Math for Wastewater Operators

NPDES Operator Webinar Series

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Presentation Outline

I. Applied Math For Pumps And Motors
II. Area, Volume and Conversions
III. Velocity and Flow

Poll Question #1

How would you describe your current ability in math?

a. Poor
b. Average
c. Good
d. Excellent

Part I.

Horsepower and Efficiency

Applied Math For Pumps And Motors

Reference: Tennessee Department of Environment & Conservation (TDEC) Training Materials

Understanding Work and Horsepower

- Work: The exertion of force over a specific distance.
  - Example: Lifting a one-pound object one foot.
- Amount of work done would be measured in foot-pounds
  - (feet) (pounds) = foot-pounds
- (1 pound object) (moved 20 ft)

  = 20 ft-lbs of work

Understanding Power

- Power is the measure of how much work is done in a given amount of time
- The basic units for power measurement is foot-pounds per minute and expressed as (ft-lb/min)
  - in electric terminology $\Rightarrow$ Watts
- This is work performed per time (work/time)
  - One Horsepower: 1 HP = 33,000 ft-lb/min
  - In electric terms: 1 HP = 746 Watts

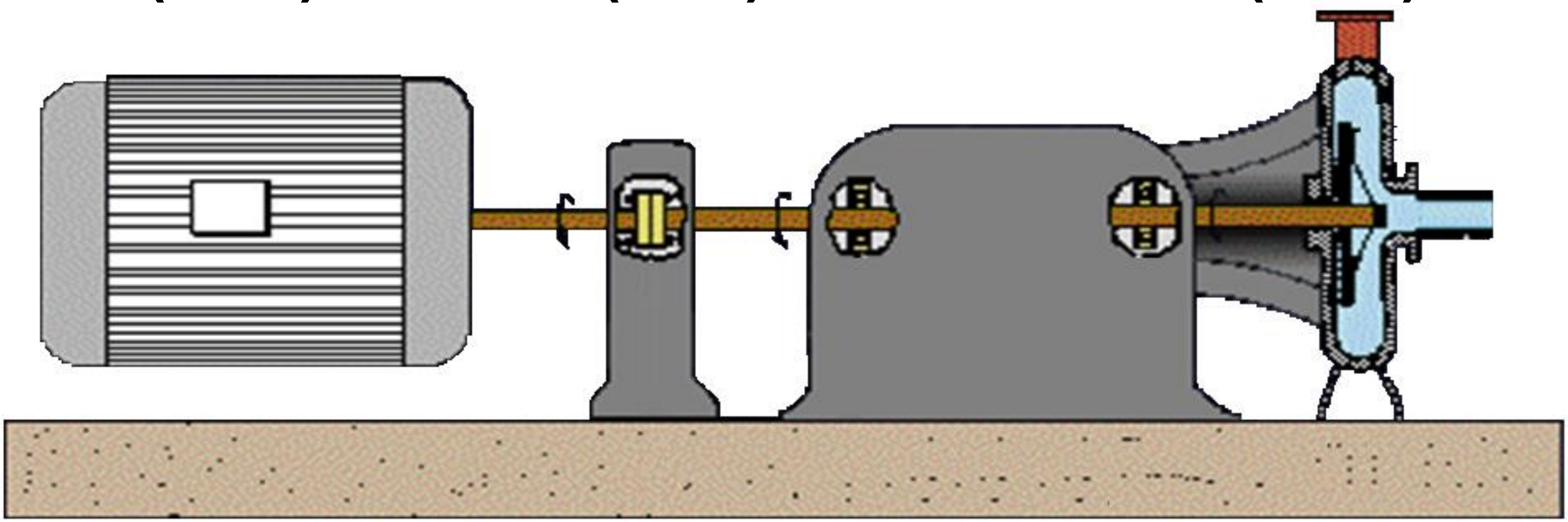
Types of Horsepower

- Motor Horsepower is related to the watts of electric power supplied to a motor
- Brake Horsepower is the power supplied to a pump by a motor
- Water Horsepower is the portion of power delivered to a pump that is actually used to lift the water
- Water horsepower is affected by elevation and location of the pump.

Motor Horsepower
(MHP)

Brake Horsepower
(BHP)

Water Horsepower
(WHP)



Computing Water Horsepower

- It is the amount of horsepower required to lift the water
- Formula for water horsepower (WHP)

$$\text{WHP} = \frac{(\text{flow gpm})\ (\text{total head feet})}{3{,}960}$$

$$\frac{33{,}000 \text{ ft-lb/min}}{(\text{HP})8.34\ ^{\text{lbs}}/_{\text{gal}}} = 3960$$

Water Horsepower

- For example: A pump must pump 3,000 gpm against a total head of 25 feet. What water horsepower will be required?

- $$\text{WHP} = \frac{(3000\text{ gpm})(25\text{ head in ft})}{3960}$$

  $$= 18.94\text{ HP}$$

Brake Horsepower

- $BHP = \dfrac{(\text{flow, gpm})\,(\text{head, ft})}{(3960)\,(\%\text{ pump efficiency})}$

