# CALCULATION SUMMARY SHEET



PROJECT NUMBER: 143418
PROJECT NAME: USEN - Trench 13 Design
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**CALCULATION TITLE: Drainage Piping Strength Analysis**

**DESCRIPTION OF CALCULATION:**

Calculation to determine the necessary drainage pipe strength and characteristics for LCRS applications.

**REFERENCES USED:**

See attached list. .

**REVIEW COMMENTS:**

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| | | | |
|---|---|---|---|
| CALCULATION MADE BY: | PAZ | DATE: | 04/28/15 |
| CALCULATION CHECKED BY: | CWK | DATE: | 05/22/15 |
| CALCULATION REVISED BY: | PAZ | DATE: | 06/02/15 |
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# CALCULATION

## PURPOSE OF CALCULATION

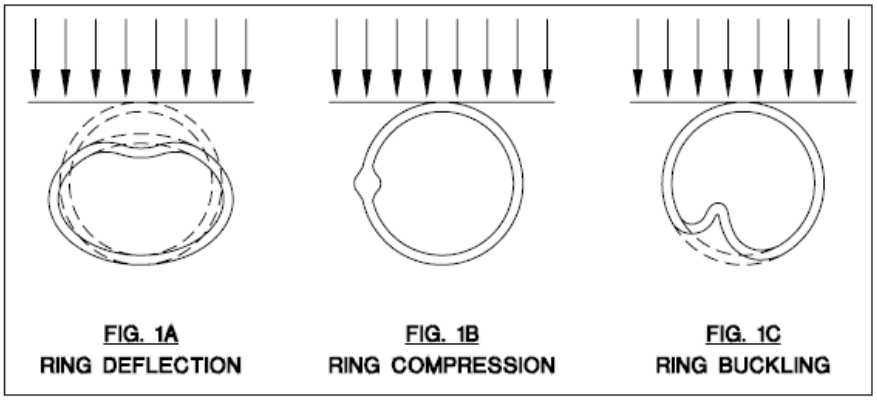
Calculate the anticipated loading scenarios to allow the selection of adequate drainage pipe used for leachate collection on the Trench floor and LDS sump piping standard dimension ratios (SDR).

## APPROACH

Evaluate the proposed High Density Polyethylene (HDPE) perforated drainage pipe for potential damage under two conditions:

- long-term, at full burial and maximum stress, and
- after construction, when cover over the piping is the thinnest and potential damage from vehicles is greatest.

HDPE pipe failure can occur through three modes. The following schematic illustrates them.




The polyethylene (PE) reference design handbook from Plastic Pipe.org (**Reference 1:** Plastic Pipe Institute, PE Design Handbook) was used for this analysis. It examines each of these failure modes.

For pipe burial depths greater than 50 feet, the design process consists of the following steps:

1. Determine the vertical soil pressure acting at the crown of the pipe due to earth, live, and surcharge loads.

2. Select a dimension ratio (DR) for the pipe to test.

3. Assign or estimate the properties for the crushed rock embedment material and degree of compaction. Since the crushed rock backfill cannot be compacted, an

estimated soil reaction value (E') roughly equivalent to a moderate Proctor compaction value will be used.

4. For the trial pipe strength and trial modulus of soil reaction, calculate the deflection due to the vertical soil pressure. Compare the pipe deflection to the deflection limit. If deflection exceeds the limit, two routes are possible:

   - increase the modulus of soil reaction, or
   - reduce the SDR

Repeat step 4 for the new E' and/or new trial pipe.

5. For the trial pipe and trial modulus of soil reaction, calculate the allowable soil pressure for wall crushing and for wall buckling. Compare the allowable soil pressure to the applied vertical pressure. If the allowable pressure is equal to or higher than the applied vertical pressure, the design is complete. If not, select a different pipe DR or heavier profile or different E', and repeat step 5.

One step that was not included in the above process was the evaluation of the impacts of perforating the pipe. The strength reduction resulting from pipe perforations was incorporated into the design analysis by reducing the unperforated HDPE material strength properties.

**Vertical Soil Pressure**

The weight of the waste, as well as surface loads above the pipe, produce pressure on the pipe. The waste weight is considered to be a "dead-load" while additional surface loads are referred to as "surcharge loads." Surcharge loads can be temporary (such as vehicle traffic) or permanent (such as structures, equipment, and piles of stored materials or debris). The soil pressures are calculated at the top of the pipe and summed together to obtain the total vertical soil pressure which is then used for calculating deflection and for comparison with wall crush and wall buckling performance limits.

**Waste Load**

The waste is assumed to be a uniform, homogeneous mass acting on the top of the pipe. A conservative assumption is that the entire waste load acts on the pipe crown ("prism load"). However, if the mass contains areas of varying stiffness (such as the difference between the pipe and surrounding gravel backfill), the weight of the mass will redistribute itself toward the stiffer areas due to internal shear resistance, and arching will occur. Arching in deep fills results in a reduction in load on the HDPE pipe.

One minor exception to this is when the pipe is only covered to a shallow depth and is exposed to dynamic loads, such as from vehicle traffic. The influence of live loads at a shallow burial depth after installation is examined in this calculation.

## LEACHATE COLLECTION PIPING ON THE TRENCH FLOOR

The following data are used in the calculation:

| Item | Value | Source |
|---|---|---|
| Perforated HDPE | 4 inch | Design drawing |
| Perforations | 3/8 inch diameter, 2 holes per lineal foot, 45 degrees below horizontal spacing | |
| SDR | 17 and 11 | |
| Minimum burial depth | 30 inches | |
| Maximum burial depth | 150 feet | |
| Maximum Waste density | 115 pcf | |
| Bedding material | 1-1.5 inch crushed rock | |
| Poisson's Ratio for crushed rock | 0.35 | CDOT, 2012 |
| Friction angle for crushed rock | 35 degrees | Clover, 1995, range from 30 to 45 degrees |
| Tensile strength, PE 4710 | 3500 psf | Iles and Eddy, 2014 |

Deep Burial with No Live Load

The deep burial mode assumes only the static waste load acts on the drainage pipe. The influence of vehicle traffic is insignificant at the final depth of waste placement.

*Failure Mode: Ring Compression*

The vertical load for the waste at maximum burial is calculated as:

$$Pe = wH = (100\ pcf)(70\ ft) + (115\ pcf)(80\ ft) = 16{,}200\frac{lb}{ft^2} = 112.5\ psi$$

Where:
Pe = earth load pressure
w = unit weight of the waste
H = height of the waste

The deep burial leads to reduction in the vertical loading from arching. The first step in the analysis is compressive ring thrust associated with the vertical arching factor (VAF). The VAF is calculated from:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5}$$

Where:
$S_A$ = hoop thrust stiffness ratio, and is calculated using:

$$S_A\frac{1.43M_s r_{cent}}{EA}$$

Where:
$M_s$ = modulus of soil, psi
$R_{cent}$ = radius to centroid of pipe, inches
E = modulus of elasticity for pipe material, psi

A = wall thickness for SDR pipe, inches

The centroid radius is calculated from the assumed material type and pipe dimensions. Pipe properties for 4 inch solid wall HDPE pipe (**Reference 1:** Plastic Pipe Institute, PE Design Handbook) are:

| OD | | | Pipe inside diameter (d) | Minimum Wall Thickness (t) | Weight (w) |
|---|---|---|---|---|---|
| Nominal in. | Actual in. | DR | in. | in. | lb. per foot |
| | | 7 | 3.14 | 0.643 | 3.384 |
| | | 7.3 | 3.19 | 0.616 | 3.269 |
| | | 9 | 3.44 | 0.500 | 2.737 |
| | | 9.3 | 3.47 | 0.484 | 2.660 |
| | | 11 | 3.63 | 0.409 | 2.294 |
| 4 | 4.500 | 11.5 | 3.67 | 0.391 | 2.204 |
| | | 13.5 | 3.79 | 0.333 | 1.906 |
| | | 15.5 | 3.88 | 0.290 | 1.678 |
| | | 17 | 3.94 | 0.265 | 1.540 |
| | | 21 | 4.05 | 0.214 | 1.262 |
| | | 26 | 4.13 | 0.173 | 1.030 |
| | | 32.5 | 4.21 | 0.138 | 0.831 |

For a 4 inch pipe with assumed SDR=17, the inside diameter is 3.94 inches. The wall thickness is 0.265 inches.

The long term modulus of elasticity for polyethylene is 28,200 psi (**Reference 1:** Plastic Pipe Institute, PE Design Handbook).

The radius to the centroid axis of the 4-inch HDPE ($D_o$ = 4.5 inches) is 2.25 inches

The $S_A$ is calculated as:

$$S_A = \frac{1.43 M_s r_{cent}}{EA} = \frac{1.43(3{,}000\ psi)(2.25\ inch)}{(28{,}200\ psi)(0.265\ inch)} = 1.29$$

The VAF is calculated as:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5} = 0.88 - 0.71\frac{1.29 - 1}{1.29 + 2.5} = 0.83$$

Therefore, the radial directed earth pressure can be found from

$$P_{RD} = (VAF)wH = (0.83)[(100\ pcf)(70\ ft) + (115\ pcf)(80\ ft)] = 13{,}450\ psf$$

For SDR 17 pipe, with no live load assumed because of the depth, the compressive stress in the pipe wall from the overlying waste is calculated to be

$$S = \frac{P_{RD} SDR}{288} = \frac{(13{,}450\ psf)(17)}{288} = 794\ psi$$

Since the backfill around the pipe is composed of sub-angular drain rock, the points impart a larger stress then calculated above. Brachman et al (2000) (**Reference 4:** Brachman, Local Strain on a Leachate Collection Pipe) suggest that a strain multiplier of 1.5 be used. The strain multiplier can be interpreted using:

$$E = \frac{\sigma}{\varepsilon} = 28{,}200 = \frac{794}{\varepsilon} = \varepsilon = 0.0282$$

$$0.0282 * 1.5 = 0.042$$

$$28{,}200 psi = \frac{\sigma}{0.042} = \sigma = 1{,}184\ psi$$

The perforation analysis involves evaluating the static and dynamic stresses that are put on the pipe. The effects of perforations weaken the pipe compared to a solid pipe, and must be considered in the design.

The Bureau of Reclamation (2009) examined pipe failure mechanisms associated with deep burial of PVC and HDPE in toe drains. The reduction in strength associated with perforations in smooth wall HDPE was found to be function of the percent open area of the perforations to the pipe surface.

The LCRS drain pipe perforation specification is for a minimal amount of open area, or 0.0057 inches$^2$ per foot of pipe. A greater amount of open area must be planned for the pipe to account for possible plugging over time. For conservatism, a 10 times multiplier is applied to the open area requirement, or 0.057 inches$^2$ per foot. While it does not matter for the strength assessment, for this calculation, the perforations are assumed to be spaced at 45 degrees below the horizontal plane. Estimating two 3/8" diameter holes drilled in the pipe, per foot. The total perforation open area would be:

$$OA = 2\pi \frac{d^2}{4} = 2\pi \frac{0.375^2}{4} = 0.22\ square\ inches$$

Where
OA = open area, square inches
d = diameter of each perforation, inches

The percent open area (POA) is calculated per linear foot of pipe as:

Where:
POA = percent open area
OA = perforation open area
$A_p$ = pipe surface area per linear foot

$D_O$ = pipe outside diameter, inches

$$POA = \frac{OA}{A_p} = \frac{0.22\ sq\ inches}{12\pi D_O} = \frac{0.22}{12\pi(4.5)} = 0.1\%$$

The Bureau of Reclamation report indicates that solid wall HDPE pipe experiences a strength reduction of about 15% for each percent open area. This value should be used cautiously, as it is only based on one test sample in the laboratory. With that in mind, for the perforation pattern in the design specifications, the reduction in strength is estimated to be:

$$S_R = POA * \frac{15\%}{1\%\ area} = 0.1 * 15 = 1.5\%$$

Where:
Sr = strength reduction from perforations

Given the following properties (**Reference 1:** Plastic Pipe Institute, PE Design Handbook)

**TABLE C.1**
**Allowable Compressive Stress for 73°F (23°C)**

| | Pe Pipe Material Designation Code (1) | | | | | |
|---|---|---|---|---|---|---|
| | PE 2406 | | PE3408 | | PE 4710 | |
| | PE 2708 | | PE 3608 | | | |
| | | | PE 3708 | | | |
| | | | PE 3710 | | | |
| | | | PE 4708 | | | |
| | psi | MPa | psi | MPa | psi | MPa |
| Allowable Compressive Stress | 800 | 5.52 | 1000 | 6.90 | 1150 | 7.93 |

(1) See Chapter 5 for an explanation of the PE Pipe Material Designation Code.

