The modeling package includes a series of R utilities in `utilities/R_utilities.R` that provide functions to run the model and process the results from R. The file `examples/basic_example.R` shows how this hypothetical abatement requirement may be run and results processed from an R script. An analogous example of how such routines can be built in GAMS is included in `examples/basic_example.gms`.

6.4.2 Additional Examples

Other examples are included in the `examples` subdirectory and are listed in Table 17. These relatively simple examples are designed to demonstrate basic features of the modeling framework and their general impact on simulation results. The examples are intended to be run from the top-level directory of the SAGE package. Most of the routines listed in Table 17 use the R programming language to conduct the simulations and process the results.[40]

[40] The scripts may be run from a development environment with an R backend or from the command line using Rscript (assuming it is available).

In addition to the simpler examples, there is a more extensive suite of simulations contained in the file `examples/scenario_analysis.R`, which conducts sensitivity analyses around hypothetical regulations implemented as productivity shocks similar to those considered in Marten et al. (2019). Note that the results in Marten et al. (2019) are based on SAGE v. 1.0.7 and a slightly different implementation of the hypothetical regulations, such that `examples/scenario_analysis.R` is not intended to replicate the quantitative results of that paper.[41]

Table 17: Additional Simulation Examples

File Name	Description
`static_vs_dynamic.R`	Compares the results from the sample abatement requirement using a dynamic vs. static version of the model.
`putty-clay_vs_putty-putty.R`	Compares the results of the sample abatement requirement under the default partial putty-clay capital framework vs the case of fully malleable capital under the putty-putty assumption.
`labor_supply_elasticity.R`	Simulates the substitution and income effect in the labor supply elasticity and plots the results over households and time against the calibration values as presented in Figure 8. This file uses `examples/labor_supply_elasticity.gms` as epilogue code.
`national_vs_regional.R`	Compares the results from the sample abatement requirement from the dynamic model with and without regional delineation.
`regulatory_modeling_approach.R`	Compares the output of the sample abatement requirement when modeled as a productivity shock vs. an explicit abatement requirement per unit of output. Produces the comparisons in Figure 10.
`scenario_analysis.R`	Runs sensitivity analyses around hypothetical regulations similar to those considered in Marten et al. (2019). Uses the file `examples/productivity_shock.gms` as the policy file to define a variety of hypothetical regulations as productivity shocks.

[41] For code to replicate the specific results of Marten et al. (2019), see the Dataverse site for the Journal of the Association of Environmental and Resource Economists.

References

Armington, P. S. (1969). A theory of demand for products distinguished by place of production. *Staff Papers (International Monetary Fund)*, 16(1):159–178.

Arora, V. (2014). Estimates of the price elasticities of natural gas supply and demand in the United States. Munich Personal RePEc Archive, MPRA Paper No. 54232.

Arora, V. and Cai, Y. (2014). US natural gas exports and their global impacts. *Applied Energy*, 120:95–103.

Babiker, M. H., Maskus, K. E., and Rutherford, T. F. (1997). Carbon taxes and the global trading system. University of Adelaide, Center for International Economic Studies, Seminar Paper 97-08.

Balistreri, E. J. and Rutherford, T. F. (2001). Dynamic general equilibrium analysis at the state level: Assessing the economic implications of the Kyoto Protocol. Presented at the 4th Annual Conference on Global Economic Analysis, Purdue University, June 27-29.

Baumeister, C. and Hamilton, J. D. (2019). Structural interpretation of vector autoregressions with incomplete identification: Revisiting the role of oil supply and demand shocks. *American Economic Review*, 109(5):1873–1910.

Beck, T., Jolly, L., and Loncar, T. (1991). Supply response in the Australian black coal industry (no. 91.1). Canberra, Australia: Australian Board of Agricultural and Resource Economics.

Beckman, J., Hertel, T., and Tyner, W. (2011). Validating energy-oriented CGE models. *Energy Economics*, 33(5):799–806.

Bjørnland, H., Nordvik, F., and Maximilian, R. (2017). Supply flexibility in the shale patch: Evidence from North Dakota. Norges Bank Research Working Paper 9.