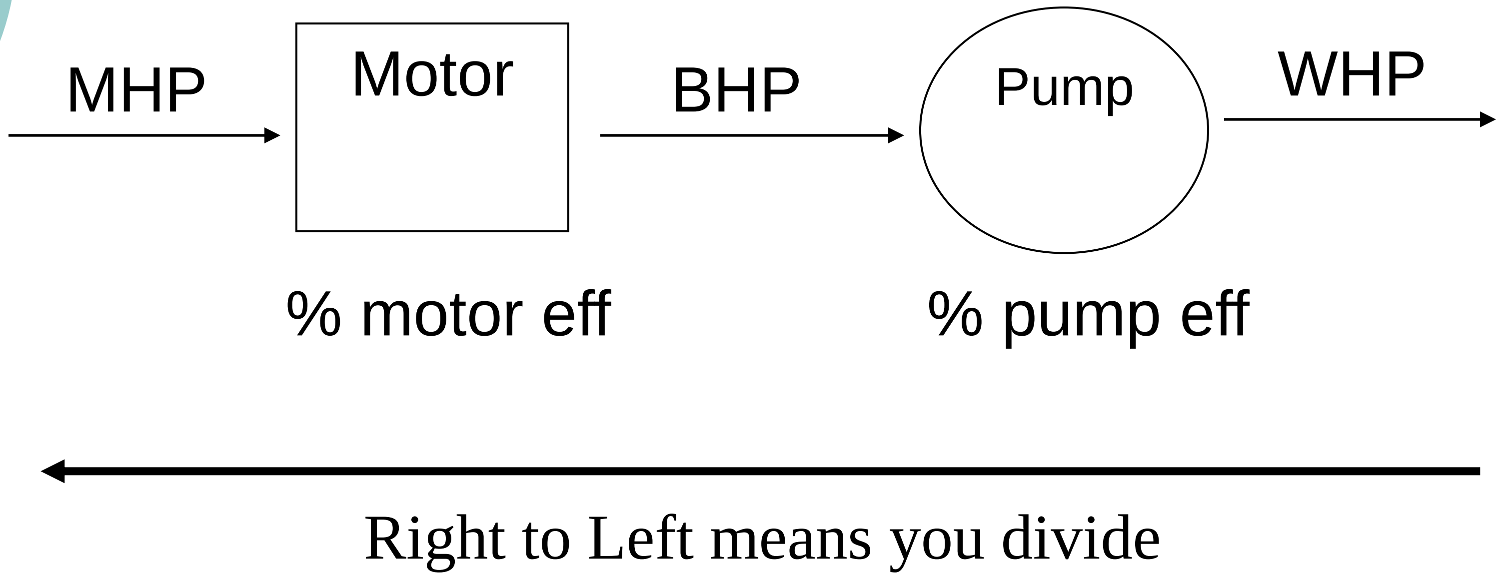
- $BHP = \dfrac{\text{water HP}}{(\%\text{ pump efficiency})}$

Motor Horsepower

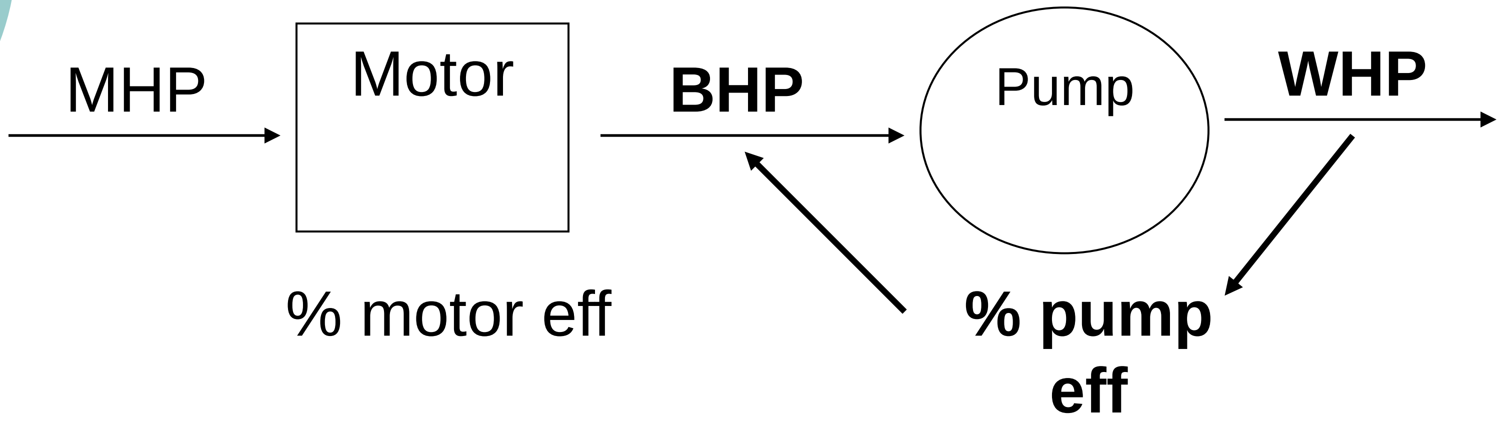
- $$MHP = \frac{(\text{flow, gpm})\ (\text{head, ft})}{(3960)(\%\ \text{pump eff.})(\%\ \text{motor eff.})}$$