It can be seen that the maximum stress allowed for the perforation pattern using strength characteristics of the strongest HDPE (PE 4710) and reducing that strength by effects of the perforations is:

$$S_{PR} = S_p(1 - S_r) = 1{,}150\ psi\ (1 - 0.015) = 1{,}133\ psi$$

Where:
Spr = Allowable reduced pipe stress from perforations
Sp = Allowable pipe stress

Since the anticipated compressive wall stress for the SDR=17 under maximum waste burial conditions and angular rock backfill was 1,184 psi which is greater than the allowable compressive stress of 1,133 psi (PE4710), a new pipe SDR must be selected.

Repeat Approach Steps with SDR 11

Choosing SDR 11, with an inner diameter of 3.63 inches and wall thickness of 0.409 inches, the computed values for the compressive stress generated at maximum waste burial become:

$S_A$ = 0.837

VAF = 0.92

$P_{RD}$ = 14,823 psf

$$S = \frac{P_{RD} SDR}{288} = \frac{(14{,}823\ psf)(11)}{288} = 566\ psi$$

With the stronger SDR, the compressive wall stress becomes S = 566 psi. The angular rock backfill increases this stress to 846 psi. Since this is less than the $S_{PR}$ (at 1,133 psi) allowable compressive strength for the perforated pipe constructed of material HDPE 4710, the SDR 11 pipe passes.

*Failure Mode: Deflection*

Evaluation done for SDR 11 only since SDR 17 failed for ring compression.

The buried drainage pipe will deform under the weight of the waste to assume somewhat of an elliptical shape. This deformation slightly reduces the pipe's flow area. Practically speaking, this phenomenon can be considered negligible as it relates to pipe flow capacity. Deflections resulting in an elliptical deformation which reduces the pipe's vertical diameter by 7% results in a flow reduction of approximately 1% (**Reference 1:** Plastic Pipe Institute, PE Design Handbook). The deflection under final waste loading is examined using the Watkins-Gaube equation.

The secant modulus is calculated assuming a Poisson's Ratio, μ=0.35 (**Reference 2:** CDOT). A table of typical values of the soil modulus ($M_s$) for various vertical soil stress values provides guidance on the effect of compaction on the material surrounding the drainage pipe.

| Vertical Soil Stress[1] (psi) | Gravelly Sand/Gravels 95% Std. Proctor (psi) | Gravelly Sand/Gravels 90% Std. Proctor (psi) | Gravelly Sand/Gravels 85% Std. Proctor (psi) |
|---|---|---|---|
| 10 | 3000 | 1600 | 550 |
| 20 | 3500 | 1800 | 650 |
| 40 | 4200 | 2100 | 800 |
| 60 | 5000 | 2500 | 1000 |
| 80 | 6000 | 2900 | 1300 |
| 100 | 6500 | 3200 | 1450 |

**Reference 1:** Plastic Pipe Institute, PE Design Handbook, Table 3-12.

A literature value of $M_S$ = 3000 psi (**Reference 3:** National Engineering Handbook, Chapter 52) for a moderately compacted gravel backfill with fill of about 150 feet thick was used in the analysis. This value is supported by the other sources, including the table above, and is used in this analysis.

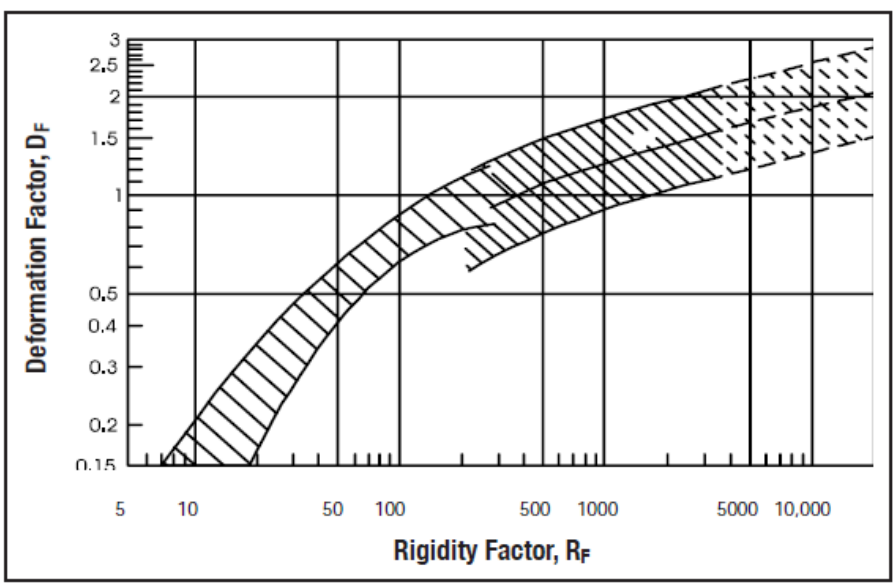
The secant modulus is calculated from:

$$E_s = \frac{M_s(1+\mu)(1-2\mu)}{1-\mu} = \frac{(3000\,psi)(1+0.35)(1-2(0.35))}{(1-0.35)} = 1{,}870\,psi$$

The rigidity factor is

$$R_F = \frac{12(E_s)(SDR-1)^3}{E} = \frac{12(1{,}870\,psi)(11-1)^3}{(28{,}200\,psi)} = 795\,psi$$

The deformation factor ($D_F$) is taken from the Watkins-Gaube graph.




$D_F$ is estimated to be about 1.2 from the graph.

The soil strain is calculated from the normal load imposed by the overlying waste. USEN reports an average placed waste density of 96.3 lbs/ft$^3$ (pcf). For conservatism in calculations, two waste densities were assumed; waste in the upper portion of the cell with a density of a 100 pcf, and waste in the lower portion of the cell that had been subjected to compaction with a slightly higher density of 115 pcf. The maximum design waste thickness in Trench 13 is 150 feet, the centerline leachate pipe must be able to withstand the combined load of the two waste densities. For this calculation, it is assumed that the upper 70 feet of waste are at the lower density and the lower 80 feet of waste are at the higher density, producing a load of about 16,200 pound per square foot. The soil strain, $\varepsilon_S$, is calculated as:

$$\varepsilon_S = \frac{wH}{144E_s} = \frac{(100\ pcf)(70\ ft) + (115\ pcf)(80\ ft)}{144(1{,}870\ psi)} = 0.060 = 6.0\%$$

So the deflections is estimated to be:

$$\frac{\Delta X}{D_M}(100) = D_F\varepsilon_s = (1.2)(6.0) = 7.2\%$$

Since the backfill around the pipe is composed of sub-angular crushed rock, the points impart a larger strain than that of a granular sand backfill. Brachman et al (2000) (**Reference 4:** Brachman, Local Strain on a Leachate Collection Pipe) suggest that a strain multiplier of 1.5 be used to account for that, yielding:

$$D_F\varepsilon_s = (1.2)(6.0)(1.5) = 10.8\%$$

This deflection would result in 2.4% reduction in flow capacity (**Reference 8:** Fisher, PVC Pipe News) of the drainage pipe, and is considered acceptable. Additionally, failures identified in the Bureau of Reclamation testing for solid wall perforated HDPE pipe occurred at 50% deflection (**Reference 5:** Bureau of Reclamation), far greater than the value calculated here.

*Failure Mode: Buckling*

Critical buckling is estimated from the Moore Selig equation:

$$P_{CR} = \frac{2.4\varphi R_H}{D_M}(EI)^{1/3}(E_s^*)^{2/3}$$

Where:
$P_{CR}$ = critical constrained buckling pressure, psi
$\varphi$ = calibration factor, 0.55 for granular soils
$R_H$ = geometry factor, (for deep burials, the Geometry Factor reverts to 1)
E = apparent modulus of elasticity of pipe material, psi
$I_{PW}$ = pipe wall moment of inertia
$E_s^*$ = Es/(1- $\mu$)
$E_s$ = secant modulus of the soil, psi
$\mu$ = Poisson's Ratio of Soil

$D_M$ = Mean diameter

Where the secant modulus

$$E_s^* = \frac{E_s}{(1-\mu)} = \frac{1{,}870\ psi}{(1-0.35)} = 2{,}877\ psi$$

And the moment of inertia on the pipe wall is calculated to be:

$$I_{PW} = \frac{t^3}{12} = \frac{0.409^3}{12} = 0.0057$$

Where t = SDR 11 wall thickness = 0.409 inch.

Substituting yields:

$$P_{CR} = \frac{2.4(0.55)(1)}{(4.5-0.409)}(28{,}200 * 0.0057)^{1/3}(2{,}877)^{2/3}$$

$$P_{CR} = 355\ psi$$

The design safety factor against buckling should be (**Reference 6:** National Engineering Handbook):

$$2.5 \text{ when } \frac{H}{\left(D_o/_{12}\right)} > 2 \text{ and } 3.0 \text{ when } \frac{H}{\left(D_o/_{12}\right)} < 2$$

For the final waste configuration, a Safety Factor =2.5 is selected because the equation value is:

$$\frac{150\ ft}{\left(4.5/_{12}\right)} = 400 > 2$$

The safety factor is then calculated against buckling to be:

$$SF = \frac{P_{CR}}{P_E} = \frac{(355)(144)}{(100\ pcf)(70\ ft) + (115\ pcf)(80\ ft)} = 3.2$$

Therefore, the 4” diameter HDPE perforated pipe with SDR 11 with material properties equal to or greater than PE 4710 is suitable for long term use.

**Shallow Burial with Live Load**

The live load from vehicles will be most significant when there is the minimum depth of daily cover and waste placed above the piping. The greatest live load will come from the heavy machinery and trucks as waste is placed and compacted. The pressure transmitted to the drainage pipe by a vehicle depends on the pipe’s burial depth, the vehicle’s weight, the tire pressure and size, vehicle speed, surface smoothness, the soil

surrounding the pipe (backfill) and cover, and the distance from the pipe to the point of loading (directly overhead, or to the side of the pipe crown).

The maximum load under a wheel occurs at the surface and diminishes with depth. Guidance suggests that HDPE pipes should be installed a minimum of one diameter or 18", whichever is greater, beneath the surface (**Reference 1:** Plastic Pipe Institute, PE Design Handbook). At this depth, the pipe is far enough below the wheel load to significantly reduce soil pressure and the pipe can fully utilize the embedment soil for load resistance. With a pipe of outer diameter of 4.5", the 30" operations layer represents almost 150% of the 18 inch minimum and more than 6 times the diameter. Vehicle loading influence is not anticipated to be an issue, as supported by the following calculations.

*Failure Mode: Deflection*

The shallow cover soil is assumed to be a compacted backfill with a density of 120 pcf. The vertical load after installation with a shallow soil cover of 30 inches (2.5 feet) is calculated as:

$$Pe = wH = (120\ pcf)(2.5\ ft) = 300\frac{lb}{ft^2} = 2.1\ psi$$

The deflection associated with the shallow cover is assumed to be negligible, so the next part of the analysis is the live load component.