Boetel, B. L., Hoffmann, R., and Liu, D. J. (2007). Estimating investment rigidity within a threshold regression framework: The case of US hog production sector. *American Journal of Agricultural Economics*, 89(1):36–51.

Böhringer, C., Carbone, J. C., and Rutherford, T. F. (2018). Embodied carbon tariffs. *The Scandinavian Journal of Economics*, 120(1):183–210.

Böhringer, C., Schneider, J., and Asane-Otoo, E. (2016). Trade in carbon and the effectiveness of carbon tariffs. University of Oldenburg, Department of Economics, Discussion Papers in Economics, No. V-388-16.

Borges, A. M. and Goulder, L. H. (1984). Decomposing the impact of higher energy prices on long-term growth. In Scarf, H. E. and Shoven, J. B., editors, *Applied General Equilibrium Analysis*. Cambridge University Press.

Brockway, P. E., Heun, M. K., Santos, J., and Barrett, J. R. (2017). Energy-extended CES aggregate production: Current aspects of their specification and econometric estimation. *Energies*, 10(2).

Brown, J. P., Fitzgerald, T., and Weber, J. G. (2016). Capturing rents from natural resource abundance: Private royalties from U.S. onshore oil & gas production. *Resource and Energy Economics*, 46:23 – 38.

Brown, S. P. and Huntington, H. G. (2003). Terms of trade and OECD policies to mitigate global climate change. *Federal Reserve Bank of Dallas Economic and Financial Policy Review*, 2(1):1–25.

Buhr, B. L. and Kim, H. (1997). Dynamic adjustment in vertically linked markets: the case of the US beef industry. *American Journal of Agricultural Economics*, 79(1):126–138.

Burniaux, J.-M. and Truong, T. P. (2002). GTAP-E: an energy-environmental version of the GTAP model. Purdue University, Center for Global Trade Analysis, GTAP Technical Paper No. 16.

Cai, Y., Newth, D., Finnigan, J., and Gunasekera, D. (2015). A hybrid energy-economy model for global integrated assessment of climate change, carbon mitigation and energy transformation. *Applied Energy*, 148:381–395.

Caldara, D., Cavallo, M., and Iacoviello, M. (2018). Oil price elasticities and oil price fluctuations. *Journal of Monetary Economics*, 103:1–20.

Capros, P., Van Regemorter, D., Paroussos, L., Karkatsoulis, P., Fragkiadakis, C., Tsani, S., Charalampidis, I., and Revesz, T. (2013). GEM-E3 model documentation. European Commission Joint Research Centre, JRC Technical Report 26034 EN.

Caron, J. and Rausch, S. (2013). A global general equilibrium model with US state-level detail for trade and environmental policy analysis–technical notes. MIT Joint Program on the Science and Policy of Global Change, Joint Program Technical Note 13.

CBO (2017). International comparisons of corporate income tax rates. U.S. Congressional Budget Office. `https://www.cbo.gov/system/files/115th-congress-2017-2018/reports/52419-internationaltaxratecomp.pdf`.

CBO (2018). The distribution of household income, 2014. U.S. Congressional Budget Office. `https://www.cbo.gov/system/files/115th-congress-2017-2018/reports/53597-distribution-household-income-2014.pdf`.

CEA (2017). Discounting for public policy: Theory and recent evidence on the merits of updating the discount rate. Council of Economic Advisers Issue Brief. `https://obamawhitehouse.archives.gov/sites/default/files/page/files/201701_cea_discounting_issue_brief.pdf`.

Chateau, J., Dellink, R., and Lanzi, E. (2014). An overview of the OECD ENV-Linkages model: Version 3. OECD Environment Working Papers, No. 65.

Coughlin, C. C. and Novy, D. (2013). Is the international border effect larger than the domestic border effect? Evidence from US trade. *CESifo Economic Studies*, 59(2):249–276.

Dahl, C. and Duggan, T. E. (1996). US energy product supply elasticities: A survey and application to the US oil market. *Resource and Energy Economics*, 18(3):243–263.

Dissou, Y., Karnizova, L., and Sun, Q. (2015). Industry-level econometric estimates of energy-capital-labor substitution with a nested CES production function. *Atlantic Economic Journal*, 43(1):107–121.