- $$MHP = \frac{\text{brake HP}}{(\%\ \text{motor efficiency})}$$

Pumps

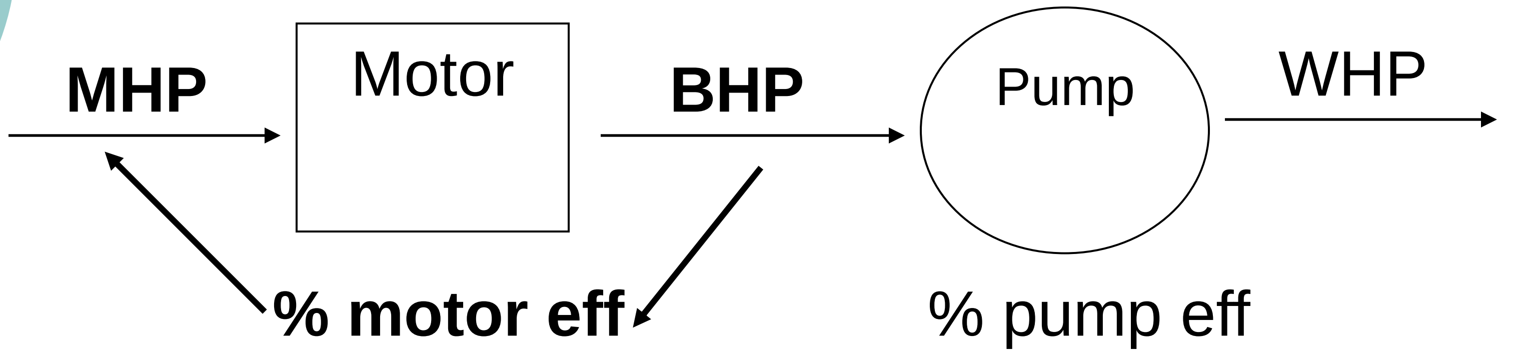

MHP
Motor
BHP
Pump
WHP
% motor eff
% pump eff
Right to Left means you divide

Pumps - BHP



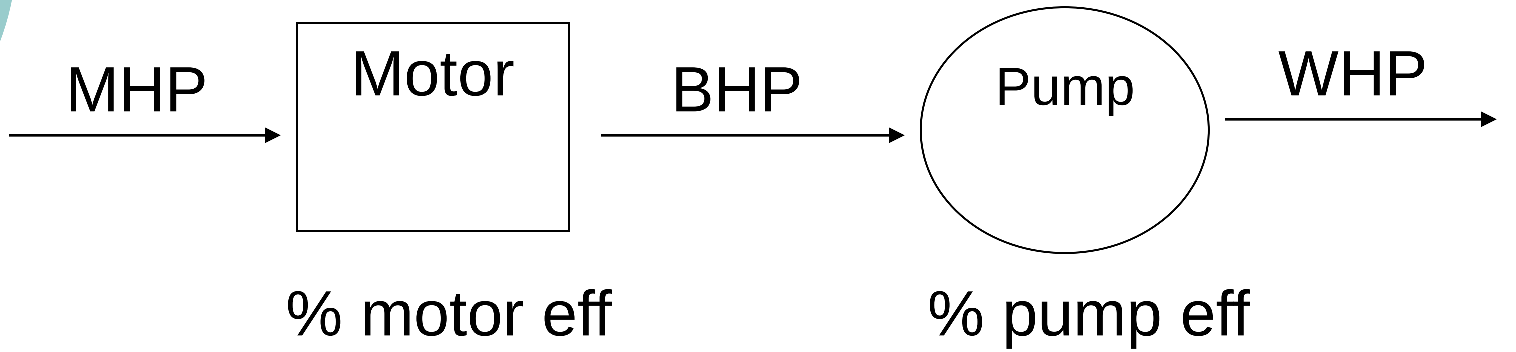
BHP = WHP ÷ (% pump efficiency/100)

Pumps - MHP



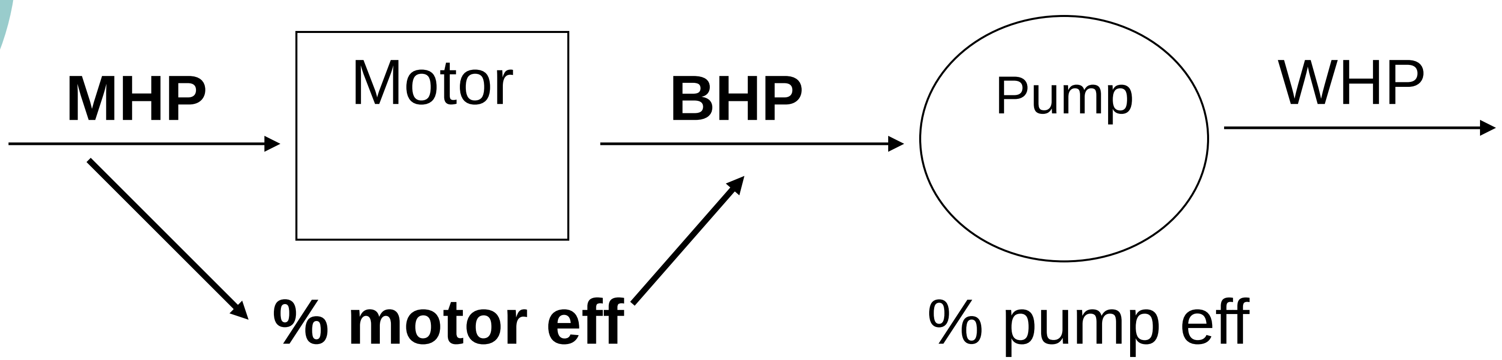
MHP = BHP ÷ (% motor efficiency/100)

Pumps



Left to Right means you multiply

Pumps - BHP



BHP = MHP x (% motor efficiency/100)

Motor and Pump Efficiency

- Neither the motor nor the pump will ever be 100% efficient
- Not all the power supplied by the motor to the pump (Brake Horsepower) will be used to lift the water (Water Horsepower)
- Power for the motor and pump is used to overcome friction
- Power is also lost when energy is converted to heat, sound, etc.

Typical Efficiency

- Pumps are generally 50-85 % efficient
- Motors are usually 80-95% efficient
- Combined efficiency of the motor and pump is called wire-to-water efficiency
- Wire-to-Water efficiency is obtained by multiplying the motor and pump efficiencies together

Typical Efficiency

- Example:
  - Motor Efficiency = 82%
  - Pump Efficiency = 67%
- Wire to Water Efficiency
  - (0.82) (0.67) = 0.55
  - 0.55 x 100% = 55%
- Note: If not given, you will have to calculate both motor and pump efficiency.

Overall Efficiency

- Must know the WHP and the MHP
  - If not given you will have to compute both.
- % Efficiency, overall = $\frac{\text{WHP}}{\text{MHP}}$
- % Overall Efficiency = $\frac{\text{18.5 WHP}}{\text{35 MHP}} \Rightarrow$ 53%
- ***In all cases, the bottom number will be larger than the top number.***

Poll Question #2

One horsepower is equivalent to:

a. 33,000 ft-lb/minute
b. 746 watts
c. 0.746 kilowatt
d. All of the above

QUESTIONS?

Determining Pumping Costs

What was your electric bill last month?

Determining Pumping Costs

- Electrical Power is sold in units of kilowatt-hours
- One Horsepower = 0.746 kilowatt
- To compute pumping costs, need to know the power requirements (power demand) of the motor and the length of time the motor runs

Determining Pumping Costs

- For example, if you have a pumping job which requires 25 HP and the cost is $0.085/kW-hr. What is the pumping cost for one hour?

Cost, $/hr = (MHP)(0.746 kW/HP)(1 hr)(cost, $/kW-hr)

= (25 HP)(0.746)(1 hr)($0.085/ kW-hr)

= $1.59/hr

A Few Electrical Terms...