For off-highway vehicles, it is generally necessary to calculate live load pressure from information supplied by the vehicle manufacturer regarding the vehicle weight or wheel load, tire footprint (contact area) and wheel spacing. The location of the vehicle's wheels relative to the pipe is also an important factor in determining how much load is transmitted to the pipe. Soil pressure under a point load at the surface is dispersed through the soil in both depth and expanse. Wheel loads not located directly above a pipe may apply pressure to the pipe, and this pressure can be significant. The load from two wheels straddling a pipe may produce a higher pressure on a pipe than from a single wheel directly above it, however, for the shallow condition on the drainage pipe after installation, the maximum soil pressure will occur when a single wheel (single or dual tire) is directly over the pipe.

The Timoshenko Equation can be used to find the pressure directly under a single wheel load. For deeper pipes, the maximum case often occurs when vehicles traveling above the pipe pass within a few feet of each other while straddling the pipe, or in the case of off-highway vehicles when they have closely space axles. The Boussinesq Equation can be used to find the pressure from wheels not directly above the pipe.

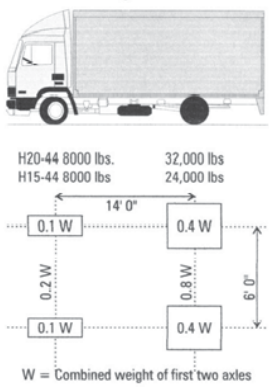
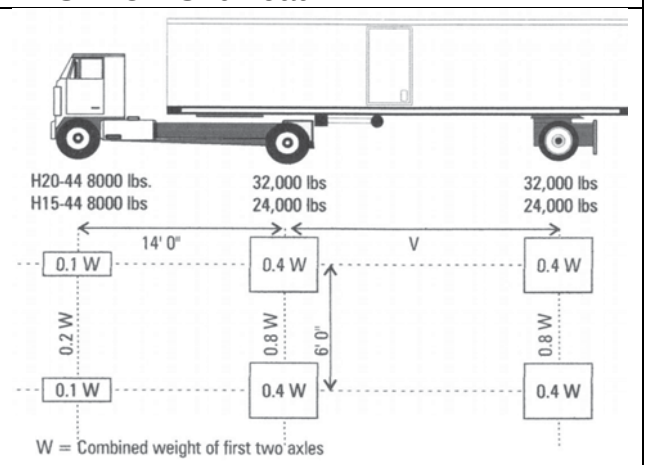
*Single Wheel Loading*

For a single wheel load, the Timoshenko Equation (**Reference 1:** Plastic Pipe Institute, PE Design Handbook) gives the soil pressure at a point directly under a tire:

$$P_L = \frac{I_f W_w}{a_c}\left(1 - \frac{H^3}{(r_T^2 + H^2)^{1.5}}\right)$$

Where:
$P_L$ = vertical soil pressure due to live load (psf)
$I_f$ = impact factor
$W_w$ = wheel load
$a_c$ = contact area, ft2
$r_T$ = equivalent radius, ft
H = depth of cover

Solving for a tractor trailer that may deliver waste to the site, the loading information is available for two different trucks

| AASHTO H20 Load | AASHTO HS20 Load |
|---|---|
|   |   |

The top of the placed waste typically is a very uneven surface, which can create impact loading as the vehicle bounces over the surface. Guidance from (**Reference 1:** Plastic Pipe Institute, PE Design Handbook) suggests an impact factor of 2.0 to 3.0 should be considered, except for slow traffic. Because the site maintains very slow traffic speeds, an impact factor $I_f$=2 was selected.

The truck tire contact area is assumed to be for a dual rear tire configuration. This has a contact area ($a_c$) of about 114 square inches, or 0.8 square feet (**Reference 7:** Yap**,** Truck and Tire Types and Road Contact Pressures). The wheel load ($W_W$) is estimated as 16000 pounds (0.4*the sum of the two axles, or 40,000 pounds).

The equivalent radius is calculated as:

$$r_T = \sqrt{\frac{a_c}{\pi}} = \sqrt{\frac{0.8}{\pi}} = 0.5\ ft$$

Substituting in the values yields:

$$P_L = \frac{(2)(16{,}000\ lbs)}{0.8}\left(1 - \frac{(2.5)^3}{((0.5\ ft)^2 + 2.5^2)^{1.5}}\right) = 2{,}285\ psf$$

*Between Axle Loading*

By comparison, the Bousinnesq equation is used to solve for the loading between the wheels:

$$P_L = \frac{3 I_f W_w H^3}{2\pi r^5}$$

Where:
$P_L$ = vertical soil pressure due to live load (psf)
$I_f$ = impact factor
$W_w$ = wheel load
H = depth of cover
r = distance from point of load to crown, ft

The distance is calculated from

$$r = \sqrt{X^2 + H^2}$$

The distance between the back two axles for a H20 vehicle is 14 feet, so for the loading to the crown pipe, the distance would be half or 7 feet.

Therefore, X = 7 feet, and r becomes

$$r = \sqrt{7^2 + 2.5^2} = 7.43\ ft$$

It is assumed that both the front axle and rear axle are contributing to the load, so from the axle configuration figure shown above:

$$W_w = 0.1W + 0.4W = 0.5(40{,}000\ pounds) = 20{,}000\ pounds$$

The live load between the wheels for a two axle waste truck is calculated as:

$$P_L = \frac{3(2)(20{,}000\ lbs)(2.5)^3}{2\pi(7.43)^5} = 13\ psf$$

This value is less than the single wheel load calculated by the Timenko equation, so that becomes the controlling live load.

The total load then is

$$P_T = P_E + P_L = 300 + 2{,}285 = 2{,}585\ psf$$

When the pipe is adequately buried, it is assumed to behave almost perfectly flexible with little ability to resist bending. At shallow cover depths, surcharge or live loads can

place a bending load on the pipe crown. In this case the pipe's flexural stiffness carries part of the load and prevents the pipe crown from dimpling inward under the load.

An analysis by Watkins (1977) gives the soil pressure that can be supported at the pipe crown by the combination of the pipe's flexural stiffness (bending resistance) and the soil's internal resistance against heaving upward. Watkins' equation applies to the condition after construction is completed, with the depth of cover soil greater than one-half of the pipe diameter and the pipe is installed at least 18 inches below the ground surface.

The solution follows:

$$P_{wat} = \frac{12w(KH)^2}{D_O} + \frac{7{,}387(I)}{D_O^2 c}\left(S_{MAT} - \frac{wD_O H}{288A}\right)$$

Where:
$P_{wat}$ = allowable live load pressure at pipe crown with thin cover, psf
w = soil weight
$D_O$ = pipe outside diameter, inches
H = depth of cover, ft
I = pipe wall moment of inertia, 0.0057 for 4-inch, IPS HDPE SDR 11
A = wall thickness for SDR pipe
c = outer fiber wall to centroid = 0.5A for SDR pipe
$S_{MAT}$ = material yield strength, psi, use 3500 psi for PE4710 material
K = passive earth coefficient

Where K is calculated as

$$K = \frac{1 + sin\theta}{1 - sin\theta}$$

And θ= angle of internal friction, degrees

For crushed stone, the low end will likely be θ = 35 degrees (Reference X: Clover, 1995, from Wikipedia), yields K = 3.69

Substituting into the equation yields

$$P_{wat} = \frac{12(120\ pcf)(3.69 * 2.5)^2}{(4.5)} + \frac{7{,}387(0.0057)}{4.5^2 0.5(0.409)}\left(3{,}500 - \frac{(120pcf)(4.5inches)(2.5ft)}{288(0.409\ inches)}\right)$$

$$P_{wat} = 27{,}230 + 10.2(3{,}488) = 62{,}813\ psf$$

The safety factor is

$$N_S = \frac{P_{wat}}{P_L} = \frac{62{,}813\ psf}{2{,}285\ psf} = 27$$

The safety factor shows that flexural stiffness of the drainage pipe is adequate to meet the live loads under shallow cover conditions. A safety factor of 3.0 for shallow cover is recommended in (**Reference 1:** Plastic Pipe Institute, PE Design Handbook).

For SDR pipe, with a live load and shallow depth, the compressive stress in the pipe wall from the overlying waste is calculated to be

$$S = \frac{P_T SDR}{288} = \frac{(2{,}585\ psf)(11)}{288} = 99\ psf$$

Since this is less than the $S_{PR}$ allowable compressive strength for pipe constructed of material HDPE 4710, it therefore passes with shallow cover soil and a live load. Therefore, PE 4710 SDR 11 is adequate to meet the live vehicle load with shallow cover on the perforated pipe.

*Failure Mode: Ring Compression*

The vertical load for the waste and live load from a single wheel is

$$P_T = P_E + P_L = 300\ psf + 2{,}285\ psf = 2{,}585\ psf$$

Because it is only shallow burial, it is not expected for significant arching to develop, so it is unlikely for there to be a reduction in the vertical stress. The first step in the analysis is compressive ring thrust associated with the vertical arching factor (VAF). The VAF is calculated from:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5}$$

Where:
$S_A$ = hoop thrust stiffness ratio, and is calculated using:

$$S_A \frac{1.43 M_s r_{cent}}{EA}$$

Where:
$M_s$ = modulus of soil, psi
$r_{cent}$ = radius to centroid of pipe, inches
E = modulus of elasticity for pipe material, psi
A = wall thickness for SDR pipe, inches

For a 4 inch pipe with assumed SDR=11, the inside diameter is 3.63 inches. The wall thickness is 0.409 inches. The radius to the centroid axis of the 4-inch HDPE ($D_o$ = 4.5 inches) is 2.25 inches.

The long term modulus of elasticity for polyethylene is 28,200 psi (**Reference 1:** Plastic Pipe Institute, PE Design Handbook). The soil modulus should be lower than the deep burial, compacted value (Ms=3000 psi), but greater than a loose dump value (Ms=1,000 psi), so an average was estimated to be Ms =2,000 psi

The $S_A$ is calculated as:

$$S_A = \frac{1.43 M_s r_{cent}}{EA} = \frac{1.43(2{,}000\ psi)(2.25\ inch)}{(28{,}200\ psi)(0.409\ inch)} = 0.56$$

The VAF is calculated as:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5} = 0.88 - 0.71\frac{0.56 - 1}{0.56 + 2.5} = 0.98$$

Therefore, the influence of arching is negligible.

For SDR pipe, with the live load assumed from the vehicle traffic, the compressive stress in the pipe wall from the overlying waste is calculated to be

$$S = \frac{P_T SDR}{288} = \frac{(2{,}585 psf)(11)}{288} = 99\ psf$$

It can be seen that the maximum stress allowed for the perforation pattern using strength characteristics of the HDPE with a strength of 1,150 psi (PE4710) is:

$$S_{PR} = S_p(1 - S_r) = 1{,}150\ psi\ (1 - 0.015) = 1{,}133\ psi$$

Since the anticipated compressive wall stress for the SDR=11 under minimum cover and focused live load conditions was 99 psi and less than the allowable compressive strength of 1,133 psi, the drainage pipe with SDR 11 meets the strength requirements.

**Summary for Leachate Collection Piping on the Trench Floor**

The approach was to evaluate the proposed High Density Polyethylene (HDPE) perforated drainage pipe necessary for leachate drainage in the floor of Trench 13 for potential damage under two conditions:

- long term, at full burial and maximum stress, and
- after construction, when soil cover over the piping is the thinnest and potential damage from vehicle surcharge loads is greatest.

The analysis showed that the long term condition under maximum burial conditions yielded the greatest potential for piping damage, and was the criteria for pipe dimensions and perforation patterns. Using a SDR=11 (PE 4710) for a 4-inch HDPE and a perforation pattern with two 3/8 inch holes per linear foot spaced at @ 45 degrees below the horizontal plane, the pipe yielded a satisfactory factor of safety.