EIA (2001). Model documentation: Coal market module of the National Energy Modeling System. DOE/EIA-M060(2001).

EIA (2011). Performance profiles of major energy producers 2009. Report Number: DOE/EIA-0206(2009).

EPA (2015). Economy-wide modeling: Social cost and welfare white paper. `https://yosemite.epa.gov/sab/SABPRODUCT.NSF/0/07E67CF77B54734285257BB0004F87ED`.

Farzin, Y. H. (1992). The time path of scarcity rent in the theory of exhaustible resources. *The Economic Journal*, 102(413):813–830.

Feenberg, D. and Coutts, E. (1993). An introduction to the TAXSIM model. *Journal of Policy Analysis and Management*, 12(1):189–194.

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The next generation of the Penn World Table. *American Economic Review*, 105(10):3150–82.

Ferris, M. C. and Munson, T. S. (2000). Complementarity problems in GAMS and the PATH solver. *Journal of Economic Dynamics and Control*, 24(2):165–188.

Fitzgerald, T. and Rucker, R. R. (2016). US private oil and natural gas royalties: Estimates and policy relevance. *OPEC Energy Review*, 40(1):3–25.

Foley, P. and Clark, J. (1981). US copper supply: An economic/engineering analysis of cost—supply relationships. *Resources Policy*, 7(3):171–187.

Golombek, R., Hagem, C., and Hoel, M. (1995). Efficient incomplete international climate agreements. *Resource and Energy Economics*, 17(1):25–46.

Greene, D. L. and Liu, C. (2015). U.S. oil dependence 2014: Is energy independence in sight? *Energy Policy*, 85:126–137.

Ha, S. J., Lange, I., Lecca, P., and Turner, K. (2012). Econometric estimation of nested production functions and testing in a computable general equilibrium analysis of economy-wide rebound effects. University of Stirling, Division of Economics, Stirling Economics Discussion Papers 2012-08.

Haggerty, M., Lawson, M., and Pearcy, J. (2015). Steam coal at an arm's length: An evaluation of proposed reform options for US coal used in power generation. Manuscript, Headwaters Economics and Montana State University.

Havranek, T., Horvath, R., Irsova, Z., and Rusnak, M. (2015). Cross-country heterogeneity in intertemporal substitution. *Journal of International Economics*, 96(1):100–118.

Hendricks, N. P., Smith, A., and Sumner, D. A. (2014). Crop supply dynamics and the illusion of partial adjustment. *American Journal of Agricultural Economics*, 96(5):1469–1491.

Hertel, T., Hummels, D., Ivanic, M., and Keeney, R. (2007). How confident can we be of CGE-based assessments of free trade agreements? *Economic Modelling*, 24(4):611–635.

Hertel, T. W., McDougall, R., Narayanan, B., and Aguiar, A. H. (2008). Behavioural parameters. In Narayanan, B. and Walmsley, T. L., editors, *Global Trade, Assistance, and Production: The GTAP 7 Data Base*. Center for Global Trade Analysis, Purdue University.

Hertel, T. W. and Tsigas, M. (2002). Primary factor shares. In Dimaranan, B. V. and McDougall, R. A., editors, *Global Trade, Assistance, and Production: The GTAP 5 Data Base*. Center for Global Trade Analysis, Purdue University.

Higgs, P. J. and Powell, A. A. (1990). Forecasting small-area agricultural incomes using a CGE model of the Australian economy. *The Annals of Regional Science*, 24(1):43–59.

Holt, M. T. and Aradhyula, S. V. (1998). Endogenous risk in rational-expectations commodity models: A multivariate generalized ARCH-M approach. *Journal of Empirical Finance*, 5(2):99–129.

Holt, M. T. and McKenzie, A. M. (2003). Quasi-rational and ex ante price expectations in commodity supply models: An empirical analysis of the US broiler market. *Journal of Applied Econometrics*, 18(4):407–426.

Huntington, H. G. (1992). Inferred demand and supply elasticities from a comparison of world oil models. In Sterner, T., editor, *International Energy Economics*. International Studies in Economic Modelling, Springer, Dordrecht.

Iqbal, M. Z. and Babcock, B. A. (2018). Global growing-area elasticities of key agricultural crops estimated using dynamic heterogeneous panel methods. *Agricultural Economics*, 49(6):681–690.