- Power (Watts) - amount of work done
- Voltage (volts) - electrical “pressure” available to cause flow of electricity
- Amperage (amps) - the amount of flow of electricity
- Power = (voltage)(amperage)
- Watts = (volts)(amps)

Motor Ratings, Volts, Amps, Single and Multiple Phases

- Power in reference to motors is in watts
  - determined by multiplying the volts and ampere spec for the particular motor used
- For example, a 220-volt motor which pulls 100 amps would have a power wattage of 22,000 watts. What would be the horsepower of this motor?
- $$HP = \frac{(volts)(amps)}{746\ watts/hp} = \frac{(220)(100)}{746} = 29\ hp$$

Wattage Power Factor of Motors

- There are two type of motors that we usually use. They are:
  - Single-Phase Motors
  - Three-Phase Motors (usually any motor over 2 hp)

- $$\text{kW, Single Phase} = \frac{\text{(volts)(amps)(power factor)}}{\text{1,000 Watts/kilowatt}}$$

- $$\text{kW, Three Phase} = \frac{\text{(volts)(amps)(power factor)(1.732)}}{\text{1,000 Watts/kilowatt}}$$

Remember, if you are asked to find watts, don't divide by 1,000

Power Factor Of Motors

- The power factor of a motor is computed by dividing the watts by the volt and amp rating of the motor
- Power Factor = $\frac{\text{watts}}{(\text{volts})(\text{amps})}$
- The power factor might be on the data plate, but should always be in the manufacturer's manual

Amperes Single and Three Phase

- amps, = $\dfrac{(746)(\text{horsepower})}{(\text{volts})(\%\ \text{eff.})(\text{power factor})}$

  Single Phase

- amps, = $\dfrac{(746)(\text{horsepower})}{(1.732)(\text{volts})(\%\ \text{eff.})(\text{power factor})}$

  Three Phase

Part II.

Area, Volume and Conversions

Reference: TDEC Training Materials

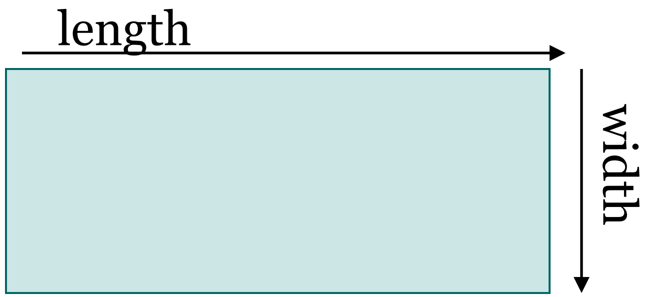
Area

- Surface of an object
- Two dimensional
- Measured in:
  - Square inches
  - Square feet
  - Square meters, etc.

Area Formulas

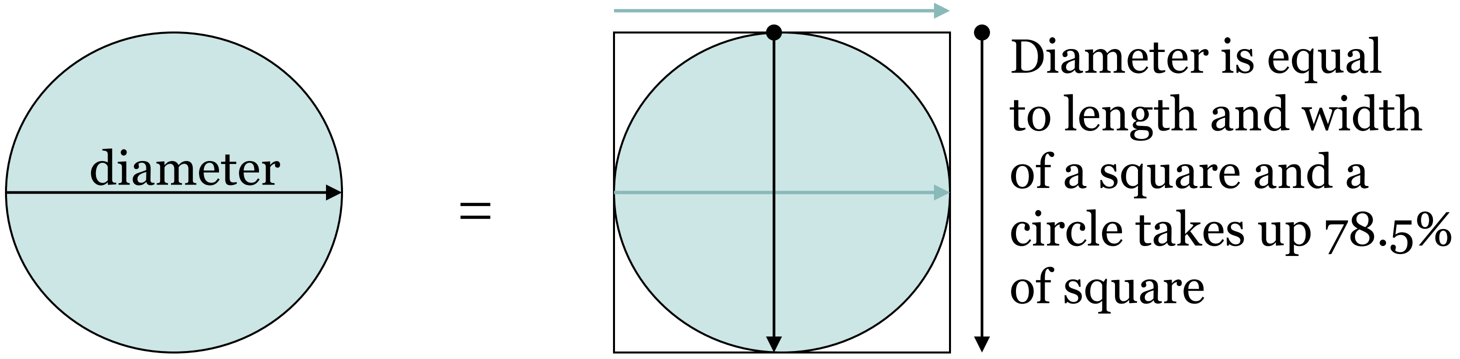
- Rectangle

A = (length, ft)(width, ft)