## LDS HORIZONTAL COLLECTION PIPE

The pipe burial analysis was repeated for the LDS collection pipe located in the base of the LDS sump. Based on the findings of the leachate collection piping on the trench floor, the long term condition under maximum burial condition yields the greatest

potential for piping damage. For the LDS horizontal collection pipe with ½-inch perforations at 24 per foot in the sump, this condition is evaluated.

| Item | Value | Source |
|---|---|---|
| Perforated HDPE | 12 inch | Design drawing |
| Perforations | 1/2 inch diameter, 24 holes per foot, 6 holes @ 60 degrees around pipe every 3 inches | Per Trench 12 design and proposed for Trench 13 design |
| SDR | 11 | |
| Minimum burial depth | 5 feet | |
| Maximum burial depth | 100 feet | |
| Maximum Waste density | 115 pcf | |
| Bedding material | 1-1.5 inch crushed rock | |
| Poisson's Ratio for crushed rock | 0.35 | CDOT, 2012 |

**Deep Burial with No Live Load**

The deep burial mode assumes only the static waste load acts on the drainage pipe. The influence of vehicle traffic is insignificant at the final depth of waste placement.

*Failure Mode: Ring Compression*

The vertical load for the waste at maximum burial is calculated as:

$$Pe = wH = (100\,pcf)(20\,ft) + (115\,pcf)(80\,ft) = 11{,}200\frac{lb}{ft^2} = 77.8\,psi$$

Where:
Pe = earth load pressure
w = unit weight of the waste
H = height of the waste

The deep burial leads to reduction in the vertical loading from arching. The first step in the analysis is compressive ring thrust associated with the vertical arching factor (VAF). The VAF is calculated from:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5}$$

Where:
$S_A$ = hoop thrust stiffness ratio, and is calculated using:

$$S_A\frac{1.43M_s r_{cent}}{EA}$$

Where:
$M_s$ = modulus of soil, psi
$r_{cent}$ = radius to centroid of pipe, inches
E = modulus of elasticity for pipe material, psi
A = wall thickness for SDR pipe, inches

The centroid radius is calculated from the assumed material type and pipe dimensions. Pipe properties for 12 inch solid wall HDPE pipe (**Reference 1:** Plastic Pipe Institute, PE Design Handbook) are:

| | | | | | |
|---|---|---|---|---|---|
| | | 7 | 9.20 | 1.886 | 29.121 |
| | | 9 | 10.09 | 1.467 | 23.548 |
| | | 11 | 10.66 | 1.200 | 19.734 |
| | | 13.5 | 11.13 | 0.978 | 16.397 |
| 12 | 13.200 | 15.5 | 11.39 | 0.852 | 14.439 |
| | | 17 | 11.55 | 0.776 | 13.250 |
| | | 21 | 11.87 | 0.629 | 10.862 |
| | | 26 | 12.12 | 0.508 | 8.863 |
| | | 32.5 | 12.34 | 0.406 | 7.151 |

For a 12 inch pipe with assumed SDR=11, the inside diameter is 10.66 inches. The wall thickness is 1.200 inches.

The long term modulus of elasticity for polyethylene is 28,200 psi (**Reference 1:** Plastic Pipe Institute, PE Design Handbook).

The radius to the centroid axis of the 12-inch HDPE ($D_o$ = 13.2 inches) is 6.51 inches

The $S_A$ is calculated as:

$$S_A = \frac{1.43 M_s r_{cent}}{EA} = \frac{1.43(3{,}000\ psi)(6.51\ inch)}{(28{,}200\ psi)(1.200\ inch)} = 0.83$$

The VAF is calculated as:

$$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5} = 0.88 - 0.71\frac{0.83 - 1}{0.83 + 2.5} = 0.92$$

Therefore, the radial directed earth pressure can be found from

$$P_{RD} = (VAF)wH = (0.92)[(100\ pcf)(20\ ft) + (115\ pcf)(80\ ft)] = 10{,}300\ psf$$

For SDR pipe, with no live load assumed because of the depth, the compressive stress in the pipe wall from the overlying waste is calculated to be

$$S = \frac{P_{RD} SDR}{288} = \frac{(10{,}300\ psf)(11)}{288} = 390\ psi$$

Since the backfill around the pipe is composed of sub-angular drain rock, the points impart a larger stress then calculated above. Brachman et al (2000) (**Reference 4:** Brachman, Local Strain on a Leachate Collection Pipe) suggest that a strain multiplier of 1.5 be used. The strain multiplier can be interpreted using:

$$E = \frac{\sigma}{\varepsilon} = 28{,}200 = \frac{390}{\varepsilon} = \varepsilon = 0.014$$

$$0.014 * 1.5 = 0.021$$

$$28{,}200 = \frac{\sigma}{0.021} = \sigma = 584\, psi$$

The perforation analysis involves evaluating the static and dynamic stresses that are put on the pipe. The effects of perforations weaken the pipe compared to a solid pipe, and must be considered in the design.

The Bureau of Reclamation (2009) examined pipe failure mechanisms associated with deep burial of PVC and HDPE in toe drains. The reduction in strength associated with perforations in smooth wall HDPE was found to be function of the percent open area of the perforations to the pipe surface.

The LDS horizontal collection pipe perforation specification is for a 24 – ½ inch holes every lineal foot, a total of 4.7 square inches per lineal foot. While it does not matter for the strength assessment, for this calculation, the perforations are assumed to be uniformly spaced 6 holes @ 60 degrees around the pipe every 3 inches. The total perforated open area would be:

$$OA = 24\pi\frac{d^2}{4} = 24\pi\frac{0.5^2}{4} = 4.71\ square\ inches$$

Where
OA = open area, square inches
d = diameter of each perforation, inches

The percent open area (POA) is calculated per linear foot of pipe as:

$$POA = \frac{OA}{A_p} = \frac{4.71\ sq\ inches}{12\pi D_O} = \frac{4.71}{12\pi(13.2)} = 1.0\%$$

Where:
POA = percent open area
OA = perforation open area
$A_p$ = pipe surface area per linear foot
$D_O$ = pipe outside diameter, inches

The Bureau of Reclamation (**Reference 5**) report indicates that solid wall HDPE pipe experiences a strength reduction of about 15% for each percent open area. This value should be used cautiously, as it is only based on one test sample in the laboratory. With that in mind, for the perforation pattern in the design specifications, the reduction in strength is estimated to be:

$$S_R = POA * \frac{15\%}{1\%\ area} = 1.0 * 15 = 15\%$$

Where:
Sr = strength reduction from perforations

Given the following properties (**Reference 1:** Plastic Pipe Institute, PE Design Handbook)

**TABLE C.1**
**Allowable Compressive Stress for 73°F (23°C)**

| | Pe Pipe Material Designation Code (1) | | | | | |
|---|---|---|---|---|---|---|
| | PE 2406 | | PE3408 | | PE 4710 | |
| | PE 2708 | | PE 3608 | | | |
| | | | PE 3708 | | | |
| | | | PE 3710 | | | |
| | | | PE 4708 | | | |
| | psi | MPa | psi | MPa | psi | MPa |
| Allowable Compressive Stress | 800 | 5.52 | 1000 | 6.90 | 1150 | 7.93 |

(1) See Chapter 5 for an explanation of the PE Pipe Material Designation Code.

It can be seen that the maximum stress allowed for the perforation pattern using strength characteristics of the strongest HDPE (PE 4710) and reducing that strength by effects of the perforations is:

$$S_{PR} = S_p(1 - S_r) = 1{,}150\ psi\ (1 - 0.15) = 980\ psi$$

Where:
Spr = Allowable reduced pipe stress from perforations
Sp = Allowable pipe stress

Since the anticipated compressive wall stress for the SDR 11 under maximum waste burial conditions was 584 psi, less than the allowable reduced pipe stress from perforations of 980 psi, therefore SDR 11 will be sufficient.

*Failure Mode: Deflection*

The secant modulus is calculated assuming a Poisson's Ratio, μ=0.35 (**Reference 2:** CDOT). A table of typical values of the soil modulus ($M_s$) for various vertical soil stress values provides guidance on the effect of compaction on the material surrounding the LDS collection pipe. The LDS sump is backfilled drain rock. For this evaluation, gravel drain rock is approximately 1-1.5 inch gravel drain rock.

A literature value of $M_S$ = 3000 psi (**Reference 3:** National Engineering Handbook, Chapter 52) for crushed rock with moderate compaction.

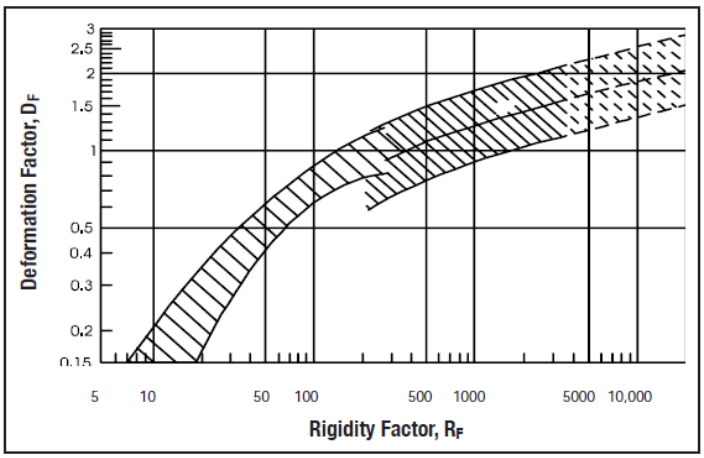
The secant modulus is calculated from:

$$E_s = \frac{M_s(1+\mu)(1-2\mu)}{1-\mu} = \frac{(3{,}000\,psi)(1+0.35)(1-2(0.35))}{(1-0.35)} = 1{,}870\,psi$$

The rigidity factor is

$$R_F = \frac{12(E_s)(SDR-1)^3}{E} = \frac{12(1{,}870\,psi)(11-1)^3}{(28200)} = 795\,psi$$

The deformation factor ($D_F$) is taken from the Watkins-Gaube graph.




$D_F$ is estimated to be about 1.2 from the graph.

As specified for the leachate collection piping, two waste densities were assumed; waste in the upper portion of the cell with a density of a 100 pcf, and waste in the lower portion of the cell that had been subjected to compaction with a slightly higher density of 115 pcf. The maximum design waste thickness in Trench 13 above he LDS horizontal drain pipe is 100 feet and the LDS horizontal drain pipe must be able to withstand the maximum design waste thickness. For this calculation, it is assumed that the upper 20 feet of waste are at the lower density and the lower 80 feet of waste are at the higher density, producing a load of about 11,200 pound per square foot. The soil strain, $\varepsilon_s$, is calculated as:

$$\varepsilon_S = \frac{wH}{144E_s} = \frac{(100\,pcf)(20\,ft) + (115\,pcf)(80\,ft)}{144(1{,}870\,psi)} = 0.042 = 4.2\%$$

So the deflections is estimated to be:

$$\frac{\Delta X}{D_M}(100) = D_F\varepsilon_s = (1.2)(4.2) = 5.0\%$$

Since the backfill around the pipe is composed of sub-angular crushed rock, the points impart a larger strain than that of a granular sand backfill. Brachman et al (2000)

(**Reference 4:** Brachman, Local Strain on a Leachate Collection Pipe) suggest that a strain multiplier of 1.5 be used to account for that, yielding.

$$D_F \varepsilon_s = (1.2)(4.2)(1.5) = 7.6\%$$

This deflection would result in 1.4% reduction in flow capacity (**Reference 8:** Fisher, PVC Pipe News) of the drainage pipe, and is considered acceptable. Additionally, failures identified in the Bureau of Reclamation testing for solid wall perforated HDPE pipe occurred at 50% deflection (**Reference 5:** Bureau of Reclamation), far greater than the value calculated here.

