Measurement Uncertainty

Module 6

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Overview

- Basic concepts (e.g., what is "uncertainty")
- Why uncertainty is important
- The role that uncertainty plays in MARLAP
- Traditional practices
- The GUM
- Causes of uncertainty
- MARLAP's recommendations

What Is Uncertainty?

- In general, "uncertainty" means a lack of complete knowledge about something of interest
- In metrology (the science of measurement) ***uncertainty*** usually means ***uncertainty of measurement***, which has a more precise definition

Definition of Uncertainty

- "Parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand" ***– Guide to the Expression of Uncertainty in Measurement* (GUM)**

- Examples might include:
  - Standard deviation
  - Multiple of a standard deviation
  - Half-width of interval with stated level of confidence

Comments on the Definition

- Associated with result of a measurement

  (Not with a measurement process or procedure)
- Measurement result and the uncertainty together allow one to place reasonable bounds on what the "true" value might be

Question

- If a lab reports that a sample of soil from a frequently used playground contains 110 pCi/g of ^{239}Pu, what actions if any would you recommend?
  - **Insist that the lab report the uncertainty of result**
- If the uncertainty is 10 pCi/g, one might conclude the playground should be closed while more tests are performed
- If the uncertainty is 300 pCi/g, the result doesn't mean much

Importance of Uncertainty

If the result of a measurement is reported without some indication of its uncertainty, the result is *useless* for decision making

Traceability

Are your results supposed to be "traceable"? If so, note that the concept of *metrological traceability* is defined as —

> "Property of a measurement result whereby the result can be related to a reference through a documented unbroken chain of calibrations, each contributing to the ***measurement uncertainty***"

Source: International vocabulary of metrology – Basic and general concepts and associated terms (VIM) 3rd edition, JCGM 200:2012

Role of Uncertainty in MARLAP

- MARLAP's approach to method evaluation and selection uses criteria based on measurement uncertainty (and the derived concept of method uncertainty)
- Criteria for **evaluating a lab's performance** based on required method uncertainty
- Criteria for **evaluating internal laboratory QC** based on measurement uncertainty
- Criteria for **making decisions about the contents of an individual sample** based on measurement uncertainty

Traditional Practices

- Radiochemists have known about uncertainty for many years, but for most of that time, there was no standard terminology or notation
- Often use the term "sigma" to mean an uncertainty expressed as a standard deviation
- Some use one sigma (1σ) , 2σ, or even 1.96σ
- Uncertainty often stated without any explanation, leaving data users to make their own assumptions

More Traditional Practices

- Incomplete uncertainty evaluations common
- Reported uncertainty might be only the "counting error"
  - It is one component of the total uncertainty
- Sometimes result might be reported with a relative uncertainty of only a fraction of 1 % **(usually unrealistic)**
- Sometimes you might even see 0 ± 0 pCi/L **(bad!)**

The GUM

- *Guide to the Expression of Uncertainty in Measurement* (GUM)
  - Published in 1993 by ISO in the name of 7 international organizations
  - Presents terminology, notation, and methods for evaluating and expressing measurement uncertainty
  - Promotes more complete uncertainty evaluations and comparability of uncertainty statements
- Current version available as a free PDF from the website of BIPM, the International Bureau of Weights and Measures

The GUM – Continued

- MARLAP's primary recommendation regarding measurement uncertainty is to

Follow the GUM

  - So we speak and write the same language about uncertainty
  - So we can interpret each other's results and uncertainty statements

MARLAP and the GUM

- If you follow the GUM, you're following the most important part of MARLAP's guidance for evaluating and expressing uncertainty
- MARLAP goes further and applies the GUM to radiochemical measurements
- Most of additional guidance is intended to be helpful, not prescriptive

Question

How can you comply substantially with MARLAP's guidance for evaluating and expressing uncertainty?