ITC (2011). Magnesium from China and Russia. Investigation Nos. 731-TA-1071-1072 (Review) Publication 4214.

ITC (2014a). Gray portland cement and cement clinker from Mexico. Investigation No. 731-TA-451 (Final) Publication 2305.

ITC (2014b). Silica bricks and shapes from China. Investigation No. 731-TA-1205 (Final) Publication 4443.

Jomini, P., Zeitsch, J. F., McDougall, R., Welsh, A., Brown, S., Hambley, J., and Kelly, J. (1991). SALTER: A general equilibrium model of the world economy, Volume 1, Model structure, database and parameters. Industry Commission, Canberra.

Jordan, B., Lange, I., and Linn, J. (2018). Coal demand, market forces, and US coal mine closures. RFF Working Paper 18-13.

Kaiser, H. M. (2012). An economic analysis of the national pork board checkoff program. Report prepared for the National Pork Board.

Kapombe, C. M. and Colyer, D. (1998). Modeling US broiler supply response: A structural time series approach. *Agricultural and Resource Economics Review*, 27(2):241–251.

Keller, W. J. (1976). A nested CES-type utility function and its demand and price-index functions. *European Economic Review*, 7(2):175–186.

Kemfert, C. (1998). Estimated substitution elasticities of a nested CES production function approach for Germany. *Energy Economics*, 20(3):249–264.

Kim, H. and Moschini, G. (2018). The dynamics of supply: US corn and soybeans in the biofuel era. *Land Economics*, 94(4):593–613.

Kiuila, O. and Rutherford, T. F. (2013). Piecewise smooth approximation of bottom–up abatement cost curves. *Energy Economics*, 40:734–742.

Koesler, S. and Schymura, M. (2015). Substitution elasticities in a constant elasticity of substitution framework—Empirical estimates using nonlinear least squares. *Economic Systems Research*, 27(1):101–121.

Krichene, N. (2002). World crude oil and natural gas: A demand and supply model. *Energy Economics*, 24(6):557–576.

Lau, M. I., Pahlke, A., and Rutherford, T. F. (2002). Approximating infinite-horizon models in a complementarity format: A primer in dynamic general equilibrium analysis. *Journal of Economic Dynamics and Control*, 26(4):577–609.

Lecca, P., Swales, K., and Turner, K. (2011). An investigation of issues relating to where energy should enter the production function. *Economic Modelling*, 28(6):2832–2841.

Lin, C.-Y. C. and Wagner, G. (2007). Steady-state growth in a Hotelling model of resource extraction. *Journal of Environmental Economics and Management*, 54(1):68 – 83.

Liu, J., Arndt, C., and Hertel, T. W. (2004). Parameter estimation and measures of fit in a global, general equilibrium model. *Journal of Economic Integration*, 19(3):626–649.

Markandya, A. and Pedroso-Galinato, S. (2007). How substitutable is natural capital? *Environmental and Resource Economics*, 37(1):297–312.

Marsh, J. M. (2003). Impacts of declining US retail beef demand on farm-level beef prices and production. *American Journal of Agricultural Economics*, 85(4):902–913.

Marten, A. and Garbaccio, R. (2018). An applied general equilibrium model for the analysis of environmental policy: Sage v1.0 technical documentation. NCEE Working Paper 2018-05.

Marten, A. L., Garbaccio, R., and Wolverton, A. (2019). Exploring the general equilibrium costs of sector-specific regulations. *Forthcoming in Journal of the Association of Environmental and Resource Economists*.

Mathiesen, L. (1985). Computational experience in solving equilibrium models by a sequence of linear complementarity problems. *Operations research*, 33(6):1225–1250.

McClelland, R. and Mok, S. (2012). A review of recent research on labor supply elasticities. Congressional Budget Office, Working Paper 2012-12.

Narayanan, B., Aguiar, A., and McDougall, R. (2016). An overview of the gtap 9 data base. *Journal of Global Economic Analysis*, 1(1):181–208.

Paltsev, S., Reilly, J. M., Jacoby, H. D., Eckaus, R. S., McFarland, J., Sarofim, M., Asadoorian, M., and Babiker, M. (2005). The MIT Emissions Prediction and Policy Analysis (EPPA) Model: Version 4. Joint Program Report Series, Report 125.