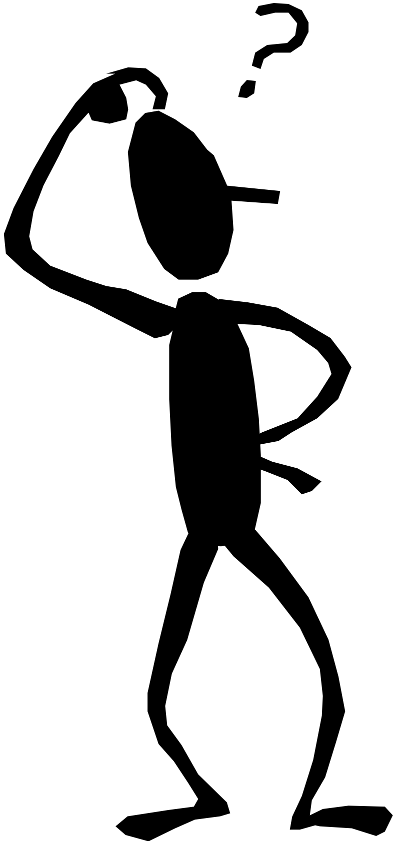


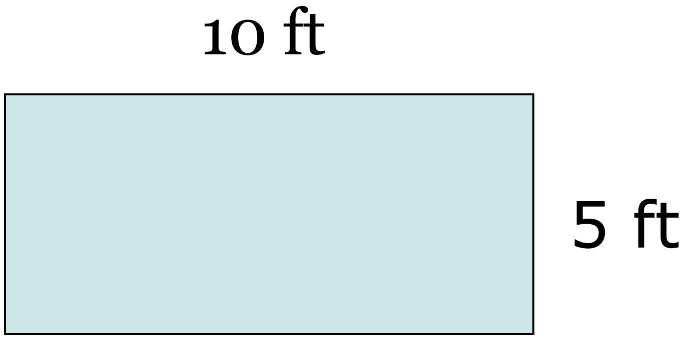
- Circle

A = (0.785)(diameter, ft)²



Area of a Rectangle



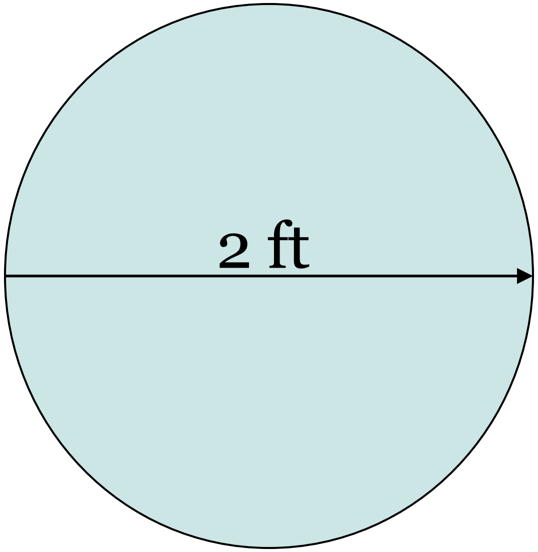


$A = (l,\ ft)(w,\ ft)$

$A = (10\ ft)(5\ ft)$

$A = 50\ ft^2$

Area of a Circle



Diameter = 2 ft

$A = (\pi/4)(D, ft)^2$

$A = (0.785)(D, ft)^2$

$A = (0.785)(2ft)(2ft)$

$A = 3.14\ ft^2$

Volume

- The amount of space an object occupies
- Volume = (area)(third dimension) or

$$V = (l)(w)(d)$$

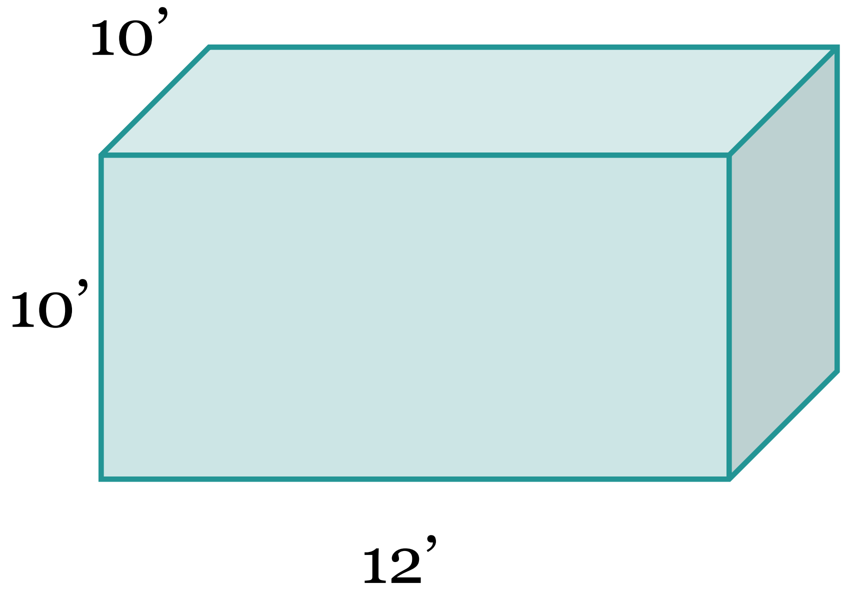
- Measured in:
  - Cubic inches
  - Cubic feet
  - Gallons
  - Acre-feet, etc.

Volume of a Rectangular Tank, ft^3

V = (length, ft)(width, ft)(depth, ft)

V = (12 ft)(10ft)(10ft)

V = 1,200 ft^3



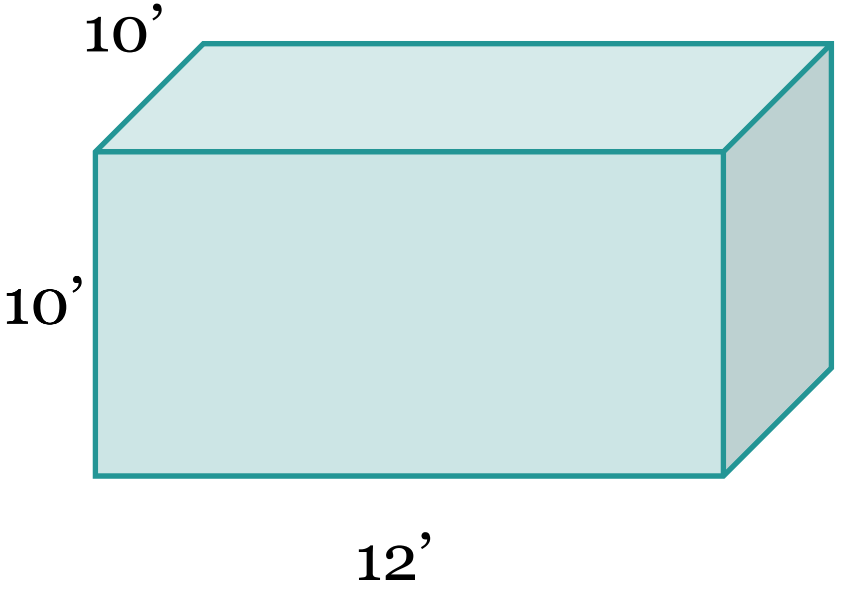
Volume of a Rectangular Tank, gal

V, ft^3 = 1200 ft^3

V, gal = (Volume, ft^3)(7.48 gal/ft^3)

V, gal = (1200 ft^3)(7.48)