*Failure Mode: Buckling*

Critical buckling is estimated from the Moore Selig equation:

$$P_{CR} = \frac{2.4\varphi R_H}{D_M}(EI)^{1/3}(E_s^*)^{2/3}$$

Where:
$P_{CR}$ = critical constrained buckling pressure, psi
$\varphi$ = calibration factor, 0.55 for granular soils
$R_H$ = geometry factor, (for deep burials, the Geometry Factor reverts to 1)
E = apparent modulus of elasticity of pipe material, psi
$I_{PW}$ = pipe wall moment of inertia
$E_s$* = Es/(1- $\mu$)
$E_s$ = secant modulus of the soil, psi
$\mu$ = Poisson's Ratio of Soil
$D_M$ = Mean diameter

Where the secant modulus

$$E_s^* = \frac{E_s}{(1-\mu)} = \frac{1{,}870\ psi}{(1-0.35)} = 2{,}877 psi$$

And the moment of inertia on the pipe wall is calculated to be:

$$I_{PW} = \frac{t^3}{12} = \frac{1.200^3}{12} = 0.144$$

Where t = SDR 11 wall thickness = 1.200 inch.

Substituting into the equation:

$$P_{CR} = \frac{2.4(0.55)(1)}{(13.2-1.200)}(28{,}200 * 0.144)^{1/3}(2{,}877)^{2/3}$$

$$P_{CR} = 355\ psi$$

The design safety factor against buckling should be (**Reference 6:** National Engineering Handbook):

$$2.5 \text{ when } \frac{H}{\left(D_o/_{12}\right)} > 2 \text{ and } 3.0 \text{ when } \frac{H}{\left(D_o/_{12}\right)} < 2$$

For the final waste configuration, a Safety Factor =2.5 is selected because the equation value is:

$$\frac{100\ ft}{\left(13.2/_{12}\right)} = 91 > 2$$

The safety factor is then calculated against buckling to be:

$$SF = \frac{P_{CR}}{P_E} = \frac{(355psi)(144)}{(100\ pcf)(20\ ft) + (115\ pcf)(80\ ft)} = 4.6$$

Therefore, the 12" diameter HDPE perforated pipe of SDR 11 with material properties equal to or greater than PE 4710 is suitable for long term use.

**Summary for LDS Horizontal Collection Pipe**

Based on the evaluation of leachate drainage piping in the floor of Trench 13, only SDR 11 pipe was considered for the horizontal collection pipe in the LDS. The calculation evaluated the proposed High Density Polyethylene (HDPE) perforated LDS pipe for potential damage under long term conditions, at full burial and maximum stress.

The analysis showed that the long term condition under maximum burial conditions yielded the greatest potential for piping damage, and was the criteria for pipe dimensions and perforation patterns. Using a SDR=11 (PE 4710) for a 12-inch HDPE and a perforation pattern with 24 ½-inch holes per linear foot spaced at @ 60 degrees around the pipe yielded a satisfactory factor of safety.

**CONCLUSION**

The LCRS centerline and lateral drain pipes shall be 4 inch diameter SDR 11 HDPE perforated pipe, with material properties equal or greater than PE 4710. The specified perforations evaluated included two 3/8-inch holes equally spaced at 45 degrees below the horizontal plane, totaling 0.22 square inches of perforation every lineal foot. The pipes shall be backfilled with drain rock and placed according to the design drawings.

The LDS horizontal collection pipe shall be 12 inch diameter SDR 11 HDPE perforated pipe, with material properties equal or greater than PE 4710. The specified perforations shall 24 ½-inch holes per lineal foot, or six ½-inch holes spaced equally at 60 degrees around the pipe and repeating every 3 inches, totaling 4.71 square inches of perforation every lineal foot. The LDS horizontal collection pipe shall be backfilled with drain rock.

## REFERENCES

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**REFERENCE 1: PLASTIC PIPE INSTITUTE, PE DESIGN HANDBOOK**



## Chapter 6

# Design of PE Piping Systems

Equations as noted

(3-1) $P_E = wH$

**WHERE**
$P_E$ = vertical soil pressure due to earth load, psf
w = unit weight of soil, pcf
$H$ = depth of cover, ft

The Timoshenko Equation gives the soil pressure at a point directly under a distributed surface load, neglecting any pavement.

(3-2)
$$P_L = \frac{I_f W_w}{a_C}\left(1 - \frac{H^3}{(r_T^2 + H^2)^{1.5}}\right)$$

**WHERE**
$P_L$ = vertical soil pressure due to live load, lb/ft$^2$
$I_f$ = impact factor
$W_W$ = wheel load, lb
$a_C$ = contact area, ft$^2$
$r_T$ = equivalent radius, ft
$H$ = depth of cover, ft

The equivalent radius is given by:

(3-3)
$$r_T = \sqrt{\frac{a_C}{\pi}}$$

**Boussinesq Equation**

The Boussinesq Equation gives the pressure at any point in a soil mass under a concentrated surface load. The Boussinesq Equation may be used to find the pressure transmitted from a wheel load to a point that is not along the line of action of the load. Pavement effects are neglected.

(3-4) $$P_L = \frac{3 I_f W_w H^3}{2\pi r^5}$$

**WHERE**

$P_L$ = vertical soil pressure due to live load lb/ft²

$W_W$ = wheel load, lb

$H$ = vertical depth to pipe crown, ft

$I_f$ = impact factor

$r$ = distance from the point of load application to pipe crown, ft

(3-5) $$r = \sqrt{X^2 + H^2}$$

Compressive Ring Thrust

Crushing occurs when the compressive stress in the wall exceeds the compressive yield stress of the pipe material. Equations 3-13 and 3-14 give the compressive stress resulting from earth and live load pressure for conventional extruded DR pipe and for ASTM F894 profile wall PE Pipe:

(3-13) $$S = \frac{(P_E + P_L)\, DR}{288}$$

**WHERE**

$P_E$ = vertical soil pressure due to earth load, psf

$P_L$ = vertical soil pressure due to live-load, psf

$S$ = pipe wall compressive stress, lb/in²

$DR$ = Dimension Ratio, $D_O/t$

$D_O$ = pipe outside diameter (for profile pipe $D_O = D_I + 2H_P$), in

$D_I$ = pipe inside diameter, in

$H_P$ = profile wall height, in

$A$ = profile wall average cross-sectional area, in²/in
(Obtain the profile wall area from the manufacturer of the profile pipe.)

## Soil Arching

Burns and Richard[8] have published equations that give the resulting stress occurring in a pipe due to arching. As discussed above, the arching is usually considered when calculating the ring compressive stress in profile pipes. For deeply buried pipes McGrath [12] has simplified the Burns and Richard's equations to derive a vertical arching factor as given by Equation 3-21.

(3-21) $$VAF = 0.88 - 0.71\frac{S_A - 1}{S_A + 2.5}$$

**WHERE**

$VAF$ = Vertical Arching Factor

$S_A$ = Hoop Thrust Stiffness Ratio

(3-22) $$S_A = \frac{1.43 M_S r_{CENT}}{EA}$$

**WHERE**

$r_{CENT}$ = radius to centroidal axis of pipe, in

$M_s$ = one-dimensional modulus of soil, psi

$E$ = apparent modulus of elasticity of pipe material, psi (See Appendix, Chapter 3)

$A$ = profile wall average cross-sectional area, in²/in, or wall thickness (in) for DR pipe

The radial directed earth pressure can be found by multiplying the prism load (pressure) by the vertical arching factor as shown in Eq. 3-23.

(3-23) $$P_{RD} = (VAF)wH$$

**WHERE**

$P_{RD}$ = radial directed earth pressure, lb/ft²

$W$ = unit weight of soil, pcf

$H$ = depth of cover, ft

## Ring Deflection

To use the Watkins-Gaube Graph, the designer first determines the relative stiffness between pipe and soil, which is given by the Rigidity Factor, $R_F$. Equation 3-24 and 3-25 are for DR pipe and for profile pipe respectively:

(3-24) $$R_F = \frac{12 E_S (DR - 1)^3}{E}$$

**WHERE**

DR = Dimension Ratio

$E_S$ = Secant modulus of the soil, psi

E = Apparent modulus of elasticity of pipe material, psi

I = Pipe wall moment of inertia of pipe, in⁴/in

$D_m$ = Mean diameter ($D_I$ + 2z or $D_O$ – t), in

The soil layer surrounding the pipe bears the entire load of the overburden above it without arching. Therefore, settlement (compression) of the soil layer is proportional to the prism load and not the radial directed earth pressure. Soil strain, $\varepsilon_S$, may be determined from geotechnical analysis or from the following equation:

(3-27)
$$\varepsilon_S = \frac{wH}{144 E_S}$$

**WHERE**

W = unit weight of soil, pcf

H = depth of cover (height of fill above pipe crown), ft

$E_S$ = secant modulus of the soil, psi

The designer can find the pipe deflection as a percent of the diameter by multiplying the soil strain, in percent, by the deformation factor:

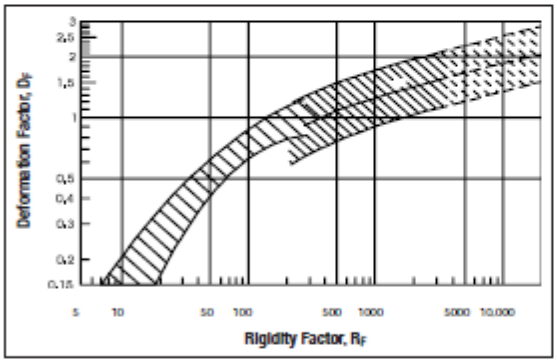


**Figure 3-6** Watkins-Gaube Graph

(3-28)
$$\frac{\Delta X}{D_M}(100) = D_F \varepsilon_S$$

**WHERE**

$\Delta X/D_M$ multiplied by 100 gives percent deflection.

## Critical Buckling

The Moore-Selig Equation for critical buckling pressure follows: (Critical buckling pressure is the pressure at which buckling will occur. A safety factor should be provided.)

(3-29)
$$P_{CR} = \frac{2.4 \varphi R_H}{D_M} (EI)^{\frac{1}{3}} (E_S^*)^{\frac{2}{3}}$$

**WHERE**

$P_{CR}$ = Critical constrained buckling pressure, psi

$\varphi$ = Calibration Factor, 0.55 for granular soils

$R_H$ = Geometry Factor

$E$ = Apparent modulus of elasticity of pipe material, psi

$I$ = Pipe wall moment of inertia, $in^4/in$ ($t^3/12$, if solid wall construction)

$E_S^* = E_S/(1-\mu)$

$E_S$ = Secant modulus of the soil, psi

$\mu_S$ = Poisson's Ratio of Soil (Consult a textbook on soil for values. Bowles (1982) gives typical values for sand and rock ranging from 0.1 to 0.4.)

The geometry factor is dependent on the depth of burial and the relative stiffness between the embedment soil and the insitu soil. Moore has shown that for deep burials in uniform fills, $R_H$ equals 1.0.