Follow the GUM

Metrology and Statistics

- What we're doing is called ***metrology***, defined as the science of measurement
- Metrology ≠ statistics, although metrology uses statistical methods and terminology
- Metrology uses lots of approximations (with no apologies) and defines new terms and symbols that a statistician wouldn't recognize

Results as Random Variables

- We consider the result of a measurement to be a ***random variable***
- The result can vary if the measurement is repeated, but it should vary in a manner that can be described probabilistically
- Can discuss its probability distribution, mean, standard deviation, etc.

Standard Uncertainty

- When we talk about the uncertainty of a result, we'll usually mean the **uncertainty expressed as a standard deviation**
- GUM calls this a ***standard uncertainty***
- Traditionally standard uncertainty often called a ***"one sigma" uncertainty***
- Standard uncertainty denoted by lower-case u
  - If x is a measured value, standard uncertainty is $u(x)$

What Causes Uncertainty?

- One of the best-known sources of uncertainty is "counting statistics"
- A nuclear counting measurement is based on the detection of rays and particles emitted by atoms of radionuclides as they decay
- Radioactive decay is inherently random
- We can describe the probability that an atom will decay during a specified time interval, but we can't be 100 % certain

Counting Uncertainty

- Radiation detection can also be random
- If you could repeat the same nuclear counting measurement over and over with the same initial conditions, you'd get a different result each time
- Uncertainty of a result due to the randomness of radioactive decay and radiation detection is what MARLAP calls the ***counting uncertainty***

Causes of Uncertainty: Subsampling

- Often the lab analyzes only a small portion of a much larger sample
- A typical sample has some heterogeneity, so one portion differs in composition from another
- Uncertainty due to subsampling is potentially very large, but may be hard to quantify

Causes of Uncertainty: Instruments

- Measuring instruments and their operators aren't perfect
- Radiation detectors usually aren't capable of detecting every particle or ray emitted from the sample
- Even volumes obtained using volumetric glassware and masses measured using precise analytical balances have uncertainty

Causes of Uncertainty: Standards

- Standards have uncertainties in their stated values
  - Including standard solutions used for instrument calibration
- Typical (standard) uncertainty for standard solution is ~ 0.5 % to 2 %
- These uncertainties may exceed the uncertainty due to counting statistics for measurements of samples with very high levels of activity

Causes of Uncertainty: Other

- Many other causes of uncertainty
  - Variable background radiation levels (e.g., cosmic)
  - Errors in mathematical models used to describe measurement process (e.g., calibration curves)
  - Errors in published values for constants (e.g., half-lives and radiation-emission probabilities)
  - Impurities in reagents
  - Contamination of glassware or instruments
  - Changing environmental conditions in the lab (temperature and humidity)

Uncertainty Propagation

- Final result typically not measured directly but calculated from other observed values
- Observed values might include volumes, masses, times, and numbers of counts
- Uncertainties of the input values combine to produce uncertainty in output value
- Mathematical operation of combining individual uncertainties to obtain the total uncertainty of final result is called ***propagation*** of uncertainty

Combined Standard Uncertainty

- Standard uncertainty of a result obtained by propagating the standard uncertainties of all the input values is called the ***combined standard uncertainty***
- "Total propagated uncertainty" (TPU) previously used to denote same concept
- Combined standard uncertainty denoted by u_c
  - CSU of a result y is written as $u_c(y)$