V, gal = 8,980 gal



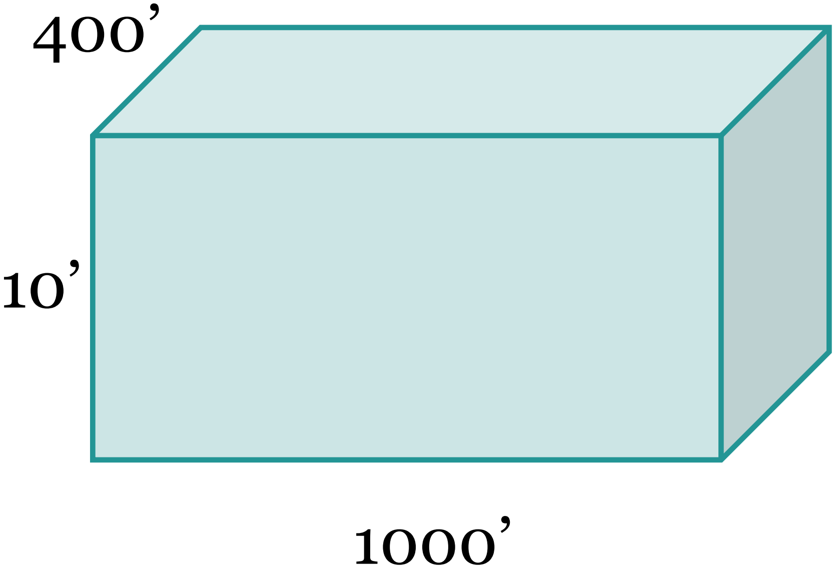
Volume of an Equalization Tank, mil gal

$V, ft^3 = (400\ ft)(1000\ ft)(10\ ft) = 4{,}000{,}000\ ft^3$

$V = (Volume, mil\ ft^3)(7.48\ gal/ft^3)$

$V = (4.00\ mil\ ft^3)(7.48)$

V, gal = 29.9 mil gal

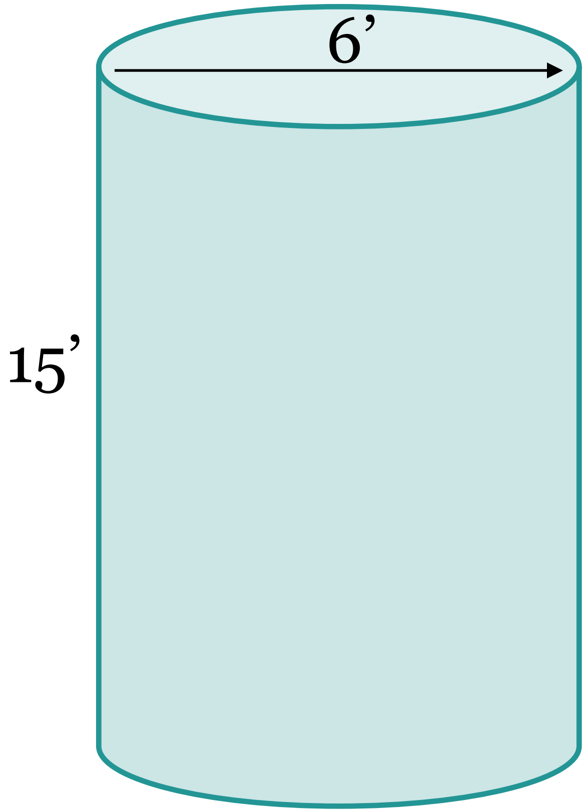


Volume of a Cylinder, ft^3

$V = (0.785)(D, ft)^2(height, ft)$

$V = (0.785)(6\ ft)(6\ ft)(15\ ft)$

$V = 424\ ft^3$



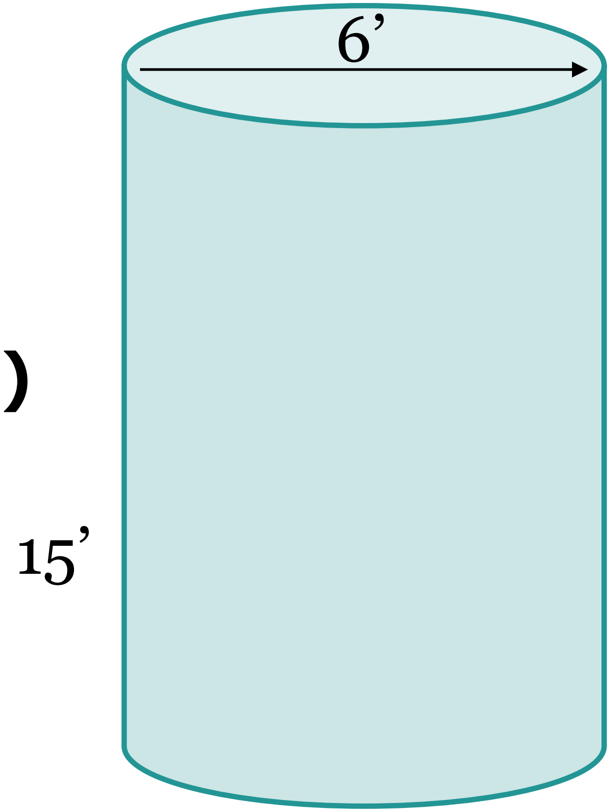
Volume of a Cylinder, gallons

$V, ft^3 = 424\ ft^3$

$V, gal = (Volume, ft^3)(7.48\ gal/ft^3)$

$V, gal = (424\ ft^3)(7.48\ gal/ft^3)$

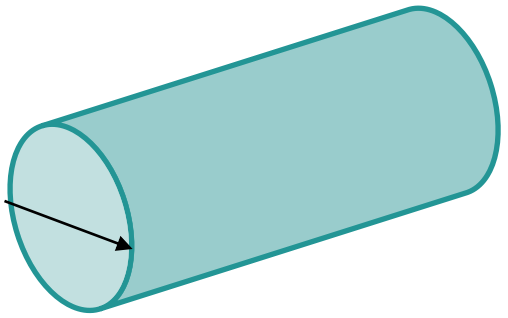
V, gal = 3,170 gal



Note

- When calculating area and volume, if you are given a pipe diameter in inches, convert it to feet.

$$8\ \cancel{\text{in}}. \times \frac{1\ \text{ft}}{12\ \cancel{\text{in}}} = 0.6667\ \text{ft}$$



Diameter = 8 in

Poll Question #3

What is the volume of a 24-inch diameter sewer pipe that is 360 feet long?

a. 640 ft^3
b. 1,130 ft^3
c. 2,260 ft^3
d. 4,520 ft^3

QUESTIONS?

Conversions

- Need to know:
- The number that relates the two units
  - Ex: 12 inches in a foot, 454 grams in a pound, 3785 mL in a gallon
- Whether to multiply or divide
  - Ex: the units must cancel out correctly

Conversions

Conversion Factors		
1 acre	=	43,560 ft^2
1 foot of head	=	0.433 psi
1 psi	=	2.31 feet of head
1 yd^3	=	27 ft^3
1 gal	=	3.785 Liters
1 gallon of water	=	8.34 lbs
1 cubic foot of water	=	7.48 gallons
1 lb	=	453.6 grams
1 mile	=	5280 feet
1%	=	10,000 $^{mg}/_{L}$

Multiply

- Just looking at the units, if you are given miles and you need feet, we are going from left to right on the page; therefore multiply

Conversions

- You have just laid ¼ mile of sewer line. How many feet is this?