**REFERENCE 2: CDOT**

## Pavement Structural Analysis and Design Training

Backcalculation of Deflection Data
Colorado Department of Transportation
25–27 June 2013

# Typical Values of Young's Elastic Modulus and Poisson's Ratio for Pavement Materials

| Material | Young's Elastic Modulus (E or $M_r$), psi | Poisson's Ratio ($\mu$ or $\nu$) |
|---|---|---|
| Asphalt concrete (uncracked) 32 F | 2,000,000 – 5,000,000 | 0.25 – 0.30 |
| 70 F | 300,000 – 500,000 | 0.30 – 0.35 |
| 140 F | 20,000 – 50,000 | 0.35 – 0.40 |
| Portland cement concrete (uncracked) | 3,000,000 – 5,000,000 | 0.15 |
| Extensively cracked surfaces | Similar to granular base course materials | Similar to granular base course materials |
| Crushed stone base (clean, well-drained) | 20,000 – 80,000 | 0.35 |
| Crushed gravel base (clean, well-drained) | 20,000 – 80,000 | 0.35 |
| Uncrushed gravel base | | |
| Clean, well-drained | 10,000 – 60,000 | 0.35 |
| Clean, poorly-drained | 3,000 – 15,000 | 0.40 |
| Cement stabilized base | | |
| Uncracked | 500,000 – 2,000,000 | 0.20 |
| Badly cracked | 40,000 – 200,000 | 0.30 |
| Cement stabilized subgrade | 50,000 – 500,000 | 0.20 |
| Lime stabilized subgrade | 20,000 – 150,000 | 0.20 |
| Gravelly and/or sandy soil subgrade (drained) | 10,000 – 60,000 | 0.40 |
| Silty soil subgrade (drained) | 5,000 – 20,000 | 0.42 |
| Clayey soil subgrade (drained) | 3,000 – 12,000 | 0.42 |
| Dirty, wet, and/or poorly-drained materials | 1,500 – 6,000 | 0.45 – 0.50 |
| Intact Bedrock<br>Note: Values greater than 500,000 have negligible influence on surface deflections. | 250,000 – 1,000,000 | 0.20 |

## REFERENCE 3: NATIONAL ENGINEERING HANDBOOK

**Table 52–2** Average values of the modulus of soil reaction for the Modified Iowa Equation

| Soil type – pipe bedding material (Unified Soil Classification – ASTM D2487) | E' for degree of compaction of bedding, lb/in$^2$ 1/ | | | |
|---|---|---|---|---|
| | Dumped | Slight, < 85% proctor, < 40% relative density | Moderate, 85-95% proctor, 40-70% relative density | High, > 95% proctor, > 70% relative density |
| Fine-grained soil (LL>50) 2/ Soil with medium to high plasticity CH, MH, CH-MH | No data available, use E' = 0 or consult with a geotechnical engineer | | | |
| Fine-grained soil (LL<50) soil with medium to no plasticity CL, ML, ML-CL, with less than 25% coarse-grained particles | 50 | 200 | 400 | 1,000 |
| Fine-grained soil (LL<50) soil with medium to no plasticity CL, ML, ML-CL, with more than 25% coarse-grained particles. Coarse-grained soil with fines GM, GC, SM, SC contains more than 12% fines | 100 | 400 | 1,000 | 2,000 |
| Coarse-grained soil with little or no fines GW, GP, SW, SP contains less than 12% fines | 200 | 1,000 | 2,000 | 3,000 |
| Crushed rock | 1,000 | 3,000 | 3,000 | 3,000 |

1/ Source ASCE Journal of Geotechnical Engineering Division, January 1977
2/ LL = liquid limit

## REFERENCE 4: BRACHMAN, LOCAL STRAIN ON A LEACHATE COLLECTION PIPE



# Local strain on a leachate collection pipe

**R.W.I. Brachman, I.D. Moore, and R.K. Rowe**

**Abstract**: Local strain measurements opposite gravel contacts and around a single isolated perforation are reported for a high-density-polyethylene pipe (320 mm outside diameter, 32 mm thick) typical of that commonly used as part of the leachate collection system in municipal solid waste landfills. Emphasis is given to examining the localized effect of coarse gravel contacts on pipe strain and the strain distribution around the perforation. The laboratory testing featured a cylindrical volume of soil with the pipe located in the middle with radially compressive stresses applied along the outer perimeter of the backfill. Two different backfill materials were used. When tested with medium sand backfill the pipe response was essentially uniform, indicative of the near-continuous support and loading provided by the sand. Large variations in local pipe strains were measured with coarse gravel backfill, such as that used in landfill leachate collection systems. Local bending induced by the discontinuous support and loading from the coarse gravel resulted in variations in circumferential and axial strains of over 40%. The local bending effects were not sufficiently large to produce circumferential tension in the pipe tested. A multiplication factor of 1.5 is suggested to account for increases in compressive strain from the coarse gravel. Measurements of surface strain around an isolated 32 mm diameter perforation revealed that a complex three-dimensional response is induced by the presence of the hole. Maximum strains near the hole were found to be 2.7 times larger than those distant from the perforation.

*Key words*: leachate collection pipes, HDPE pipes, leachate collection systems, landfill design.

**Résumé** : Les deformations locales mesurées sur les contacts opposés au gravier et autour d'une simple perforation isolée sont rapportées pour un tuyau de polyéthylène à haute densité (d'un diamètre extérieur de 320 mm et d'une épaisseur de 32 mm) typique de ce qui est couramment employé dans un système de collecte d'eau de percolation dans des sites de disposition de déchets solides municipaux. L'emphase est mise sur l'examen de l'effet localisé de contacts du gravier grossier sur les deformations au tuyau, et de la distribution de deformations autour de la perforation. Les tests en laboratoire ont impliqué un volume cylindrique de sol avec le tuyau situé au milieu, et avec des contraintes compressives radiales appliquées le long du périmètre extérieur du matériel de remblayage. Deux matériaux de remblayage différents ont été utilisés. Lorsque testé avec du sable de remblayage moyen, la réponse du tuyau a été essentiellement uniforme, ce qui est indicatif du support et du chargement quasi continus produits par le sable. De large variations dans les deformations locales sur le tuyau ont été mesurées lorsque le matériel de remblayage est un gravier grossier, tel que celui utilisé dans les systèmes de collecte d'eau de percolation des sites de disposition. Le fléchissement local induit par le support et le chargement discontinus provenant du gravier grossier a résulté en des variations de deformations radiale et axiale de plus de 40 %. Les effets du fléchissement local n'ont pas été suffisamment large pour produire une tension radiale dans le tuyau testé. Un facteur de multiplication de 1,5 est suggéré pour tenir compte des augmentations de la tension compressive provenant du gravier grossier. Les mesures de déformations de surface autour d'une perforation isolée d'un diamètre de 32 mm ont révélé qu'une réponse tridimensionnelle est induite par la présence du trou. Les deformations maximales près du trou ont été trouvées comme étant 2,7 fois plus grandes que celles éloignées de la perforation.

*Mots clés* : tuyau de collecte d'eau de percolation, tuyau de polyéthylène à haute densité, système de collecte d'eau de percolation, conception de sites de disposition.

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Written discussion of this article is welcomed and will be received by the Editor until April 30, 2001.

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## Introduction

Leachate collection systems are an important component of modern municipal solid waste landfills. They are intended to minimize the hydraulic gradient across the liner system (e.g., compacted clay and (or) geomembrane liner), and hence an operating leachate collection system is one factor that controls the migration of contaminants from the landfill. Collection systems also remove leachate from the facility, thereby reducing the contaminating lifespan of the landfill. A typical modern leachate collection system consists of perforated plastic pipes surrounded by coarse uniformly graded gravel (e.g., see Fig. 1*a*). Ensuring adequate structural per-



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**REFERENCE 5: BUREAU OF RECLAMATION**

RECLAMATION
*Managing Water in the West*

*Report DSO-09-01*

## Physical Properties of Plastic Pipe Used in Reclamation Toe Drains

**Dam Safety Technology Development Program**

U.S. Department of the Interior
Bureau of Reclamation
Technical Service Center
Denver, Colorado

September 2009

3. For solid-wall pipe, the perforated pipe was significantly weaker than the non-perforated pipe. The strength reduction was directly proportional to the percent open area (POA). Solid-wall PVC pipe (drainpipe and pressure pipe) showed a 2.5 percent decrease in pipe stiffness for every 1 percent open area. Solid-wall HDPE pipe lost more strength, with a 15.0 percent decrease in pipe stiffness for every 1 percent open area.

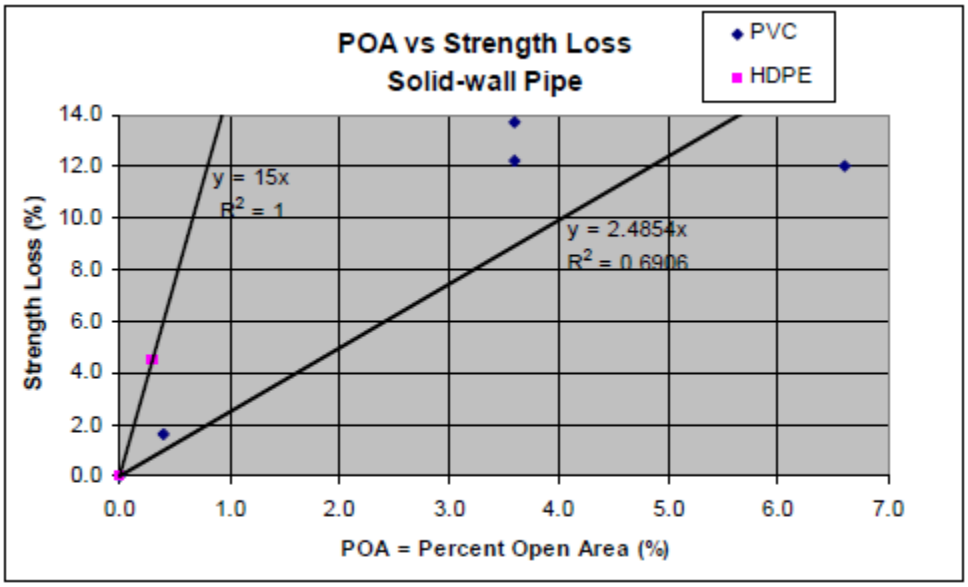


Figure 5-4.—Loss of pipe strength (pipe stiffness) is directly proportional to POA. Sampling from different lots accounts for the low correlation of PVC pipe.

# REFERENCE 6: NATIONAL ENGINEERING HANDBOOK



## (a) Plastic pipe

Plastic pipe materials consist of poly-vinyl chloride (PVC), acrylonitrile-butadiene-styrene (ABS), and polyethylene (PE). Each type of material is supplied in several grades as shown in appendix 52C.

Design of buried plastic pipe includes analyses of the wall crushing, buckling resistance, allowable long-term deflection, and allowable strain.

At a constant load, the plastic modulus of elasticity of the plastic pipe decreases with time. With any increase in load, the plastic reacts with the short-term modulus of elasticity. The ratio of the short-term to long-term modulus of elasticity varies from approximately 3 for PVC to 5 for PE. The short-term modulus of elasticity is recommended for conditions that change through time, such as deflection. The pipe-soil interaction that occurs as discrete events is similar to a new load (Chevron Chemical, 1998). The long-term modulus of elasticity is often recommended for buckling since the loads and reaction of the pipe are considered static.

### (1) Wall crushing

The design pressure and ring compression thrust in the pipe wall is determined by:

$$P = P_s + P_w + P_v \qquad (52\text{–}25)$$

where:

$P$ = pressure on pipe, lb/ft$^2$
$P_s$ = pressure due to weight of soil, lb/ft$^2$
$P_w$ = pressure on pipe due to wheel load, lb/ft$^2$
$P_v$ = internal vacuum pressure, lb/ft$^2$

$$T_{pw} = \frac{P \times \frac{D_o}{12}}{2} \qquad (52\text{–}26)$$

where:

$T_{pw}$ = thrust in pipe wall, lb/ft
$D_o$ = outside pipe diameter, in

The required wall cross-sectional area is determined by:

$$A_{pw} = \frac{\frac{T_{pw}}{12}}{\sigma} \qquad (52\text{–}27)$$

where:

$A_{pw}$ = required wall area, in$^2$/in
$T_{pw}$ = thrust in pipe wall, lb/ft
$\sigma$ = allowable long-term compressive stress, lb/in$^2$ (see appendix 52C, table 52C–1)

The area of a solid-wall pipe wall may be computed as:

$$A_{pw} = \frac{(D_o - D_i)}{2} \text{ or } t \qquad (52\text{–}28)$$

where:

$A_{pw}$ = area of pipe wall, in$^2$/in
$D_o$ = outside pipe diameter, in
$D_i$ = inside pipe diameter, in
$t$ = pipe wall thickness, in

The average area of pipe wall for corrugated and profile wall pipe should be obtained from the manufacturer.