Uncertainty Propagation

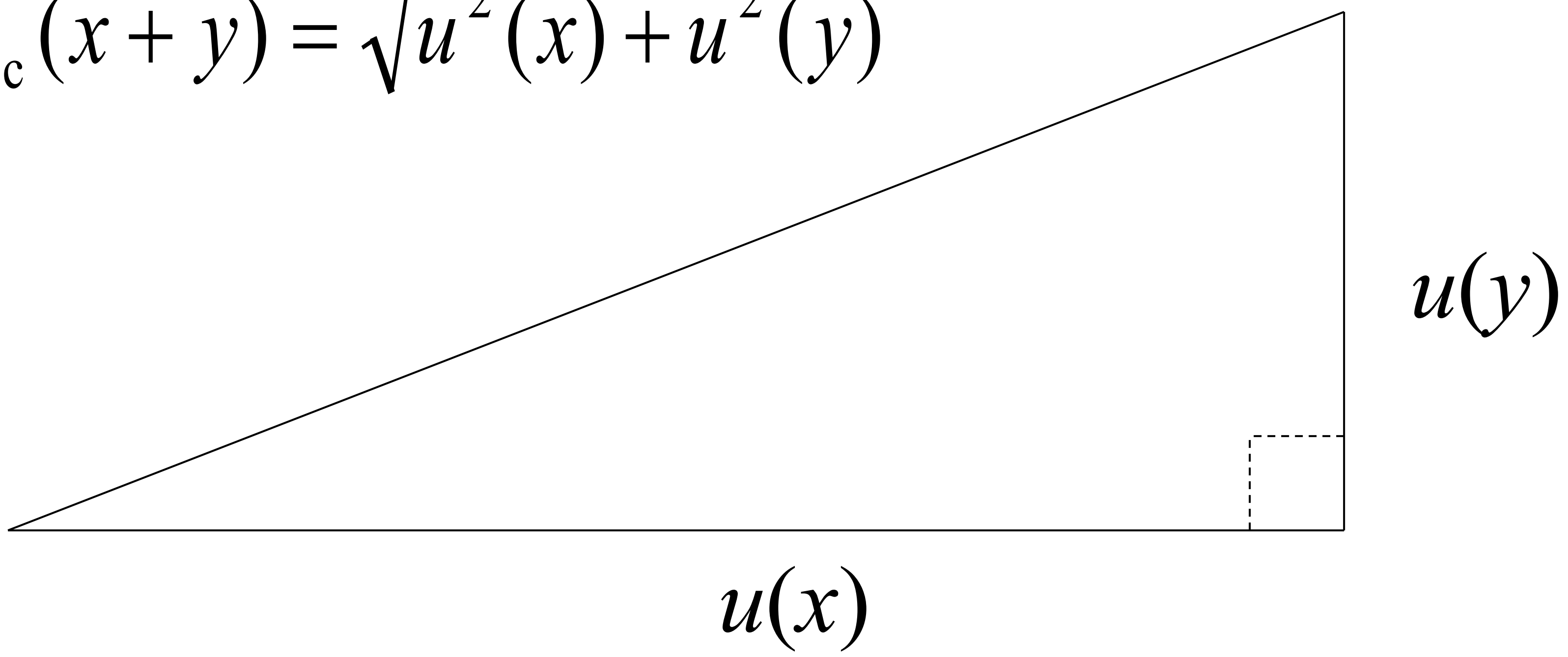
- Propagating uncertainty not the simple addition of uncertainty components
- If you multiply a value x by a constant c, the standard uncertainty of the product is $|c| \times u(x)$
- If you add two values x and y, the standard uncertainty of their sum is the square root of the sum of the squares of $u(x)$ and $u(y)$

$$u(x+y) = \sqrt{u^2(x) + u^2(y)}$$

  – Think of the Pythagorean Theorem (next slide)

The Uncertainty of a Sum

$$u_c(x+y) = \sqrt{u^2(x) + u^2(y)}$$



Large and Small Components

- A consequence of rules for uncertainty propagation:
  - Small uncertainty components tend to contribute even less to the total uncertainty than one might think
- Combine two uncertainty components 10 and 3 – the total uncertainty is only 10.4, not 13

Expanded Uncertainty

- The lab might report the combined standard uncertainty for each result…
- Or multiply CSU by k to obtain a larger uncertainty, producing a wider interval about the result with a greater probability of containing the true value
- Product of $k \times$ CSU = ***expanded uncertainty***
- Factor k called ***coverage factor***
- Probability that the interval about the result contains the true value is the ***coverage probability***
- Expanded uncertainty denoted by upper-case U