¼ = 0.25 miles

(0.25 miles)(5,280 feet/mile) = **1,320 feet**

Percent to Decimal

Percent = per one hundred

20%	=	20/100	=	0.20
5%	=	5/100	=	0.05
12.25%	=	12.25/100	=	0.1225
0.5%	=	0.5/100	=	0.005

Move decimal 2 places to the left.

Flow Conversions

- Convert a flow of 2.7 cfs to gpm.

$(2.7\ ft^3/sec)(7.48\ gal/ft^3)(60\ sec/min) =$ **1,210 gpm**

Flow Conversions

- Convert a flow of 2.7 cfs to mgd.

$(2.7 \text{ ft}^3/\text{sec})(0.6463 \text{ mgd/cfs}) = \mathbf{1.75\ mgd}$

Flow Conversions

- Convert a flow of 1.0 MGD to gpm.

(1.0 mgd)(1,000,000 gpd/mgd)(day/1440 min) = **694 gpm**

Part III.

Velocity and Flow

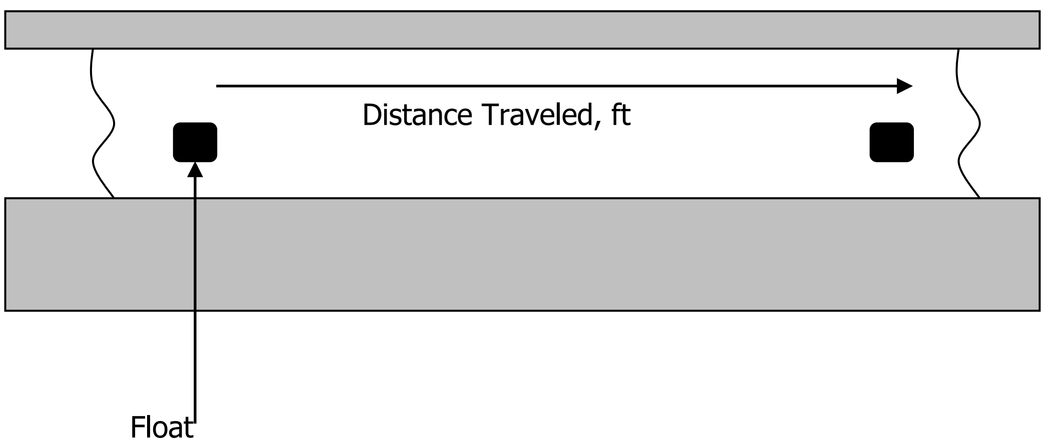
Reference: TDEC Training Materials

Velocity

- Distance per time
- Measured in:
  - Miles per hour
  - Feet per second
  - Feet per minute

Velocity Formulas

- Velocity, ft/sec = $\frac{\text{distance traveled, ft}}{\text{time, sec}}$
- Velocity, ft/min = $\frac{\text{distance traveled, ft}}{\text{time, min}}$



Velocity

- A cork is placed in a channel and travels 400 feet in 2 minutes and 25 seconds. What is the velocity of the wastewater in the channel, ft/min?

- 25 seconds/60 = 0.4167 min
- Vel = $\dfrac{400\ \text{ft}}{2.4167\ \text{min}}$ = 165.5 ft/min

Flow Conversions

- Express a flow of 2.00 ft^3/sec in terms of gal/day.
  - (2.00 cfs)(7.48 gal/ft^3)(60 sec/min)(1440 min/day)
  - = 1,290,000 gpd

Flow Conversions

- Express a flow of 2.00 ft^3/sec in terms of mgd.
  - (2.00 cfs)(0.6463 mgd/cfs)
  - = 1.293 mgd

Flow Conversions

- Express a flow of 10 mgd in terms of ft^3/sec.
  - (10 mgd)(1.547 cfs/mgd)
  - = 15.5 cfs

Flow and Velocity Conversions

- A circular pipe (18 inches in diameter) is flowing full at a flow rate of 6.0 mgd. What is the velocity in the pipe in ft/sec.

  - Flow rate = (6.0 mgd)(1.547 cfs/mgd)