### (2) Deflection

The Modified Iowa Equation may be transposed and rewritten to compute the percent deflection of each type of pipe. The properties of a pipe section are expressed as the standard dimension ratio (SDR) or standard inside dimension ratio (SIDR) for solid wall pipe, pipe stiffness (PS) for corrugated plastic pipe, and the ring stiffness constant (RSC) for profile wall pipe.

Solid-wall plastic pipe as:

$$\frac{\%\Delta X}{D} = \frac{(D_L P_s + P_w + P_v)\left(\frac{1}{144}\right)K(100)}{\left[\left(\frac{2E}{3(SDR-1)^3}\right) + 0.061E'\right]} \qquad (52\text{–}29)$$

or

$$\frac{\%\Delta X}{D} = \frac{(D_L P_s + P_w + P_v)\left(\frac{1}{144}\right)K(100)}{\left[\left(\frac{2E}{3(SIDR+1)^3}\right) + 0.061E'\right]} \qquad (52\text{–}30)$$

Corrugated-plastic pipe as:

$$\frac{\%\Delta X}{D} = \frac{\left(D_L P_S + P_w + P_v\right)\left(\frac{1}{144}\right)K(100)}{\left[0.149PS + 0.061E'\right]} \qquad (52\text{–}31)$$

Profile-wall pipe:

$$\frac{\%\Delta X}{D} = \frac{\left(D_L P_S + P_w + P_v\right)\left(\frac{1}{144}\right)K(100)}{\left[\left(\frac{1.24(RSC)}{D_i}\right) + 0.061E'\right]} \qquad (52\text{–}32)$$

where:

$\frac{\%\Delta X}{D}$ = percent deflection
$D_L$ = deflection lag factor (1.0 to 1.5)
K = bedding constant (0.1)
$P_s$ = pressure on pipe from soil (lb/ft$^2$)
$P_w$ = pressure on pipe from wheel load (lb/ft$^2$)
$P_v$ = internal vacuum pressure (lb/ft$^2$)
E = modulus of elasticity of pipe material (as shown below)
SDR = $D_o$ dimension ratio
SDR = $D_o/t$
$D_o$ = pipe outside diameter, in
t = minimum wall thickness, in
SIDR = $D_i$ dimension ratio
SIDR = $D_i/t$
$D_i$ = pipe inside diameter, in
t = minimum wall thickness, in
PS = pipe stiffness
RSC = ring stiffness constant
$D_i$ = inside pipe diameter, in
E' = modulus of soil reaction, lb/in$^2$ (see table 52–2)

| Material | Modulus of elasticity* (lb/in$^2$) |
|---|---|
| PVC | 400,000 (short term) |
| ABS | 300,000 (short term) |
| Polyethylene | 110,000 (short term) |

* Short-term modulus of elasticity varies with the cell class of each plastic. Specific values may be obtained from the manufacturer.

The modulus of soil reaction, E', is an interactive modulus representing support of the soil in reaction to the lateral pipe deflection under load. Amster Howard of the Bureau of Reclamation (Howard, 1977) developed recommended E' values based on the soil prism load described above. The recommended values are provided in table 52–2.

The allowable deflections for plastic pipe typically are limited to 5 percent for a spillway/outlet conduit in embankment dam practice and 7.5 percent in water or liquid conveyance practice and drains in embankment dam practice.

### (3) Wall buckling

Plastic pipe embedded in soil may buckle because of excessive loads and deformations. The total permanent pressure must be less than the allowable buckling pressure. The permanent load should consist of the soil pressure, groundwater pressure, and any internal long-term vacuum pressures. The allowable buckling pressure may be determined from:

$$q_a = \frac{1}{FS}\left(32R_w B'E' \frac{E_{long} I_{pw}}{D_o^3}\right)^{1/2} \qquad (52\text{–}33)$$

(Moser, 2001)

where:

$q_a$ = allowable buckling pressure, lb/in$^2$
FS = design factor of safety
= 2.5 for $(h/(D_o/12) > 2$
= 3.0 for $(h/(D_o/12) < 2$
where:
h = height of ground surface above top of pipe, ft
$D_o$ = outside diameter of the pipe, in
$R_w$ = water buoyancy factor
= $1–0.33(h_w/h)$, $0<h_w<h$
where:
h = height of ground surface above top of pipe, ft
$h_w$ = height of water above top of pipe, ft
B'= empirical coefficient of elastic support

$$= \frac{4\left(h^2 + \left(\frac{D_o}{12}\right)h\right)}{1.5\left(2h + \left(\frac{D_o}{12}\right)\right)^2}$$

$E_{long}$ = long term modulus of elasticity, lb/in$^2$ (see table below)

The long term modulus of elasticity is recommended if the pipe is subject to the pressure in the normal operations. If the pipe is subject to the pressure for short time periods and infrequently, the use of the short-term modulus of elasticity is acceptable.

$E'$ = modulus of soil reaction, lb/in$^2$ (table 52–2)

$I_{pw}$ = pipe wall moment of inertia

$= \frac{t^3}{12}$, in$^4$ / in (for solid wall pipe)

where:

t = pipe wall thickness, in

$D_o$ = outside pipe diameter, in

| Material | Modulus of elasticity* (lb/in$^2$) |
|---|---|
| PVC | 140,000 (long term) |
| ABS | 65,000 (long term) |
| Polyethylene | 22,000 (long term) |

* Long-term modulus of elasticity varies with the cell class of each plastic. Specific values may be obtained from the manufacturer.

Pipes that are out-of-round or deflected increase in bending moment and have less allowable buckling pressure. The allowable buckling pressure should be reduced by the following factor:

$$C = \left[ \frac{\left(1 - \frac{\%\Delta X}{D}\frac{1}{100}\right)}{\left(1 + \frac{\%\Delta X}{D}\frac{1}{100}\right)^2} \right]^3 \qquad (52\text{–}34)$$

where:

C = reduction factor for buckling pressure

$\frac{\%\Delta X}{D}$ = percent deflection

**Table 52–2** Average values of the modulus of soil reaction for the Modified Iowa Equation

| Soil type – pipe bedding material (Unified Soil Classification – ASTM D2487) | E' for degree of compaction of bedding, lb/in$^2$ 1/ Dumped | Slight, < 85% proctor, < 40% relative density | Moderate, 85-95% proctor, 40-70% relative density | High, > 95% proctor, > 70% relative density |
|---|---|---|---|---|
| Fine-grained soil (LL>50) 2/ Soil with medium to high plasticity CH, MH, CH-MH | No data available, use E' = 0 or consult with a geotechnical engineer | | | |
| Fine-grained soil (LL<50) soil with medium to no plasticity CL, ML, ML-CL, with less than 25% coarse-grained particles | 50 | 200 | 400 | 1,000 |
| Fine-grained soil (LL<50) soil with medium to no plasticity CL, ML, ML-CL, with more than 25% coarse-grained particles. Coarse-grained soil with fines GM, GC, SM, SC contains more than 12% fines | 100 | 400 | 1,000 | 2,000 |
| Coarse-grained soil with little or no fines GW, GP, SW, SP contains less than 12% fines | 200 | 1,000 | 2,000 | 3,000 |
| Crushed rock | 1,000 | 3,000 | 3,000 | 3,000 |

1/ Source ASCE Journal of Geotechnical Engineering Division, January 1977
2/ LL = liquid limit

### (4) Strain

Total strain in a pipe wall can be caused by two actions: (1) flexure of the pipe as it deforms, and (2) hoop stress caused by internal or external pressure in the pipe wall. If a homogeneous wall is assumed and pressure concentrations are neglected, the formula follows:

Hoop strain:

$$\varepsilon_h = \frac{\frac{P}{144} D}{2 A_{pw} E} \qquad (52\text{–}35)$$

For solid wall pipe, the equation becomes:

$$\varepsilon_h = \frac{\frac{P}{144} D_M}{2tE} \qquad (52\text{–}36)$$

where:

$\varepsilon_h$ = maximum strain in pipe wall because of ring bending, in/in
P = pressure on/in pipe (may be internal and/or external pressure with the appropriate sign), lb/ft$^2$
$D_M$ = mean pipe diameter, in
$A_{pw}$ = area of pipe wall, in$^2$/in
E = modulus of elasticity of the pipe material, lb/in$^2$
t = pipe wall thickness, in

Maximum strains because of deflection or flexure may be determined by assuming the pipe remains an ellipse during deflections. The resulting equations are:

$$\varepsilon_f = \frac{t}{D_M}\left(\frac{\frac{3\Delta Y}{D_M}}{1 - 2\frac{\Delta Y}{D_M}}\right) = \frac{1}{SDR}\left(\frac{\frac{3\Delta Y}{D_M}}{1 - 2\frac{\Delta Y}{D_M}}\right) \qquad \text{(solid wall pipe)} \qquad (52\text{–}37)$$

or

$$\varepsilon_f = 6\frac{t}{D_M}\frac{\Delta Y}{D_M} \qquad \text{(corrugated or profile wall pipe)} \qquad (52\text{–}38)$$

where:

$\varepsilon_f$ = maximum strain in pipe wall because of ring deflection, in/in
$\Delta Y$ = vertical decrease in diameter, in
$D_M$ = mean pipe diameter, in
$\Delta Y/D_M = \Delta X/D$ = percent deflection expressed as a decimal
t = pipe wall thickness, in
SDR = standard dimension ratio

In a buried pipeline, these strain components act simultaneously. The maximum combined strain in the pipe wall can be determined by summing both components.

$$\varepsilon = \varepsilon_f \pm \varepsilon_h \qquad (52\text{–}39)$$

where:

$\varepsilon$ = maximum combined strain in pipe wall, in/in

In calculating the maximum combined strain, the hoop strain, $\varepsilon_h$, resulting from applied internal pressure, if any, should be added to the maximum strain due to deflection, $\varepsilon_f$. If the hoop strain is due to external load or internal vacuum pressure, the ring hoop strain should be substracted to obtain the maximum combined strain, $\varepsilon$.

The maximum combined strain in the pipe should be limited to:

$$\varepsilon \le \varepsilon_{all} \qquad (52\text{–}40)$$

where:

$\varepsilon_{all}$ = allowable strain for the pipe material

The allowable strain should be no more than 5 percent for polyethylene and ABS pipes.

The allowable deflection for PVC pipe limits strain in standard PVC pipes to an acceptable value. Therefore, computation of strain and comparison to an allowable strain limit is not required for PVC pipe.

In polyethylene pressure pipe with pressure near the pipe pressure rating, the strain may be limited by limiting the deflection to the values shown in table 52–3.