Questions

- What is standard uncertainty?
  - **Uncertainty expressed as a standard deviation**
- What is combined standard uncertainty?
  - **Standard uncertainty obtained by uncertainty propagation**
- How do you denote the combined standard uncertainty of *y*?
  - $u_c(y)$
- What is expanded uncertainty?
  - **Uncertainty describing an interval about the result with high probability of containing the true value**

Rounding Results

- Consider a result reported as 15.381 pCi/g with CSU 4.076 pCi/g
- Final digits in the result don't mean much because of the uncertainty
- More sensible to report the result as 15 with uncertainty of 4, or 15.4 with uncertainty of 4.1

Rounding Rules

- There is a widely accepted method for rounding results with uncertainty
- Regardless of whether you report the CSU or an expanded uncertainty, round the uncertainty to either 1 or 2 figures
  - MARLAP prefers 2 in all cases – Others may differ
- Then round the result to the same number of decimal places

Example: Rounding

- Suppose a measurement result is 17.93602 Bq/L, and lab reports the result with a CSU of 0.37301 Bq/L.
- How would you round the result and the CSU according to MARLAP?
  - **CSU: 0.37301 → 0.37**
  - **Result: 17.93602 → 17.94**
  - Round the CSU to two figures; then round the result to the same power of 10

Shorthand Notations

- There are common shorthand notations for reporting results with uncertainty
- If reporting CSU, place the digits of the rounded uncertainty in parentheses just after the digits of the rounded result:

17.94(37) Bq/L

- This format is not commonly used by radiochemists
- May be encountered in published documents

Other Shorthand Notations

- For expanded uncertainty, report the numerical values of the result and uncertainty in parentheses followed by the unit of measurement, with the result and uncertainty separated by ± (or +−):

(17.94 ± 0.75) Bq/L

- This format is more familiar to radiochemists

Explain the Uncertainty

- Even if you use an accepted shorthand notation, explain what it means

Always explain the uncertainty

- In particular, state whether it is a CSU or an expanded uncertainty, and in the latter case, state the coverage factor and the approximate coverage probability

Summary of MARLAP's Recommendations

- Use the terminology, notation, and methodology of GUM
- Report all results – even if zero or negative – unless believe they are invalid
- Report either combined standard uncertainty or an expanded uncertainty for each result
- Explain the uncertainty – in particular, state coverage factor for an expanded uncertainty

(continued)

Summary of MARLAP's Recommendations
(Continued)

- Consider all sources of uncertainty, and evaluate and propagate all that are believed to be potentially significant in final result
- Do not ignore subsampling uncertainty (for solid samples) just because hard to evaluate
- Round reported uncertainty to 1 or 2 figures (we suggest 2) and round the result to match

Final Recommendation

- All preceding recommendations are severable
- Do as much as you can
- At least use GUM's terminology and notation so that we all speak and write the same language
- Make further progress as time and resources permit

Question

- Does MARLAP prefer that a lab report the combined standard uncertainty of each result, or an expanded uncertainty?

  - MARLAP has no preference
  - Explain the uncertainty, whatever it is
  - State whether combined standard uncertainty or expanded uncertainty
  - For expanded uncertainty, state coverage factor and approximate coverage probability

Question

- When a lab reports an expanded uncertainty, what coverage factor does MARLAP prefer?
  - **MARLAP has no preference.**
  - **The factor must be > 1 to produce an expanded uncertainty, and coverage factors between 2 and 3 are most common**
  - **MARLAP has no preference as long as the lab states coverage factor and approximate coverage probability**

Question

- What does the notation 12.34(56) Bq/g mean?
  - **A measurement result of 12.34 Bq/g with combined standard uncertainty 0.56 Bq/g**

Question

- What does the notation (12.34 ± 0.56) Bq/g mean?
  - **A measurement result of 12.34 Bq/g with expanded uncertainty 0.56 Bq/g**
  - **The coverage factor must also be stated**

For More Information

For more information on this subject, see Module 7, "Evaluating Measurement Uncertainty"