Rausch, S., Metcalf, G. E., Reilly, J. M., and Paltsev, S. (2011). Distributional impacts of a U.S. greenhouse gas policy: A general equilibrium analysis of carbon pricing. In Metcalf, G. E., editor, *US Energy Tax Policy*. Cambridge University Press.

Rausch, S. and Rutherford, T. F. (2009). Tools for building national economic models using state-level implan social accounts. Unpublished manuscript. Available online at: http://www.cepe.ethz.ch/people/profs/srausch/IMPLAN2006inGAMS.pdf.

Roberts, M. J. and Schlenker, W. (2013). Identifying supply and demand elasticities of agricultural commodities: Implications for the US ethanol mandate. *American Economic Review*, 103(6):2265–95.

Ross, M. T. (2005). Documentation of the Applied Dynamic Analysis of the Global Economy (ADAGE) Model. RTI International Working Paper 05-01.

Ross, M. T. (2014). Structure of the Dynamic Integrated Economy/Energy/Emissions Model: Electricity Component, DIEM-Electricity. Duke University, Nicholas Institute for Environmental Policy Solutions, Working Paper NI WP 14-11.

Rucker, R. R., Burt, O. R., and LaFrance, J. T. (1984). An econometric model of cattle inventories. *American Journal of Agricultural Economics*, 66(2):131–144.

Russo, C., Green, R. D., and Howitt, R. E. (2008). Estimation of supply and demand elasticities of California commodities. Agriculture and Resource Economics Working Papers, Department of Agricultural and Resource Economics, UCD, UC Davis.

Rutherford, T. F. (1999). Applied general equilibrium modeling with MPSGE as a GAMS subsystem: An overview of the modeling framework and syntax. *Computational economics*, 14(1-2):1–46.

Sarmiento, C. and Allen, P. G. (2000). Dynamics of beef supply in the presence of cointegration: A new test of the backward-bending hypothesis. *Review of Agricultural Economics*, 22(2):421–437.

Serletis, A., Timilsina, G. R., and Vasetsky, O. (2010a). Interfuel substitution in the United States. *Energy Economics*, 32(3):737–745.

Serletis, A., Timilsina, G. R., and Vasetsky, O. (2010b). International evidence on sectoral interfuel substitution. *The Energy Journal*, 31(4):1–29.

Shideed, K. H. and White, F. C. (1989). Alternative forms of price expectations in supply analysis for US corn and soybean acreages. *Western Journal of Agricultural Economics*, 14(1836-2016-150808):281.

Simola, A. (2015). Intensive margin of land use in CGE models–Reviving CRESH functional form. *Land Use Policy*, 48:467–481.

Stern, D. I. (2012). Interfuel substitution: A meta-analysis. *Journal of Economic Surveys*, 26(2):307–331.

Su, X., Zhou, W., Nakagami, K., Ren, H., and Mu, H. (2012). Capital stock-labor-energy substitution and production efficiency study for China. *Energy Economics*, 34(4):1208–1213.

Sue Wing, I. (2001). *Induced technical change in computable general equilibrium models for climate-change policy analysis*. PhD thesis, Massachusetts Institute of Technology.

Sue Wing, I. (2006). The synthesis of bottom-up and top-down approaches to climate policy modeling: Electric power technologies and the cost of limiting US CO2 emissions. *Energy Policy*, 34(18):3847–3869.

Sue Wing, I., Daenzer, K., Fisher-Vanden, K., and Calvin, K. (2011). Phoenix Model Documentation.

Van der Werf, E. (2008). Production functions for climate policy modeling: An empirical analysis. *Energy Economics*, 30(6):2964–2979.

Young, A. T. (2013). US elasticities of substitution and factor augmentation at the industry level. *Macroeconomic Dynamics*, 17(04):861–897.

Zahniser, S., Hertz, T., Dixon, P., and Rimmer, M. (2012). The potential impact of changes in immigration policy on U.S. agriculture and the market for hired farm labor: A simulation analysis. U.S. Department of Agricuture, Economic Research Service, Economic Research Report Number 135.