Corrugated-plastic pipe as:

$$\frac{\%\Delta X}{D} = \frac{\left(D_L P_S + P_w + P_V\right)\left(\frac{1}{144}\right)K(100)}{\left[0.149PS + 0.061E'\right]} \qquad (52\text{–}31)$$

Profile-wall pipe:

$$\frac{\%\Delta X}{D} = \frac{\left(D_L P_S + P_w + P_V\right)\left(\frac{1}{144}\right)K(100)}{\left[\left(\frac{1.24(RSC)}{D_i}\right) + 0.061E'\right]} \qquad (52\text{–}32)$$

where:

$\frac{\%\Delta X}{D}$ = percent deflection
$D_L$ = deflection lag factor (1.0 to 1.5)
K = bedding constant (0.1)
$P_s$ = pressure on pipe from soil (lb/ft$^2$)
$P_w$ = pressure on pipe from wheel load (lb/ft$^2$)
$P_v$ = internal vacuum pressure (lb/ft$^2$)
E = modulus of elasticity of pipe material (as shown below)
SDR = $D_o$ dimension ratio
SDR = $D_o/t$
$D_o$ = pipe outside diameter, in
t = minimum wall thickness, in
SIDR = $D_i$ dimension ratio
SIDR = $D_i/t$
$D_i$ = pipe inside diameter, in
t = minimum wall thickness, in
PS = pipe stiffness
RSC = ring stiffness constant
$D_i$ = inside pipe diameter, in
E' = modulus of soil reaction, lb/in$^2$ (see table 52–2)

| Material | Modulus of elasticity* (lb/in$^2$) |
|---|---|
| PVC | 400,000 (short term) |
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| Polyethylene | 110,000 (short term) |

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The modulus of soil reaction, E', is an interactive modulus representing support of the soil in reaction to the lateral pipe deflection under load. Amster Howard of the Bureau of Reclamation (Howard, 1977) developed recommended E' values based on the soil prism load described above. The recommended values are provided in table 52–2.

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where:

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= 3.0 for $(h/(D_o/12) < 2$
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B'= empirical coefficient of elastic support

$$= \frac{4\left(h^2 + \left(\frac{D_o}{12}\right)h\right)}{1.5\left(2h + \left(\frac{D_o}{12}\right)\right)^2}$$

$E_{long}$ = long term modulus of elasticity, lb/in$^2$ (see table below)

The long term modulus of elasticity is recommended if the pipe is subject to the pressure in the normal operations. If the pipe is subject to the pressure for short time periods and infrequently, the use of the short-term modulus of elasticity is acceptable.

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|---|---|---|---|---|
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1/ Source ASCE Journal of Geotechnical Engineering Division, January 1977
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Total strain in a pipe wall can be caused by two actions: (1) flexure of the pipe as it deforms, and (2) hoop stress caused by internal or external pressure in the pipe wall. If a homogeneous wall is assumed and pressure concentrations are neglected, the formula follows:

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$$\varepsilon_h = \frac{\frac{P}{144} D}{2A_{pw}E} \qquad (52\text{–}35)$$

For solid wall pipe, the equation becomes:

$$\varepsilon_h = \frac{\frac{P}{144} D_M}{2tE} \qquad (52\text{–}36)$$

where:

$\varepsilon_h$ = maximum strain in pipe wall because of ring bending, in/in
P = pressure on/in pipe (may be internal and/or external pressure with the appropriate sign), lb/ft$^2$
$D_M$ = mean pipe diameter, in
$A_{pw}$ = area of pipe wall, in$^2$/in
E = modulus of elasticity of the pipe material, lb/in$^2$
t = pipe wall thickness, in

Maximum strains because of deflection or flexure may be determined by assuming the pipe remains an ellipse during deflections. The resulting equations are:

$$\varepsilon_f = \frac{t}{D_M}\left(\frac{\frac{3\Delta Y}{D_M}}{1-2\frac{\Delta Y}{D_M}}\right) = \frac{1}{SDR}\left(\frac{\frac{3\Delta Y}{D_M}}{1-2\frac{\Delta Y}{D_M}}\right) \qquad \text{(solid wall pipe)} \quad (52\text{–}37)$$

or

$$\varepsilon_f = 6\frac{t}{D_M}\frac{\Delta Y}{D_M} \qquad \text{(corrugated or profile wall pipe)} \quad (52\text{–}38)$$

where:

$\varepsilon_f$ = maximum strain in pipe wall because of ring deflection, in/in
$\Delta Y$ = vertical decrease in diameter, in
$D_M$ = mean pipe diameter, in
$\Delta Y/D_M = \Delta X/D$ = percent deflection expressed as a decimal
t = pipe wall thickness, in
SDR = standard dimension ratio

In a buried pipeline, these strain components act simultaneously. The maximum combined strain in the pipe wall can be determined by summing both components.

$$\varepsilon = \varepsilon_f \pm \varepsilon_h \qquad (52\text{–}39)$$

where:

$\varepsilon$ = maximum combined strain in pipe wall, in/in

In calculating the maximum combined strain, the hoop strain, $\varepsilon_h$, resulting from applied internal pressure, if any, should be added to the maximum strain due to deflection, $\varepsilon_f$. If the hoop strain is due to external load or internal vacuum pressure, the ring hoop strain should be substracted to obtain the maximum combined strain, $\varepsilon$.

The maximum combined strain in the pipe should be limited to:

$$\varepsilon \le \varepsilon_{all} \qquad (52\text{–}40)$$

where:

$\varepsilon_{all}$ = allowable strain for the pipe material

The allowable strain should be no more than 5 percent for polyethylene and ABS pipes.

The allowable deflection for PVC pipe limits strain in standard PVC pipes to an acceptable value. Therefore, computation of strain and comparison to an allowable strain limit is not required for PVC pipe.

In polyethylene pressure pipe with pressure near the pipe pressure rating, the strain may be limited by limiting the deflection to the values shown in table 52–3.

## REFERENCE 7: YAP, TRUCK AND TIRE TYPES AND ROAD CONTACT PRESSURES

# TRUCK TIRE TYPES AND ROAD CONTACT PRESSURES

by

Pedro Yap
Senior Design Engineer
The Goodyear Tire & Rubber Company

A Paper Prepared for Presentation
at the
Second International Symposium on
Heavy Vehicle Weights and Dimensions

Kelowna, British Columbia

June 1989

11R24.5 DUALS

385/65R22.5 SINGLE

| SIZE | AIR VOLUME ($inch^3$) | HOT INFLATION (psi) | LOAD (lbs.) | CONTACT AREA ($inch^2$) |
|---|---|---|---|---|
| 11R24.5 | 13,613 | 105 | 4,250 | 114 (*) |
| 385/65R22.5 | 9,644 | 130 | 8,500 | 87 |
| 425/65R22.5 | 12,359 | 120 | 8,500 | 90 |

(*) Dual Tires

Figure 18 Single Wide-Base Tire versus Dual Tire Assembly

**REFERENCE 8: FISHER, C. THE EFFECTS OF DEFLECTION ON SEWER HYDRAULICS. WWW.UNI-BELL.ORG. PVC PIPE NEWS. SUMMER 2007.**

PVC PIPE news

# THE EFFECTS OF DEFLECTION ON SEWER HYDRAULICS

## The Sequel

by Craig Fisher, P.E.
Technical Director

The sequel tag to the title of this article stems from the fact that *The Effects of Deflection on Sewer Hydraulics* was featured in the Summer 1989 issue of this publication. Those of you who enjoy analytical geometry and integrals should download the earlier version from the newspaper section of our website. If your skills in those areas are a bit rusty, the sequel may be more to your liking.

The topic then, as it is now, is how to tackle the elliptical cross section of a flexible pipe when performing hydraulic calculations for gravity sewers. Every text on hydraulics discusses circular pipe, but those that also cover pipes with an elliptical cross section are in the extreme minority.

### Circular Cross Sections

First, a quick review of pipes with a circular cross section. The Manning's equation is the one most often used for calculating the flow velocity. Equation 1 is the imperial version of that formula. After determining the velocity, the flow rate is calculated using Equation 2. In this form, the flow is in units of $ft^3/s$.

Equation 1

$$v = \frac{1.486}{n} r^{\frac{2}{3}} s^{\frac{1}{2}}$$

Equation 2

$$Q = vA$$

Where:
- v – velocity of flow, ft/s
- n – Manning's roughness coefficient
- r – hydraulic radius, ft
- s – slope
- A – flow cross sectional area, $ft^2$

Calculating the hydraulic radius of a circular pipe flowing full is a simple matter. Take the ratio of the cross sectional area of the flow to the wetted perimeter of the pipe in contact with the flow. Let *d* be the inside diameter of the circular pipe and let *p* be the perimeter. Also, use the subscript *c* to signify that these geometric properties are for a circle.

Equation 3

$$A_c = \frac{\pi}{4} d^2$$

Equation 4

$$p_c = \pi d$$

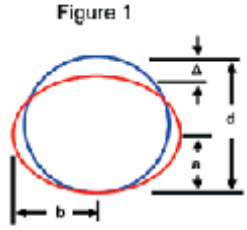
Equation 5

$$r_c = \frac{A_c}{p_c} = \frac{\frac{\pi}{4}(d^2)}{\pi d} = \frac{d}{4}$$

### Elliptical Cross Sections

Figure 1 shows the circular cross section of the pipe in blue and the elliptical cross section in red. Let *a* denote the altitude of the ellipse and *b* denote the base.

Figure 1




With this terminology, the cross sectional area and perimeter for an ellipse may be defined. The subscript *e* will be used to identify elliptical geometric properties. The formula for the perimeter for the ellipse is the one listed in the sixth edition of the Civil Engineering Reference Manual, which is a good approximation. The 1989 article goes into more details on calculating the perimeter of an ellipse (using integrals and analytical geometry).

Equation 6

$$A_e = \pi ab$$

Equation 7

$$p_e = 2\pi\sqrt{\frac{1}{2}(a^2 + b^2)}$$

As the pipe deforms from a circle into an ellipse, the perimeter stays the same for reasonable levels of deflection. Thus, $p_c$ is equal to $p_e$.

### 5% Deflection

Let's use a deflection of 5% to demonstrate the concept. With that ring deflection, *d* is decreased by 5%, *2a* is equal to *0.95d*, which means *a* equals *0.475d*. Next, increase the value of *b* until the perimeter of the ellipse matches that of the circle. That occurs when *b* is *0.524d*. Both *a* and *b* may now be written in terms of *d* in Equation 6. This is done below. $A_C$ is also shown for reference.

$$A_e = \pi(0.475D)(0.524D) = 0.249\pi D^2$$

$$A_c = 0.250\pi D^2$$

Taking the ratio of the two areas, one finds that the cross sectional area of this ellipse is only slightly less than that of a circle - just 0.4%. Getting the perimeter and hydraulic radius of this ellipse in terms of *d* takes a bit more effort.

$$p_e = 2\pi\sqrt{\frac{1}{2}\left\{(0.475d)^2 + (0.524d)^2\right\}} = 1.002\pi d$$

$$r_e = \frac{A_e}{P_e} = \frac{0.249\pi d^2}{1.002\pi d} = 0.249d$$

The ratio of flow in the deflected pipe to the circular pipe is shown below. From that ratio, one determines that the reduction in flow is also minimal - just 0.7%.

$$\frac{A_e(r_e)^{\frac{2}{3}}}{A_c(r_c)^{\frac{2}{3}}} = \frac{(0.249\pi d^2)(0.249d)^{\frac{2}{3}}}{(0.250\pi d^2)(0.250d)^{\frac{2}{3}}} = 0.993$$

**SUMMARY**

Table 1 was developed using the more rigorous approach for the perimeter as described in the 1989 article. As the table shows, in the allowable deflection range (7.5% or less), the effects of deflection on sewer hydraulics are not a concern.

## Table 1
## EFFECTS OF DEFLECTION ON SEWER HYDRAULICS

| Deflection (%) | Reduction in $A_e$ Compared to $A_c$ (%) | Reduction in $Q_e$ Compared to $Q_c$ (%) |
|---|---|---|
| 5 | 0.37 | 0.6 |
| 7.5 | 0.81 | 1.4 |
| 10 | 1.43 | 2.4 |
| 15 | 3.15 | 5.2 |
| 20 | 5.47 | 9.0 |
| 25 | 8.38 | 13.6 |
| 30 | 11.8 | 18.9 |
| 35 | 15.8 | 24.9 |