    = 9.282 cfs

    Area of the pipe = (0.7854) (1.5 ft)^2 = 1.767 ft^2

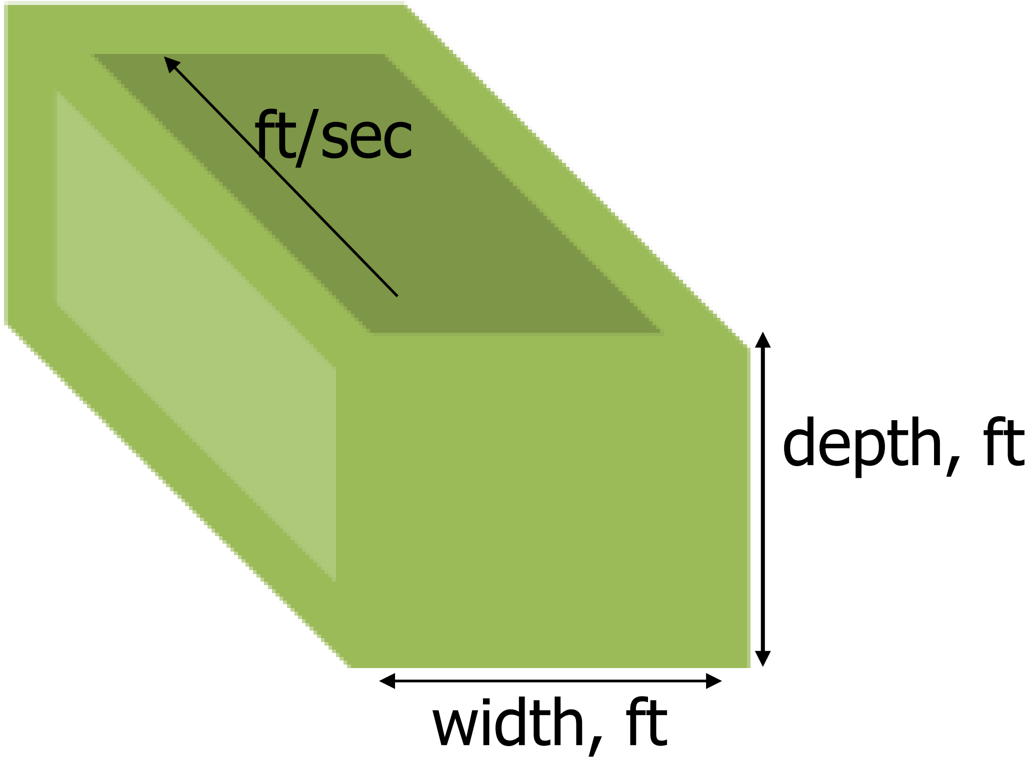
    Velocity = Flow rate ÷ area

    = 9.282 cfs ÷ 1.767 ft^2

    = 5.25 ft/sec

Flow in a Channel

- Q, ft^3/sec = (Area, ft^2)(Velocity, ft/sec)
- Q, ft^3/sec = (width, ft)(depth, ft)(velocity, ft/sec)



Flow in a Channel

- A channel 36 inches wide has water flowing to a depth of 2 feet. If the velocity of the water is 1.2 ft/sec, what is the flow in the channel in ft^3/sec?

- Q = (3 ft)(2 ft)(1.2 ft/sec)

  = 7.2 ft^3/sec

Flow in a Pipe Flowing Full

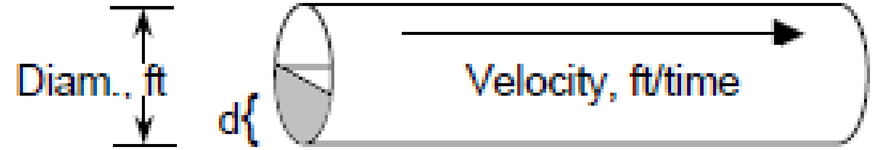
- Q, ft^3/sec = (Area, ft^2)(Velocity, ft/sec)
- Q, ft^3/sec = (0.785)(Diameter, ft)^2(velocity, ft/sec)

Flow in a Pipe Flowing Full

- The flow through a 10-inch diameter sewer is flowing full at 2.5 ft/sec. What is the flow rate in ft^3/sec and gal/day?

- Q = (0.785)(0.8333 ft)(0.8333 ft)(2.5 ft/sec)
  = 1.36 ft^3/sec

- (1.36 ft^3/sec)(7.48 gal/ft^3)(60 sec/min)(1440 min/day) = 879,000 gal/day



Flow in a Partially Full Pipe

- Q = (factor from d/D table)(Diameter, ft)^2(vel, fps)

depth/Diameter Table							
0.01	0.0013	0.26	0.1623	0.51	0.4027	0.76	0.6404
0.02	0.0037	0.27	0.1711	0.52	0.4127	0.77	0.6489
0.03	0.0069	0.28	0.1800	0.53	0.4227	0.78	0.6573
0.04	0.0105	0.29	0.1890	0.54	0.4327	0.79	0.6655
0.05	0.0147	0.30	0.1982	0.55	0.4426	0.80	0.6736
0.06	0.0192	0.31	0.2074	0.56	0.4526	0.81	0.6813
0.07	0.0242	0.32	0.2167	0.57	0.4625	0.82	0.6893
0.08	0.0294	0.33	0.2260	0.58	0.4724	0.83	0.6969
0.09	0.0350	0.34	0.2355	0.59	0.4822	0.84	0.7043
0.10	0.0409	0.35	0.2450	0.60	0.4920	0.85	0.7115
0.11	0.0470	0.36	0.2546	0.61	0.5018	0.86	0.7186
0.12	0.0534	0.37	0.2642	0.62	0.5118	0.87	0.7254
0.13	0.0600	0.38	0.2739	0.63	0.5212	0.88	0.7320
0.14	0.0668	0.39	0.2836	0.64	0.5308	0.89	0.7384
0.15	0.0739	0.40	0.2934	0.65	0.5404	0.90	0.7445
0.16	0.0811	0.41	0.3032	0.66	0.5499	0.91	0.7504
0.17	0.0885	0.42	0.3130	0.67	0.5594	0.92	0.7560
0.18	0.0961	0.43	0.3229	0.68	0.5687	0.93	0.7612
0.19	0.1039	0.44	0.3328	0.69	0.5780	0.94	0.7662
0.20	0.1118	0.45	0.3428	0.70	0.5872	0.95	0.7707
0.21	0.1199	0.46	0.3527	0.71	0.5964	0.96	0.7749
0.22	0.1281	0.47	0.3627	0.72	0.6054	0.97	0.7785
0.23	0.1365	0.48	0.3727	0.73	0.6143	0.98	0.7816
0.24	0.1449	0.49	0.3827	0.74	0.6231	0.99	0.7841
0.25	0.1535	0.50	0.3927	0.75	0.6318	1.00	0.7854

Flow in a Partially Full Pipe

- A 10-inch diameter pipeline has water flowing at a depth of 4 inches. What is the gal/min flow if the velocity of the wastewater is 3.1 fps?

- d/D = 4 inches of water ÷ 10-inch diameter
  = 4/10 = 0.4 ~ table factor is 0.2934
- Q = (0.2934)(0.8333)(0.8333)(3.1) = 0.632 ft^3/sec
- (0.632 ft^3/sec)(7.48 gal/ft^3)(60 sec/min) = 284 gpm

Detention Time Calculations

- A WWTP has a flow of 4.0 mgd, half of which is flowing into each of two primary clarifiers. Each clarifier is 60 ft in diameter with an average depth of 12 ft. What is the detention time (hours) in each clarifier?
  - Detention time = volume ÷ flow rate
  - Volume of each clarifier = $(0.7854)(60\ ft)^2(12\ ft)$
    $= 33{,}930\ ft^3 = 0.254$ mil gal

Detention time = 0.254 mil gal ÷ 2.0 mgd
= 0.127 day = 3.05 hrs

QUESTIONS?